Journal of Civil Engineering (IEB), 34 (2) (2006) 103-114

Journal of
Civil Engineering
IEB

Shear stress for initiation of motion of median sized sediment of no uniform sediment mixtures

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Received on 20 September 2006

Abstract

Critical shear stress of median sized sediment of nonuniform sediment mixtures utilizing experimental data is studied in the paper. Four sediment mixtures with varying gradation and median size are used in the experiment. The critical shear stress of median sized sediment of the four sediment mixtures are found lower than the values obtained from the Shields' diagram. The reason may be due to the effect of mixture sorting and efforts have been made to relate critical shear stress with mixture sorting. An empirical relation is developed by rearranging the dimensionless parameters of grain Reynolds number and mixture sorting. A formula is derived for the computation of critical shear stress of median sized sediment of nonuniform sediment mixtures by regression technique using with the experimental data and data available in the literature.

Keywords: Critical shear stress, median sized sediment, nonuniform sediment mixtures.

1. Introduction

Determination of shear stress for initiation of motion of streambeds is of considerable interest to river engineers. Natural streambeds are invariably nonuniform in character and their behavior differs from that of the beds composed of uniform sediment. The shear stress at which the sediment particles just start moving is known as the critical shear stress or critical tractive stress. It is used in determining bed load transport rates, predicting bed level changes undergoing degradation or aggradation and in stable channel design. There is no generally accepted method for predicting the value of critical shear stress of median sized fraction ($\tau*_{c50}$) of nonuniform sediment mixtures. Attempts have been made to relate $\tau*_{c50}$ to the Shields' threshold curve derived for uniform sediment (Wilcock and Southard, 1988, 1989; Kuhnle, 1994).

In many studies, however, large departures of τ^*_{c50} from the Shields' curve have been observed (e.g., Parker and Klingeman, 1982; Misri et al., 1983; Komar, 1987; Ashworth and Ferguson, 1989; Andrews, 1984, 1994; Patel and Ranga raju, 1999). Garde and Ranga Raju (1985) recommended an average value of 0.03 for τ^*_{ca}. They also found τ^*_{ca} varied from 0.02 to 0.05 for the data used. Patel and Ranga Raju (1999) proposed a relation in calculating τ^*_{ca} as $\tau^*_{ca} = 0.045(\sigma_g)^{-0.60}$, where σ_g is the standard deviation of the sediment mixtures. Parker et al. (1982) recommended τ^*_{c50} equals 0.0876 based on the subpavement grain size distribution of the mixture. Wilcock and Southard (1988) and Wilcock (1992) recommended that τ^*_{c50} be calculated using modified Shields' method (Miller et al., 1977) or 88% of the above value. Shvidchenko and Pender (2000a, 2000b) demonstrated that the Shields' curve is an inappropriate means of accurately evaluating the threshold for nonuniform sediment and therefore can not be used as a basis for analyzing the behavior of sediment mixtures. The authors were prompted by the above mentioned fact of uncertainty of computing τ^*_{c50} of sediment mixtures and thus efforts were made for the development of a method predicting τ^*_{c50}. This was possible by the availability of data from recent experiments conducted by the first author and data available from other sources.

2. Experimental methods

2.1 Bed materials

Natural sands of density 2650 kg/m^3 are sieved into different size ranges at first, and then they are compounded and mixed properly in required proportions to get mixtures of the required gradations. The grain size distributions of the four mixes used in the present experiments is shown in Fig. 1 and their characteristics are summarized in Table 1. In Table 1, d_{50} = median size of the sediment, d_g = geometric mean size, d_a = arithmetic mean size and σ_g = geometric standard deviation defined as, $\sigma_g = \sqrt{\left(\frac{d_{84}}{d_{16}}\right)}$ in which d_{84} and d_{16} are sizes such that 84% and 16% of the material respectively, by weight, are finer than these sizes. The sediment mixtures designated by S1, S2, S3 and S4 having log – normal distribution and narrow range of median size and mixture sorting.

