Journal of Civil Engineering (IEB), 34 (2) (2006) 91-101

Journal of
Civil Engineering
IEB

Development of an expansion transition in open channel sub critical flow

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Received on 12 September 2006

Abstract

For most of the constricted structures like bridges, weirs, barrages, river training works, cross-drainage works etc., downstream expansion transitions are the common requirement. In these structures, the flow tends to separate while subjected to the adverse pressure gradient associated with flow deceleration. The eddying of the flow as a result of flow separation may damage the bed and sides of the downstream channel. The head loss produced by the transition is most important as it is reflected as increased upstream stages. This investigation is concerned with the simpler and more dependable method of design, and behavior of downstream expansion transitions for subcritical free surface flows. The velocity distributions of flow through the sudden as well as gradual expansion models are made, thus, a transition profile for expansion of flow with minimum separation has been evolved by streamlining the boundary shape of the transition, and the performance of the transition is evaluated to compare with the existing profiles.

Keywords: Expansive transition, abrupt expansion, subcritical flow, velocity distribution, efficiency of transition.

1. Introduction

When a change of channel shape, variation of bed elevation, or contraction or expansion of channel width occurs, transition is provided as a link between the original and the new channels. Such changes are often required in natural and artificial channels at syphons, aqueducts, weirs, falls, bridges, barrages etc. for economic as well as practical reasons. This paper has relevance only to expansive transitions in subcritical flow. In the structures mentioned above, the flow tends to separate while subjected to the positive pressure gradient associated with flow deceleration, thus resulting in a considerable loss of energy. As a result of separation, a certain portion adjacent to the boundaries has a reverse flow and eddies are continuously formed in the region. Because of intermittent shedding of eddies, the body of the hydraulic structure is also subjected to lateral vibrations, which are dangerous and hence undesirable. The present study involves design of an expansive transition in a rectangular open-channel subcritical flow such that, the flow expands with minimum separation and energy loss as small as possible, and evaluation of the present transition to compare with others, evolved by different investigators. For efficient design of expanding transition boundaries, the primary aim must be to reduce the tendency for separation, and to minimize head loss appreciably. This can be achieved by streamlining the boundary shape of the transition. To evaluate the transition profile, efficiency of the transition model is determined in a laboratory setup flow, defining this as the ratio of gain of potential energy to loss of kinetic energy.

2. Present state of the work

Hinds (1928) assumed the water-surface profile in the transition to be composed of two reverse parabolas of equal length connected at the centre of the transition, and found the bed-width profile corresponding to the assumed water-surface profile. For this purpose, Eq. (1) is taken as a form loss equation. The loss due to surface resistance is neglected, as it is small. The form loss, h_L, is assumed to vary uniformly along the transition length and is expressed as:

$$h_L = K_H\left(\frac{V_0^2 - V_L^2}{2g}\right) \tag{1}$$

where, V_0 and V_L = the velocity at the inlet and outlet of expansion respectively, K_H = the loss coefficient lying between 0.3 and 0.75 (Morris and Wiggert, 1972), and g = the acceleration due to gravity. The equations of the two reverse parabolas representing the water depth y at a distance x from the inlet are given by

$$y = y_0 + 2(y_L - y_0)\xi^2;\ 0 \le \xi \le 0.5 \tag{2a}$$

and

$$y = y_L - 2(y_L - y_0)(1-\xi)^2;\ 0.5 \le \xi \le 1 \tag{2b}$$

in which y_0 and y_L = the depth of flow at inlet and outlet of channels respectively; and $\xi = \dfrac{x}{L}$.

Equating total energies at the inlet and at a section $0 \le \xi \le 0.5$, and using Eq. (1) and (2a), the bed width profile is obtained as

$$b = \frac{Q\sqrt{1-K_H}}{y_0 + 2(y_L - y_0)\xi^2}\left[\frac{Q^2(1-K_H)}{b_0{}^2 y_0{}^2} + 2g\,S_0 L\xi - 4g(y_L - y_0)\xi^2\right]^{-0.5} \tag{3a}$$

in which Q = the discharge, and S_0 = the channel-bed slope. Similarly, applying energy equation at the outlet and at a section $0.5 \le \xi \le 1$ and using Eq. (1) and (2b), the corresponding bed-width profile is

$$b = \frac{Q\sqrt{1-K_H}}{y_L - 2(y_L - y_0)(1-\xi)^2}\left[\frac{Q^2(1-K_H)}{b_L{}^2 y_L{}^2} - 2g\,S_0 L(1-\xi) + 4g(y_L - y_0)(1-\xi)^2\right]^{-0.5} \tag{3b}$$

But, the model studies have shown that separation of flow occurs downstream of such transitions and the jet is unstable.

