Chaos in the Ganges runoff

M.F.K. Pasha[1], A. Nishat[2] and M. Rahman[3]

[1]*Department of Environmental Engineering and Pollution Control*
Shahjalal University of Science and Technology, Sylhet 3114, Bangladesh
[2]*Country Representative, IUCN, Bangladesh*
[3]*Department of Civil Engineering*
Bangladesh University of Engineering and Technology, Dhaka 1000, Bangladesh

Abstract

"Chaos" denotes irregular but never repeating behavior of non-linear dynamical systems arising from deterministic time evolution, which is sensitive to initial condition. Prediction is possible due to existence of determinism in chaos. Not long ago it was thought that these types of natural processes (seem to be irregular) are completely random (or stochastic) and unpredictable even for a short-term, but the interesting mixture of determinism and sensitive dependence on initial condition helps one achieve a more subtle understanding of complex nonlinear processes. As a result, prediction is possible with high accuracy. The chaos science furnishes important information such as number of variables required to describe the dynamics of the system that is under investigation of this study. This number of variables is required to predict the system precisely. Ganges runoff together with a uniformly distributed random time series has been studied under this investigation. The result shows that the Ganges runoff time series exhibits chaotic behavior with the sufficient number of dominant variables 5 and time delay 8 unlike stochastic random time series.

1. Introduction

The discover of chaos has motivated researchers to investigate the existence of this process in many natural and physical systems as prediction with high accuracy is possible using chaotic technique to these systems which seem to be irregular in general. So chaos identification is given due consideration by the researchers in the field of meteorology and hydrology. Though ordinarily chaos is a disorder or confusion, but in the scientific sense, it is something special. It is a particular class of study of how something changes in its time history. Chaos occurs only in deterministic, nonlinear, dynamical systems. Based on these qualifications chaos can be defined as, " chaos is sustained and disorderly-looking long-term evolution that satisfies certain special

mathematical criteria and that occurs in a deterministic non-linear system" (Williams 1997).

2.   Deterministic, non-linear and dynamic system

A system is said to be deterministic if knowledge of the time-evolutions, the parameters that describe the system, and the initial conditions in principle completely determine the subsequent behavior of the system. On the other hand a system in which the input parameters are in general unknown or only some statistical measures of the parameters are known the system is known as stochastic. Not only long-term prediction, but even short-term prediction also is not guaranteed for this stochastic system, the reverse condition is sustained in the deterministic system (Sivakumar et al. 1998).

Nonlinear means that output isn't directly proportional to input, or that a change is one variable doesn't produce a proportional change or reaction in the related variable(s). In other words, a system's values at one time aren't proportional to the values at an earlier time. Three Ways in which linear and non-linear phenomena can be differed from one another:

- *Behavior over time*: Linear processes are smooth and regular, whereas nonlinear ones may be regular at first, but often change to erratic-looking.
- *Response to small changes in the environment or to stimuli*: A linear process changes smoothly and in proportion to the stimulus; in contrast, the response of a non-linear system is often much greater than the stimulus.
- *Persistence of local pulses*: In linear systems decay and may even die out over time. In non-linear systems, on the other hand, they can be highly coherent and can persist for long times, perhaps forever (Williams 1997).

The word dynamics implies force, energy, motion or change. A dynamical system is anything that moves, changes, or evolves in time. Hence, chaos deals with the experts who like to refer to as dynamical-systems theory or non-linear dynamics. Dynamical systems fall into one of the two categories, depending on whether the system losses energy. A conservative dynamical system has no friction; it doesn't lose energy over time. In contrast, a dissipative dynamical system has friction; it losses energy over time and therefore always approaches some asymptotic or limiting condition. That asymptotic or limiting state, under certain conditions, is where chaos occurs. There are also some other characteristics, which can differentiate chaotic process from others, such as:

- In spite of its disjointed appearance, it includes one or more types of order or structure.
- The ranges of the variables of the system have finite bounds. The bounds restrict the attractor (dissipative dynamical systems are characterized by the attraction of all trajectories toward a geometric object called an attractor) to a certain finite region in the phase space (a phase space is an abstract space whose co-ordinates are the degrees of freedom of the system's motion) (Williams 1997).
- Details of the chaotic behavior are hypersensitive to changes in initial conditions (minor changes in the starting values of the variables)
- A short-term prediction with high accuracy is possible in chaotic system.