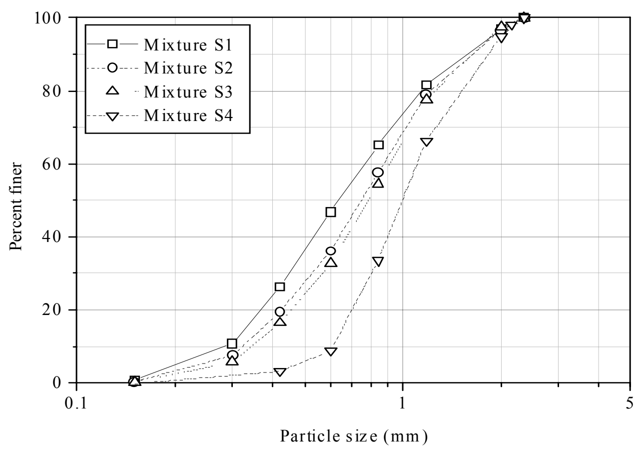

Fig. 1. Grain Size distribution of sediment mixture

Table 1

Properties of Sediment Mixtures used in the experiment

Mixture Designation	d_a (mm)	d_g (mm)	d_{50} (mm)	σ_g	n_b
S1	0.78	0.66	0.64	1.96	0.0128
S2	0.85	0.72	0.74	1.90	0.0131
S3	0.88	0.75	0.79	1.84	0.0132
S4	1.09	1.00	1.02	1.53	0.0136

2.2 Experimental channel

The experiments were carried out in a straight 11m long channel having a width of 1.0m and a depth of 0.6m. The sides of the channel were made of concrete with enamel painted and thus much smoother than the bed, thereby making the influence of sidewall rather small. Water was allowed to flow smoothly from the upstream reservoir through the pipes placed horizontally at the inlet. A tailgate was located at the downstream end of the channel. For a particular discharge, if the tailgate is raised it increases the water level and vice versa. A Rehbok weir was attached at the end of the channel that delivers water to the downstream reservoir. The Rehbok weir was calibrated to measure the discharge directly by observing the head above the weir only. The water level and bed level were measured with the help of a point gauge. A removable sediment trap at the downstream end of the channel was used to collect the transported material. Photograph of the experimental setup is shown in Fig. 2.

Fig. 2. View of experimental setup from downstream

2.3 Experimental procedure

For each sediment type at least four runs with different hydraulic conditions have been made in the present study. For each run, sand was placed in the flume bed by giving a pre-determined slope to bed and filled with the required sediment mixture to a uniform depth of 15 cm or more. Care was taken to avoid segregation of the material during placements and it was ensured that the composition of the surface layer was practically the same as that of the entire mixture. A very low discharge was allowed into the flume at first, so that the sediment bed becomes fully saturated.

After filling with water the run began by gradually increasing flow to the desired value. The tailgate could be adjusted to maintain the flow in the flume uniform. Several slopes were used and a number of runs were being done with the flow varying between a very low one with almost no sediment transport and one which moves a substantial portion of the bed material. A total of 19 runs were carried out in the experiment. The duration of single run was governed by sediment transport rate (the larger the transport rate is, the shorter is the run and vice versa) and varies from 3 hrs to 8.30 hrs. Care had been taken to ensure that the flow was uniform and that constant rate of sediment transport condition was attained. Constant rate of sediment transport was supposed to have been attained when three successive samples collected yields a sediment transport rate that is practically invariant with time. Sediment was fed in the flume at upstream with the help of a board placed at upstream and this was done manually by measuring the transported sediment for the time period of 5 or 10 minutes. After attainment of equilibrium conditions, sediment had been collected for desired purpose. The collected sediment was dried, weighed and sieved to get the fractional transport rate. All the runs were conducted in such a condition that there was no pronounced bed form. Mean hydraulic and transport parameters for the experiment are shown in Table 2.