Hartley et al (1940) assumed the following linear variation of the velocity:

$$V = V_0 + (V_L - V_0)\xi \tag{4}$$

in which V = the velocity of flow at a distance x and further assumed constant depth throughout the transition,

$$b_0 V_0 = b_{\mathrm{L}} V_{\mathrm{L}} = bV \tag{5}$$

Combining Eq. (4) and (5), the bed-width profile was obtained as:

$$b = \left[b_0^{-1} + \left(b_{\mathrm{L}}^{-1} - b_0^{-1}\right)\xi\right]^{-1} \tag{6}$$

Interestingly it can be mentioned here that, (6) has been used in most of the texts as "A. C. Mitra's Hyperbolic Transition"; probably the same has been used inadvertently by some text book writers, which has been followed by the later investigators. As per the original report available in U. P. Irrigation Research Institute, Roorke (India), the authorship goes to Hartley, Jain and Bhattacharya (1940).

Formica (Chow, 1959) suggested the energy loss, h_{L} for an open channel expansion as

$$h_{\mathrm{L}} = K_{\mathrm{F}} \frac{\left(V_0 - V_{\mathrm{L}}\right)^2}{2g} \tag{7}$$

Here the loss can not be minimized as this is function of inlet and outlet velocities only. Hence this is not considered for comparison with the present study.

Chaturvedi (1963) generalized Eq. (6) in designing the rectangular expansion transition in the following manner:

$$b = \left[b_0^{-n} + \left(b_{\mathrm{L}}^{-n} - b_0^{-n}\right)\xi\right]^{-1/n} \tag{8}$$

where, from experimental investigation, the best value of n was claimed to be 1.5.

Applying the momentum and energy equations for an abrupt expansion, for a bed width b_1 to a bed width b_2 in a rectangular channel, Henderson (1966) gave the following equation for head loss:

$$h_{\mathrm{L}} = \frac{V_1^2}{2g}\left[\left(1 - \frac{b_1}{b_2}\right)^2 + \frac{2F_1^2 b_1^3 \left(b_2 - b_1\right)}{b_2^4}\right] \tag{9}$$

in which, V_1 = pretransition average velocity.

This was used in optimization process to develop optimal transition profile by Swamee and Basak (1993), and the optimal profile has been compared with the present profile.

Vittal and Chiranjeevi (1983) found the following equation of average separating streamline by considering an abrupt expansion from a rectangular channel to a trapezoidal channel:

$$b = b_0 + \left(b_{\mathrm{L}} - b_0\right)\xi[1 - \left(1 - \xi\right)^{0.8 - 0.2\sigma\sqrt{m_{\mathrm{L}}}}] \tag{10}$$

For the variation of side slope m, the following relationship satisfying boundary conditions at $\xi = 0$ and $\xi = 1$ was suggested,

$$m = m_{\mathrm{L}}[1-(1-\xi)^{0.5}] \tag{11}$$

So the comparison of this profile is not made here, since the present profile is applicable for rectangular channel only.