Thus the chaos science furnishes important information such as number of variables required to describe and predict the dynamics of the system precisely. If some irregular dynamical system is given, one is able to show that the system is dominated by low-

dimensional chaos. Then an important physical result is that the system dynamics can be described by only a few (nonlinear and collective) modes or variables. Therefore, our primary object is to identify low-dimensional chaos in runoff time series of the Ganges.

3.   Chaos identification

For an available mathematical formulation of a deterministic system, the identification of chaos is easy. A broadband noise spectrum is sufficient enough to identify chaos of that system (Sivakumar et al. 1998). As there is a number of known variables, the construction of the phase-space as well as the estimation of the various invariants are straightforward. A phase space is an abstract space whose co-ordinates are the degrees of freedom of the system's motion. So a dynamical system can be described by a phase space diagram, which is essentially a coordinate system, whose coordinates are all the variables that enter the mathematical formulation of the system. The trajectories of the phase space diagram describe the evolution of the system from initial state that is assumed to be known and hence represent the history of the system.

Again if there are some controlled systems but one can not record all the variables or there are any uncontrolled systems whose mathematical formulation and total number of variables can not be known exactly, then the problem will be complicated one and in that case chaos identification of this complicated process of unknown variables or random variables can not be possible by Fourier analysis. The difficulties of using Fourier analysis to identify chaos led to the emergence of several methods. Among these methods, (a) Correlation dimension method (Grassberger et al. 1983a,b) (b) Lyapunov exponent method (Wolf et al. 1985), (c) Kolmogorov entropy method (Grassberger et al. 1983c) and (d) Nonlinear prediction method (NLP) are well known.

Correlation dimension method is the very widely used method for chaos identification. In this study Correlation dimension method is used for chaos identification of runoff time series. Lyapunov exponents measures the long-term average exponential rate of divergence or convergence of initially adjacent phase-space trajectories on the attractor, and thus quantify average predictability properties. When at least one Lyapunov exponent is positive, the attractor is chaotic and initially nearby trajectories diverges exponentially, on the average (Sivakumar et al. 1998). For small amount of available data, nonlinear prediction method is found to be successful for chaos identification, especially for real data. So researchers have given a great consideration on nonlinear prediction method. The chaos identification by this approach in which prediction accuracy is used to identify chaos is also very simple.

3.1   Correlation dimension method

Correlation Integral function (equation 2) is used in this method to identify chaotic behavior in the process. The process will have a limited number of degrees of freedom equal to the smallest number of first order differential equations that capture the most important features of the process, if it comes from a deterministic dynamics even though the process may look irregular. Therefore there will be a point where the dimension equal to the number of degrees of freedom. So the aim of using this method is to find the dimension of the attractor, which furnishes information on the number of dominant variables present in the dynamical process. It can be said that to embed that attractor on a phase-space the required least number of coordinates is the dimension of the attractor. A system, which is governed by stochastic process, has infinite value of the dimension. On the other hand a finite, low dimension value indicates chaotic process. Despite of

many algorithms for calculating correlation dimension of a time series the Gassberger-Procaccia [3,4] algorithm is very popular one.

Phase space construction of the time series is the first attempt to calculate correlation dimension. For a scalar time series Xi (where i=1,2,3,………,N) the phase space can be constructed using the method of delays. In this methods Xi and its successive time shifts are combined and assigned as coordinates of a new vector time series given by

$$Y_j = (X_j, X_{j+\tau}, X_{j+2\tau}, \ldots \ldots X_{j+(m-1)\tau})$$

(1)

where $j = 1,2,3,\ldots\ldots N-(m-1)\tau$. m is called dimension of the vector Yj (also called embedding dimension, i.e. no. of coordinates to embed the attractor); τ is called time delay (it is the time at which Auto Correlation Function (ACF) has its first zero, which can also be calculated by Average Mutual Information (AMI)) which makes the coordinates linearly uncorrelated. For an m dimensional phase space the correlation function C(r) is given by:

$$C(r) = \frac{2}{N(N-1)} \sum_{i,j} H(r - |Y_i - Y_j|) \qquad 1 \le i < j \le N$$

(2)

where H is the Heaviside step function with H(u)=1 for u>0 and H(u)=0 for $u \le 0$, $u = r - |Y_i - Y_j|$; r is the radius of sphere centered on Yi or Yj; $|$Yi-Yj$|$ is the distance between the m dimensional delay vectors obtained from equation 1 and N is the total number of data points. For a time series which is characterized by an attractor, for positive values of r

$$C(r) \cong \alpha r^\nu \quad \text{where} \quad r \to 0 \quad N \to \infty$$

(3)

where α is a constant and ν is the correlation exponent or the slope of the log C(r) vs. log(r) plot given by

$$\nu = \frac{\log Cr}{\log r}$$

(4)

Plotting of correlation exponent vs. embedding dimension is needed to have evidence of chaotic behavior and to quantify the dominant variables. A distinct sustained saturation level of correlation exponent for embedding dimension proves the existence of chaotic behavior in the process and minimum number of embedding dimension of this saturation level is the sufficient number of degrees of freedom (Sivakumar et al. 1998).