3. Analytical method

3.1 Review of different methods

Methodological problems have always haunted the study of incipient motion of sediments. Even in the relatively simple case of sediments that are nearly uniform in size, it has long been realized that different methods, or even variations of the same method, give different values of the critical shear stress for initiation of grain motion (Wilcock, 1988). It has long been recognized that a basic problem encountered when determining the critical shear stress is that, it can be estimated only with data from flows with some grain motion, for which the bed shear stress already exceeds critical. A second and more fundamental problem is that the bed shear stress is a fluctuating quantity, and one can not precisely define a value below which there is no motion. Both problems lead naturally to a definition of τ_c in terms of a small but finite number of grains in motion. But the number of grains displaced depends on the area of the bed examined and the length of time over which grain displacements may occur. An initial-motion criterion must therefore be defined so that the critical shear stress determined for different sediments, or for different fractions in a sediment mixture are comparable, so that empirical data on critical shear stress can be combined into a general model or compared to theoretical results.

There are two general methods for determining the critical shear stress for individual fractions in mixed-size sediment. One associates the critical shear stress with the largest grain in the mixture that can be moved by a given flow. The other approximates the critical shear stress as that shear stress that produces a small reference transport rate of a given fraction. The former is known as the Largest Grain Method (LGM) and the latter as Reference Transport Method (RTM).

Table 2
Mean hydraulic and transport parameters for the experiment

Run No.	Total Runtime (hr)	Sed. Collection Time (hr)	Mixture Type	Avg. Discharge, Q (m^3/s)	Avg. Depth, d (cm)	Avg. Velocity, U (cm/s)	Slope,$S \times 10^4$
1	3:45	3.00	S1	0.05071	15.12	33.53	3.199
2	3:40	0.5	S1	0.06373	17.33	36.76	3.336
3	3:15	2.00	S1	0.07062	21.47	32.88	2.151
4	4:00	3.00	S1	0.04764	15.24	31.26	2.759
5	3:30	0.58	S1	0.03454	10.37	33.31	4.78
6	3:00	1.25	S1	0.02754	8.98	30.67	4.781
7	5:00	3.00	S1	0.05047	15.73	32.09	2.81
8	6:45	5.00	S2	0.05065	15.97	31.72	2.888
9	5:00	3.00	S2	0.04337	13.48	32.17	3.563
10	4:00	1.50	S2	0.03527	10.72	32.90	4.81
11	8:30	8.25	S2	0.06252	20.01	31.24	2.22
12	4:00	1.75	S3	0.03527	10.78	32.72	4.79
13	4:00	3.00	S3	0.06254	18.32	34.14	2.937
14	4:30	2.00	S3	0.05008	14.74	33.98	3.658
15	3:45	1.00	S3	0.04688	13.68	34.27	4.16
16	4:00	1.00	S4	0.04939	14.00	35.28	4.386
17	4:00	1.50	S4	0.0435	12.15	35.80	5.285
18	7:00	6.00	S4	0.04513	14.20	31.78	3.201
19	4:00	3.00	S4	0.04631	14.14	32.75	3.416

3.2 Method used in the present analysis

Parker et al.'s (1982) Reference Transport criterion is used in the present analysis. In this method the critical shear stress is estimated from a fitted relationship of the dimensionless bed load parameter, W_i^*, and the dimensionless grain shear stress, τ'_{*i}, as the shear stress corresponding to W_i^* = 0.002, wherein

$$W_i^* = \frac{i_B q_B (G-1)}{i_b \gamma_s \sqrt{g\left(R_b^{/} S\right)^3}} \tag{1}$$