Nashta and Garde (1988), based on minimization of the form loss and friction loss recommended the following equation for the transition:

$$b = b_0 + (b_{\mathrm{L}} - b_0)\xi[1-(1-\xi)^{0.55}] \tag{12}$$

However, in the minimization process, Nashta and Garde (1988), assuming K_{H} to be constant used Eq. (1) to obtain the following elementary head loss, dh_{L}:

$$dh_{\mathrm{L}} = -K_{\mathrm{H}}\frac{VdV}{g} \tag{13}$$

in which the decrease in velocity $-dV$ corresponds to the increase in the bed width from b to $b+db$. The head loss h_{L} between inlet and outlet of the expansion was obtained as

$$h_{\mathrm{L}} = -\frac{K_{\mathrm{H}}}{g}\int_{V_0}^{V_{\mathrm{L}}} VdV \tag{14}$$

The bed width profile was obtained by minimizing h_{L} subjected to the differential equation of gradually varied flow. Any minimization process using Eq. (1) or (14) is spurious as according to Eq. (1) head loss between inlet and outlet of the transition depends on the inlet and outlet velocities (i.e. inlet and outlet bed widths) only. Thus, by dividing the transition into various sections 0, 1, 2, 3, …........n, one obtains

$$h_{\mathrm{L}} = \frac{K_{\mathrm{H}}}{2g}\left(V_0^2 - V_1^2 + V_1^2 - V_2^2 + V_2^2 - V_3^2 + V_3^2 - \ldots\ldots\ldots\ldots - V_{\mathrm{n-1}}^2 + V_{\mathrm{n-1}}^2 - V_{\mathrm{n}}^2\right) \tag{15}$$

Eq. (15) reduces to (1). It is therefore; evident that Eq. (12) is based on minimization of friction loss, which is of secondary importance. The performance of these transitions was not found satisfactory (Alauddin, 2006).

Based on optimal control theory, a methodology has been presented for optimal design of a rectangular subcritical expansive transition by Swamee and Basak (1993). Analyzing a large number of optimal profiles, an equation for the design of rectangular transitions is presented as

$$b = b_0 + (b_{\mathrm{L}} - b_0)\left[2.52\left(\frac{L}{x}-1\right)^{1.35}+1\right]^{-0.775} \tag{16}$$

and they obtained the head-loss equation as

$$h_{\mathrm{L}} = \int_0^L \frac{Q^4}{g^2 b^5 y^5}\frac{db}{dx}dx \tag{17}$$

This equation can be nondimensionalized by putting

$B = \frac{b - b_0}{b_L - b_0}$, $\eta = \frac{y}{b_0}$, $G = \frac{Q}{b_0^2 \sqrt{g b_0}}$, and $H_L = \frac{h_L}{b_0}$. Denoting $\frac{dB}{d\xi} = W_1$, the nondimensional form of above head-loss equation is written as

$$H_L = \int_0^1 \frac{G^4 (r-1) W_1}{\left[1 + (r-1)B\right]^5 \eta^5} d\xi \tag{18}$$

in which, $r = b_L / b_0$ (called expansion ratio).

Table 1
Overall hydraulic efficiency of the present model

Discharge, Q, cumec	Froude No., F_1	Efficiency, ε	Average Efficiency		Overall Efficiency, ε
			For Avg. Q	For Avg. F_1	
	0.25	87.3			
0.0015	0.40	81.1			
	0.55	79.2			
	0.25	87.4			
0.0045	0.40	79.3			
	0.55	77.2			
	0.25	80.6			
0.0075	0.40	77.1			80.3
	0.55	73.5			
	0.25		85.1		
	0.40		79.2		
	0.55		76.6		
0.0015				82.5	
0.0045				81.3	
0.0075				77.1	

3. Experimental setup and procedure

To develop an expansive transition profile, investigation was made on an abrupt expansion in a laboratory flume. A narrow channel of width, 9.525 cm, and length, 2.50 m was constructed and placed at upstream portion of the flume, the walls of which were of varnished wood. Then the straight-line headwalls normal to the direction of flow are provided as sudden expansion to have the flume width, 25.4 cm (Fig. 1) and continued for more 2.0 m. A tail gate was provided at the downstream end of the flume for depth regulation. Water circulation was continued by a centrifugal pump mounted beneath the flume, lifting water from the floor-standing reservoir tanks run parallel to flume, top of which was used for providing access walkway to the working section.

To test the transition model, the gradual expansion of length 55.56 cm, governed by side splay of 7:1, which is claimed to be the optimum value, (Mazumder, 1967), was provided after the contracted reach in the same flume to have the normal channel width, and then continued for remaining 1.50 m length. The models were made of wooden bed and the side walls of Perspex sheet.