4.   Chaos identification in the Ganges runoff

4.1   *The Ganges and its runoff data*

The Ganges, which is one of the largest international river in the world with its basin spread over China, Nepal, India, and Bangladesh. The river originates at an elevation of about 23,000 feet in Gangotri on the southern slope of the Himalayan range and traverses south and southeastward in India for about 1,400 miles. After crossing the Indo-Bangladesh border, the Ganges forms the boundary of the two countries for a distance of

about eighty miles. After the river enters Bangladesh wholly, it flows for another seventy miles before joining the river the Brahmaputra-Jamuna at Goalando. The combined course of the Ganges and Brahmaputra-Jamuna taking the name of the Padma joins lower down another river, the Meghna at Chandpur. From this point, the combined course of the three rivers continues as the lower Meghna into the Bay of Bengal. The total course of Ganges before falling to the Bay of Bengal is about 1,600 miles (2,500 km).

It has economic, cultural and religious significance for many people. Study on the Ganges is thus very important as it is used for a number of different purposes. Study to recognize the runoff system of the Ganges is also most important in the sense that a major portion of flood in Bangladesh is created due to runoff of this river and it is obvious that flood is the worst one among other natural disasters in Bangladesh. 10 days sampling time of runoff at the Hardinge Bridge point in Bangladesh of 62 years from 1934 to 1996 has been analyzed for this study. The statistics of the data is following with maximum 92300 m3/sec, minimum 2830 m3/sec, average 11145.5 m3/sec and standard deviation 14082.4 m3/sec.

5.   Results and discussion

Though there is a lot of approaches to find out the chaotic parameters like embedding dimension and time delay, in this study a widely used Correlation dimension and Autocorrelation function (ACF) method have been used to find out these parameters respectively. A time delay 8 is found from the Ganges runoff data using ACF. A random time series with minimum and maximum value same as runoff time series with uniform distribution is used so that comparison would be easy between stochastic and chaotic system. A chaotic system is recognized with a well-identified scaling region defined from a correlation exponent vs. logr graph. It is known that a random time series, which is stochastic in nature, could not be predicted, as a result there should not be any particular number of degrees of freedom so that for this degree of freedom one can reconstruct the phase space. Therefore the relation between correlation exponent and embedding dimension for this random time series should be to some extent proportional. It describes that correlation exponent will be increasing with increase of embedding dimension. But reverse situation would happen in the chaotic system. With the increase of embedding dimension the correlation exponent should increase up to a certain point and saturates beyond that dimension for a chaotic system. This saturation level indicates the required dominant number of degrees of freedom in the process. The first value of embedding dimension for which the saturation starts is the sufficient number of dominant variables to describe the system. Figure 1 shows both the runoff and random time series. It is shown in Figure 2(a) that for runoff time series though there is not well identified but still a definitive scaling region between 4.35 to 4.70 of logr value, but there is no such scaling region in figure 2(b). Figure 3(a) and 3(b) can clearly identify the chaotic and stochastic process respectively. In figure 3(a) there is a well-identified saturation level of correlation exponent for the embedding dimension 5 to 10 but such level of saturation is not found in figure 3(b). The reason why there is no distinct scaling region in figure 2(a) may be higher sampling time. It is well known that with the increase of sampling time the correlation between data becomes less, instead of hourly or even daily 10 days data has been used in the study it may cause less correlated data. Again, the Ganges runoff data is not fully natural flow; there is some effect of hydraulic structures such as barrage and bridge constructed at the upstream of the river on data set. Anyway after considering all the limitations such as effect of hydraulic structures and higher sampling time the chaotic process is still identified with a time delay 8 and