in which i_B, i_b - proportions of size fraction d_i in the transported material and the sediment bed respectively, q_B= total bed load transport rate per unit width by weight, γ_s= unit weight of sediment, γ_f = unit weight of water, G = relative density of sediment $= {\gamma_s}/{\gamma_f}$, g = acceleration due to gravity, R'_b = hydraulic radius corresponding to grain resistance, and S = longitudinal slope of the channel bed. The use of the grain shear stress in preference to the total shear stress is logical because the former is the effective shear stress for bed load transport in case of an undulated bed. The dimensionless grain shear stress, τ'_{*i}, is defined as

$$\tau'_{*i} = \frac{\tau'_0}{\Delta \gamma_s d_i} \tag{2}$$

in which $\Delta\gamma_s = \gamma_s - \gamma_f$, and τ'_0 is the grain shear stress which can be computed using the equation

$$\tau'_0 = \gamma_f R'_b S \tag{3}$$

Here R'_b is the corrected hydraulic radius corresponding to grain resistance. A sidewall correction is applied to R_b to get the corrected hydraulic radius, R'_b. *S* is the slope of the energy line. Computation of the reference shear stress for individual fractions requires a number of choices concerning the appropriate technique for fitting the data. Not only must a reference transport criterion be chosen, but also an appropriate curve of transport rate versus shear stress must be fitted to the data, and the method of fitting the curve to the data must be chosen. The relation chosen here is a power approximation of the Einstein's bed load function at low stresses derived by Parker (1979)

$$W_i^* = 11.2\left(1 - \frac{0.8531\tau_{ci}^*}{\tau_i^*}\right)^{4.5} \tag{4}$$

A typical plot of W_i^* and τ'_{*i} for computing dimensionless critical shear stress of individual fractions in case of sediment mixtures is shown in Fig . 3.

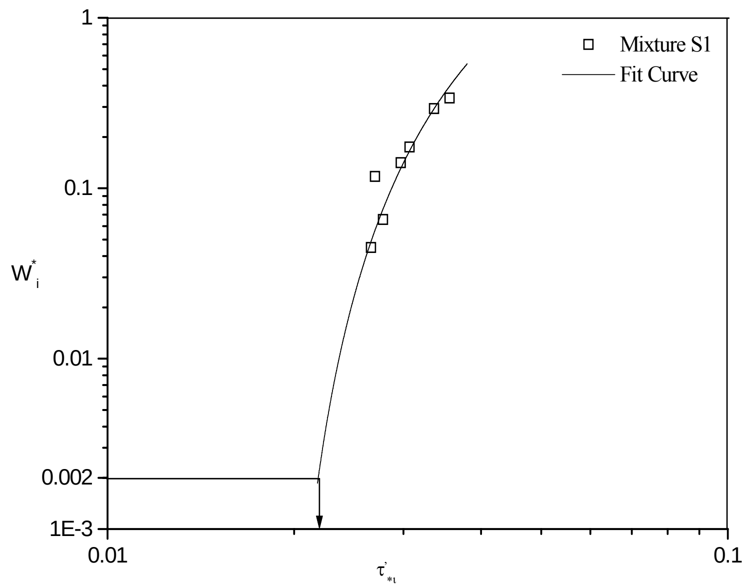

Fig. 3. Variation of W_i^* with τ_{*i}

4. Additional data

The present analysis uses both experimental results from this study and the data of other researchers from previous investigations. The main objective to select the additional data is that the fractional transport rates were measured over a wide range of bed mobility, including very low transport rates near to incipient motion and the data should cover a wide range of bed material gradation and median sediment sizes. Although the data were collected using different operational and measurement procedures and differ in accuracy and reliability, all refer to the transport of sand/ gravel mixture on a reasonably flat bed by steady, uniform flow. Summary of additional data are given in Table 3.