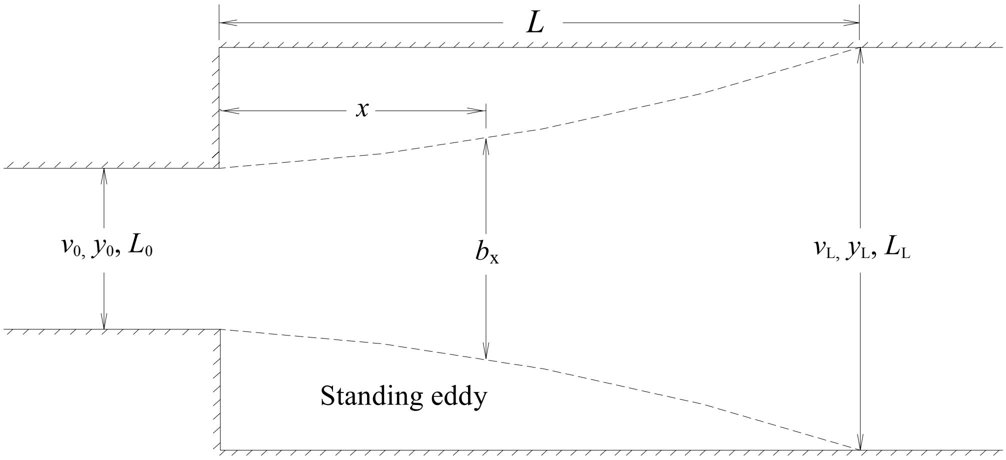

Fig. 1. Definition sketch: abrupt expansion

To know the velocity distribution of flow through abrupt and gradual expansions, the (*i*) velocity at various points, and (*ii*) depth of flow at various sections were to be known. Three runs were carried out to collect data in abrupt expansion for analysis. The data were collected at a section, 20 cm upstream of expansion and at more six sections after expansion, 10 cm apart. Velocity measurements were made across the width at 1/6, ½, and 5/6 times the width in upstream channel, and at 1/8, 1/4, 3/8, ½, 5/8, 3/4 and 7/8 times the width in downstream channel and vertically at near surface, 0.20y, 0.40y, 0.60y, 0.80y, and near bottom. The velocities were measured with Pitot tube, and measurements were made only when the velocity was in the forward direction. Water level along the centreline of the channel was measured with a pointer gauge mounted on a moving carriage.

The velocity distributions were made along the length of transition at inlet, mid-length, and outlet sections for performance-study of the transition model. The transition was tested for three different discharges, Q as 0.0015, 0.0045, and 0.0075 cumec. For each discharge, experiments were conducted at three different depths so that Froude number F_1, at entry were 0.25, 0.40, and 0.55.

4. Analysis of data

To develop a transition profile, the separating streamline in a sudden expansion was determined. The velocities measured at different depths in the vertical indicated the usual turbulent boundary layer profile; as such, only the average velocity over a vertical was used in further analysis. The average velocity was plotted across the width of abrupt expansion to detect the separating points at the sections. The two points, A and B, on the separating streamlines were determined by trial and error such that the product of the depth and the area of velocity diagram equaled the discharge at the inlet (Fig. 2). By repeating this procedure at various sections along the length, the coordinates of separating streamline were determined. These velocity distributions were made for three different discharges and same expansion ratio, and were found to be nearly independent of the flow conditions. By a method of curve fitting, empirical equation of streamline was obtained from average separating points. A close observation of these data obtained from streamline profile reveals that an equation of the type

$$b = b_0 + (b_L - b_0)\left[a\left(\frac{L}{x} - 1\right)^n + 1\right]^{-m}$$

could be fitted to find the constants a, n and m. A computer programme in FORTRAN has been developed adopting *grid-search algorithm* to find the optimal value of a, n and m. Having known the constants a, n and m, the final form of the transition equation is obtained as

$$b = b_0 + (b_L - b_0)\left[2.52\left(\frac{L}{x} - 1\right)^{1.35} + 1\right]^{-0.80} \tag{19}$$