sufficient embedding dimension 5 to reconstruct the phase space for the purpose of prediction with higher accuracy.
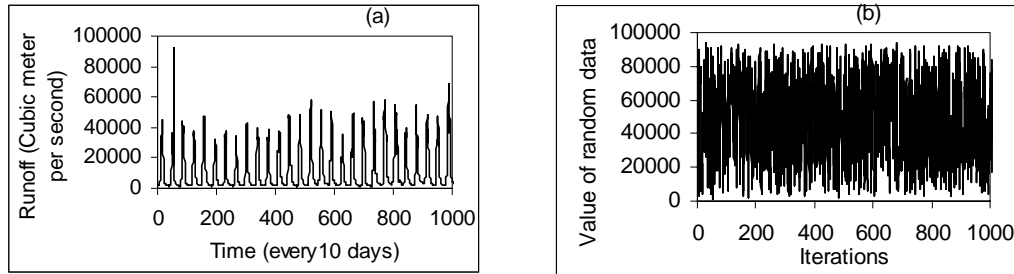
Fig. 1. Time series (a) Ganges runoff (b) uniformly distributed random data
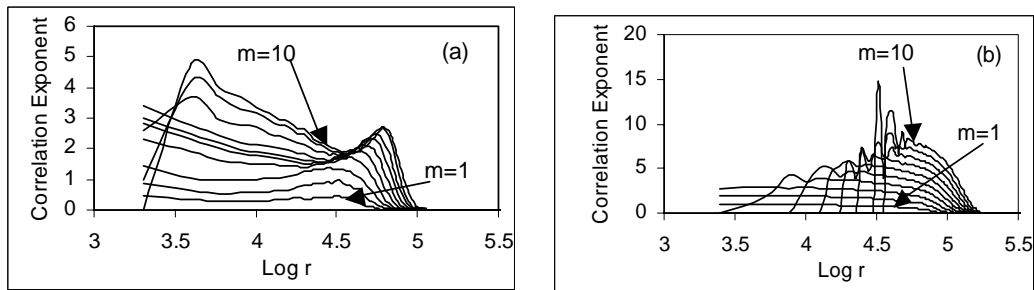
Fig. 2. Scaling region defined by correlation exponent vs Log r graph for (a) Ganges runoff (b) uniformly distributed random data
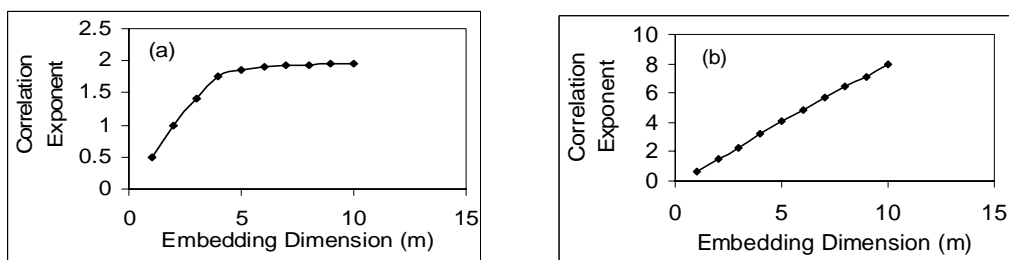
Fig. 3. Relationship between correlation exponent and embedding dimension (m) for (a) Ganges runoff (b) uniformly distributed random data

6.   Conclusions

Study of chaos identification on the Ganges runoff data shows that there is a definite scaling region, which in turn proves that the Ganges runoff data exhibits chaotic behaviors with sufficient dominant variables 5 and time delay 8. It is also found from the study on uniformly distributed random data that there is no saturation level of correlation exponent in other words sufficient number of variables can not be found from the random process. Because of space limitation it is not possible to describe how to play with embedding dimension and time delay to predict the system. So our next paper will content prediction procedure using chaotic technique.

References

Williams, G. P. (1997). "Chaos Theory Tamed", Taylor & Francis, London, pp. 9-15, 209.

Sivakumar, B., Liong, S. Y. and Liaw, C. Y. (1998). Evidence of chaotic behavior in Singapore rainfall, *J. American Water Res. Assoc.*, 34(2), 302-303.

Grassberger, P. and Procaccia, I. (1983a). Measuring the strangeness of strange attractors, *Physica D*, 9, 189-208.

Grassberger, P. and Procaccia, I. (1983b). Characterisation of strange attractors, *Physical Review Letters*, 50(5), 346-349.

Wolf, A., Swift, J. B., Swinney, H. L. and Vastano, A. (1985). Determining Lyapunov exponents from a time series, *Physica D*, 16, 285-317.

Grassberger, P. and Procaccia, I. (1983c). Estimation of the Kolmogorov Entropy from a Chaotic Signal, *Physical Review A*, 28, 2591-2593.