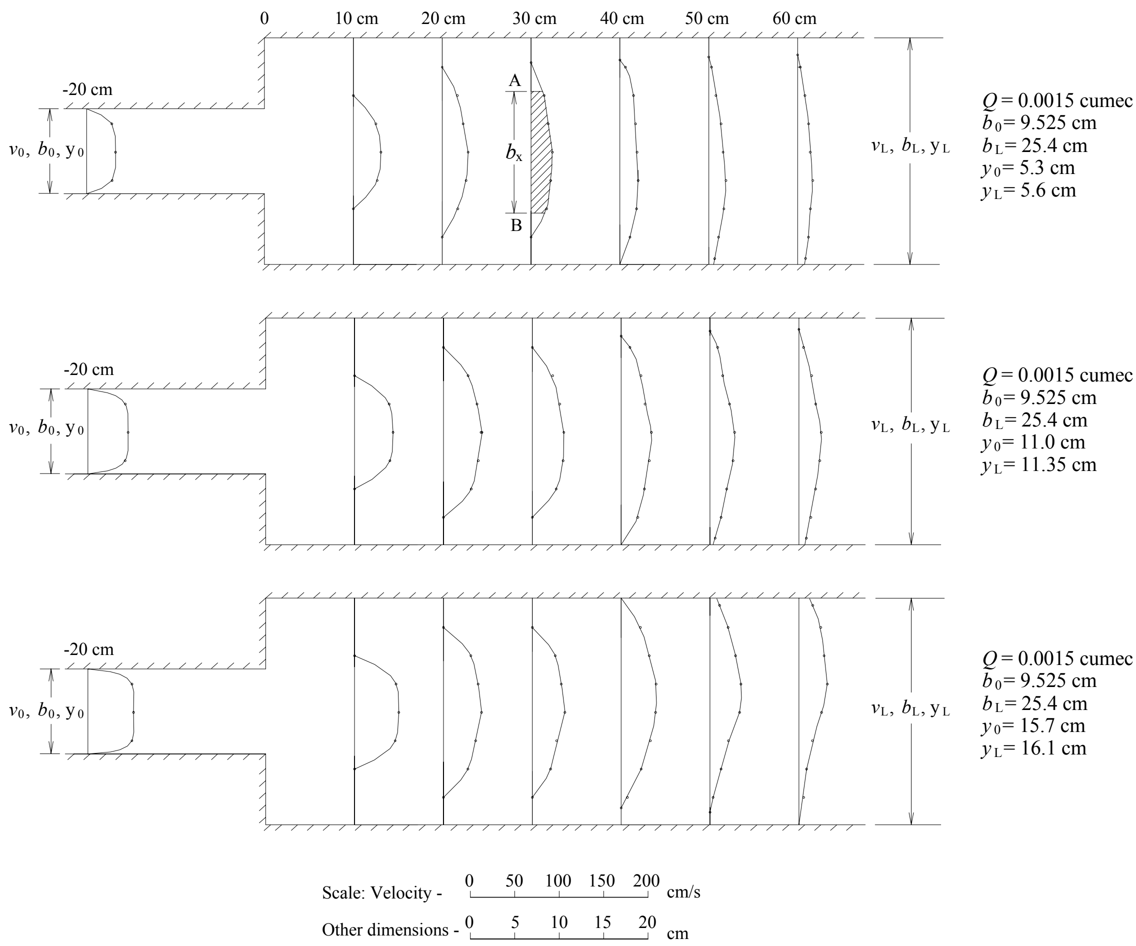

Fig. 2. Velocity distributions across the width and along the length of abrupt expansion

The present profile (Model V) is tested for performance and compared with the other transition profiles used commonly in the field, designed by Hartley et al (1940), Chaturvedi (1963), Nashta and Garde (1988), Swamee and Basak (1993); Model I, II, III, and IV respectively. To evaluate the transition models, efficiency and head loss of the transitions were determined in a laboratory setup flow. Using the data for depth of flow and local velocity, the flow area and average velocity at the sections were known, and thus energy correction factors were calculated. After then these data were used to determine the efficiency of transition from the expression as,

$$\varepsilon = \frac{Q\rho g\left(y_2 - y_1\right)}{(\frac{1}{2}\rho Q V_1^2)\alpha_1 - \left(\frac{1}{2}\rho Q V_2^2\right)\alpha_2} = \frac{\left(y_2 - y_1\right)}{\left(\alpha_1 \frac{V_1^2}{2g} - \alpha_2 \frac{V_2^2}{2g}\right)}$$

(20)

The values of α_1 and α_2 were calculated from the following expressions: $\alpha_1 = \frac{\sum v^3 dA}{A_1 V_1^3}$, for inlet section and $\alpha_2 = \frac{\sum v^3 dA}{A_2 V_2^3}$, for outlet section. Head loss was determined using the equation evolved by Swamee and Basak (1993), mentioned in Article 2.

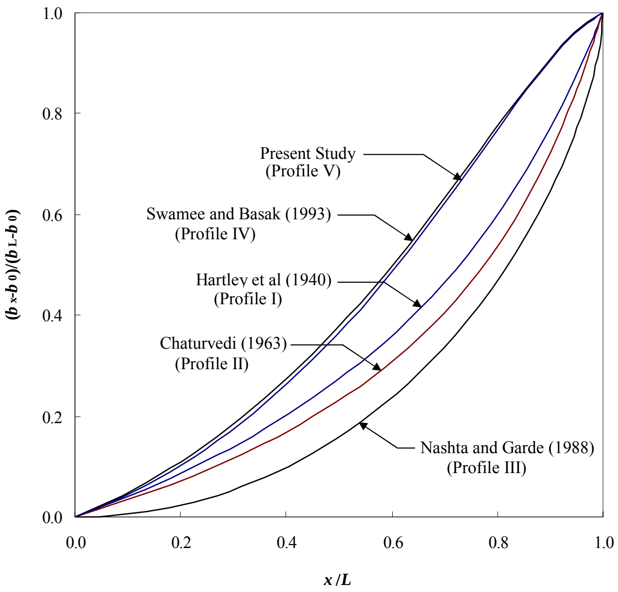

Fig. 3. Comparison of bed-width profiles

5. Results and discussion

The transition profile obtained in the present study conforms closely to the shape of separating streamline in sudden expansion, which is logical. The present profile is different from the bed-width profiles of Hartley et al (1940), Chaturvedi (1963), Nashta and Garde (1988), but conforms mostly to the profile evolved by Swamee and Basak (1993) (Fig. 3). The velocity distributions for Model I, II, and III depict central deformations indicating one sidedness of maximum velocity thread, but these are close to flat and near ideal for model IV and V (Fig. 4). Provision of smooth outlet in the present profile eliminates the separation and the chances of eddy formation to a significant amount. The eddy with reverse flow is looked strong for Transition I to III. The overall hydraulic efficiency of the transition models I, II, III, IV, and V are 75.8, 74.7, 71.4%, 78.7% (Alauddin, 2006), and 80.3% (Table 1) respectively. Except present profile and Profile IV, all other transitions have abrupt ending at downstream part of transition, which may be responsible for the separation noticed at the exit section in these transitions. From the comparison of overall hydraulic efficiency of the transition models, dominance of the present profile is observed over others (Fig. 5). The comparison of head loss of the transition models is made and minimum loss is found in case of profile IV and present profile (Fig. 6).

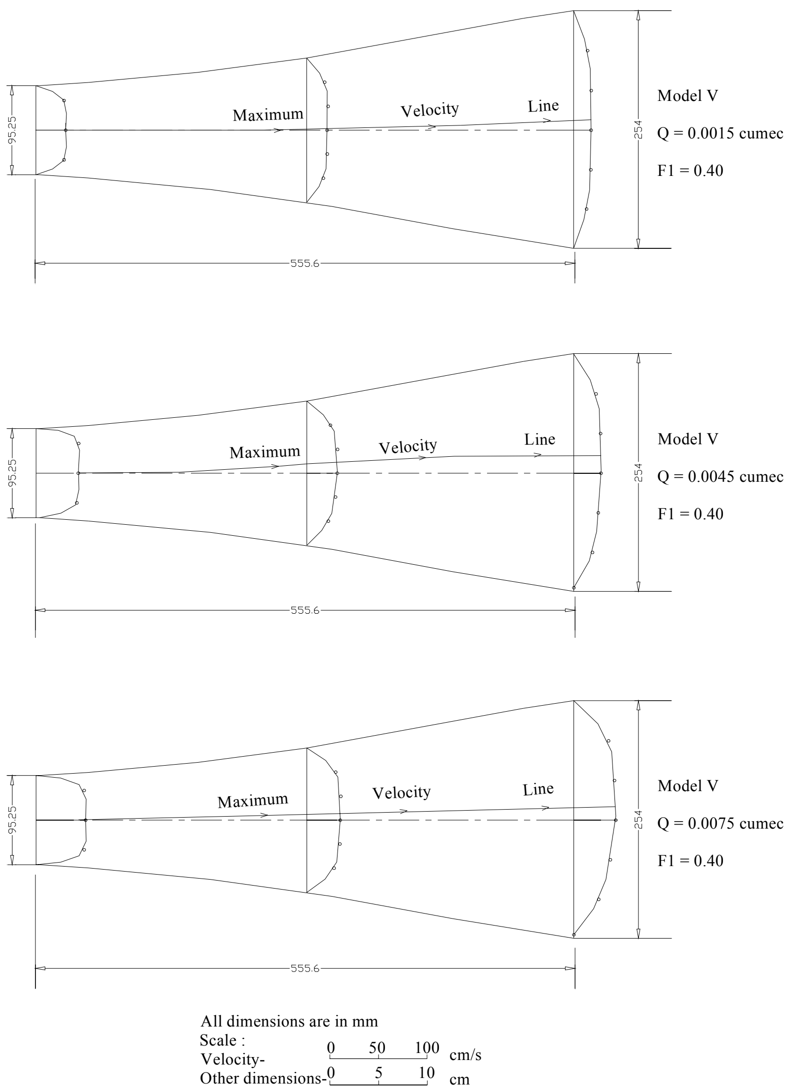

Fig. 4. Velocity distributions in the New Transition Model

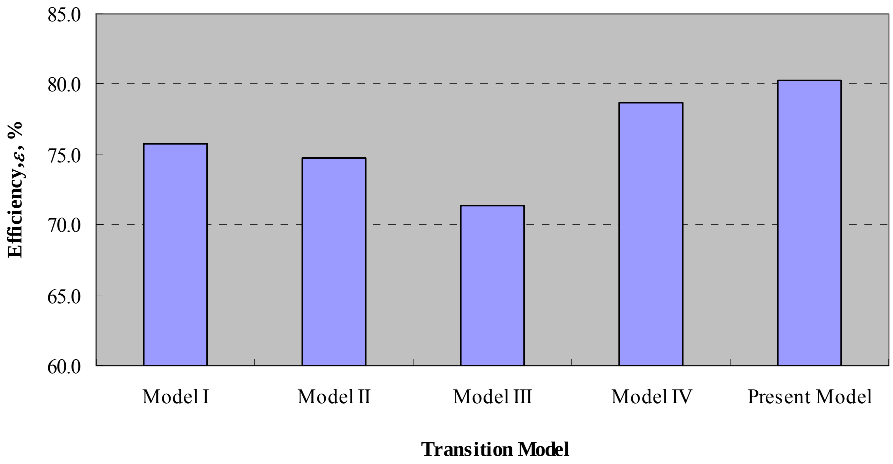

Fig. 5. Overall hydraulic efficiency of the transition models

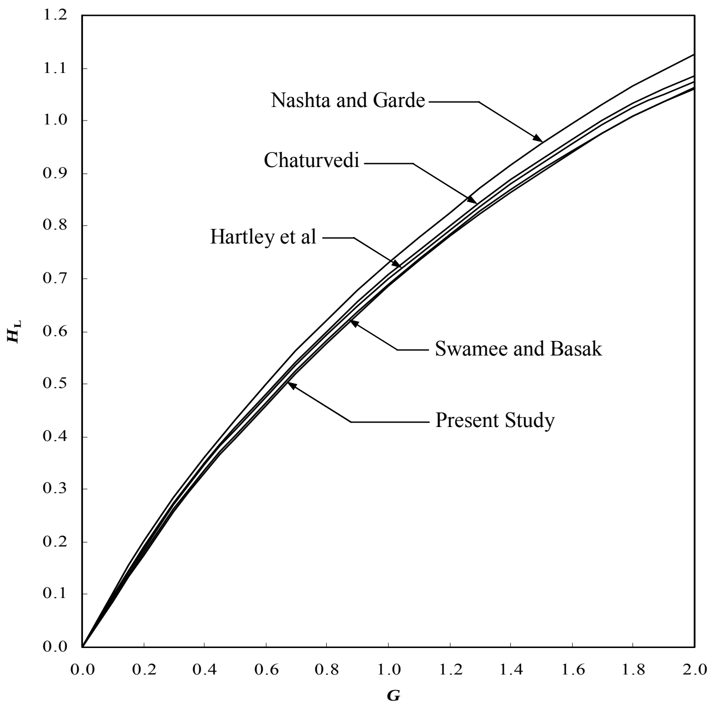

Fig. 6. Comparison of head loss of the models

6. Conclusions

The conclusions from the present study could be summarized as follows:

1. A methodology for design of expansive transition in open channel subcritical flow has been presented, and an empirical equation for the expansion transition is evolved.
2. The present profile yields a design that produces less energy loss than others do, and its (Model V) hydraulic efficiency is the highest among the existing profiles.

References

Alauddin. M. (2006). “Efficiency of an expansive transition in an open channel subcritical flow”. *Unpublished M. Sc. Engg. (Civil) Thesis,* DUET, Gazipur, 41-63.

Chaturvedi, R. S. (1963). “Expansive subcritical flow in open channel transitions”. *J. Inst. of Engrs.,* India, 43(9), 447-487.

Hartley, G. E., Jain, J. P. and Bhattacharya, A. P. (1940). “Report on the model experiments of fluming of bridges on Purwa branch”. *Technical Memorandum*. 9, United Provinces Irrigation Res. Inst., Lucknow (now at Roorkee), India, 94-110.

Henderson, F. M. (1966). *Open-channel flow*. The MacMillan Co., New York, N. Y., 235-250.

Hinds, J. (1928). “The hydraulic design of flume and siphon transitions”. *Trans*., ASCE, 92, 1423-1459.

Mazumder, S. K. (1967). "Optimum length of transition in open channel expansive subcritical flow". *J. Inst. of Engrs.*, India, 48(3), 463-478.

Morris, H. M. and Wiggert, J. M. (1972). *Applied Hydraulics in Hngineering*. Second Ed., The Ronald Press Co., New York, N. Y., 184-188.

Nashta, C. F. and Garde, R. J. (1988). "Subcritical flow in rigid-bed open channel expansions". *J. Hydr. Res.*, 26(1), 49-65.

Swamee, P. K. and Basak, B. C. (1993). “A comprehensive open-channel expansion transition design”. *J. of Irrg. and Drainage Div.,* ASCE, 119(1), 1-17.

Vittal, N. and Chiranjeevi, V. V. (1983). “Open channel transitions: Rational method of design”. *J. Hydr. Engrg.*, ASCE, 109 (1), 99-115.

Notations

The following symbols have been used in this study:

A	= cross-sectional area of flow;
b	= bed width of transition;
b_0	= bed width at inlet;
b_L	= bed width at outlet;
F_1	= $Q/(b_1 y_1 \sqrt{gy_1})$;
G	= $Q/(b_0^2 \sqrt{gb_0}$;
g	= gravitational acceleration;
h_L	= head loss;
K_H	= head-loss coefficient (Hinds);
L	= length of transition;
Q	= discharge;
r	= $\frac{b_L}{b_0}$
S_0	= channel-bed slope at inlet;
V	= velocity of flow;
V_0	= velocity at inlet;
V_L	= velocity at outlet;
x	= longitudinal distance from inlet;
y	= depth of flow;
y_0	= depth of flow at inlet;
y_L	= depth of flow at outlet;
α	= energy correction factor;
ε	= efficiency of transition;
ρ	= density of water;
ξ	= x/L

Note: *Additional notations are defined locally wherever they occur.*