Table 3

Values of R^*_{c50} and τ^*_{c50} for different sediment types

Mixture designation	d_{50}	σ_g	τ^*_{c50}	R^*_{c50}	$\tau^*_{c50(uni)}$	Reference
S1	0.64	1.96	0.02595	9.9	0.033	Experiment
S2	0.74	1.90	0.02593	12.3	0.0326	do
S3	0.79	1.84	0.02629	13.7	0.0326	do
S4	1.02	1.53	0.02267	18.6	0.0329	do
M1	3.35	1.79	0.057	175.7	0.04819	Patel and Ranga Raju (1999)
M2	3.70	2.29	0.054	198.5	0.04917	do
M3	2.59	1.73	0.049	110.7	0.04438	do
M4	2.65	2.61	0.047	112.3	0.0450	do
M5	2.80	2.90	0.056	133.1	0.0459	do
MIT 1φ	1.83	1.99	0.0309	52.2	0.03882	Wilcock and Southard (1988)
MIT 1/2φ	1.83	1.41	0.0295	51.0	0.03868	do
DAY B	1.62	3.26	0.0269	40.6	0.03736	do
SAF	2.16	2.60	0.0646	96.8	0.043303	do
MISRI N1	2.37	2.0	0.0457	93.6	0.043036	do
MISRI N2	3.85	3.0	0.0363	172.7	0.04805	do
MISRI N3	3.12	2.36	0.0407	133.4	0.04592	do
Oak Creek	20.0	4.29	0.0490	2376.4	0.06	do

5. Comparison with shields curve and development of a new relationship

Values of $\tau*_{c50}$ obtained by different researchers are plotted on the Shields' diagram in Fig. 4 and compared with the shields' thresholds curve (as given by Vanoni et al., 1966). It is seen that the scatter of the reported data is considerable and is hardly acceptable for practical use. The ratio of the critical Shields' stress of size d_{50} in a mixture (τ^*_{c50}) to that in uniform sediment ($\tau^*_{c50(uni)}$) versus grain Reynolds number Re^* for the experimental data and other available data sets is shown in Fig. 5. For the experimental sediment mixtures $\frac{\tau^*_{c50}}{\tau^*_{c50(uni)}}$ values vary from 0.69 to 0.81 and for all available data sets its value fall within a factor of 0.70 to 1.2.

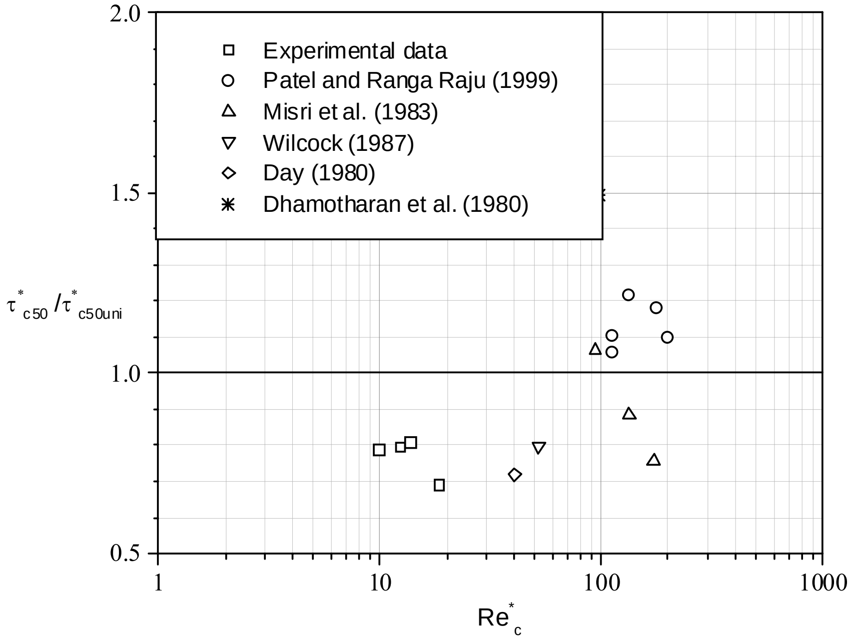

Fig. 4. Comparison of experimental and others τ^*_{c50} value with Shields threshold curve

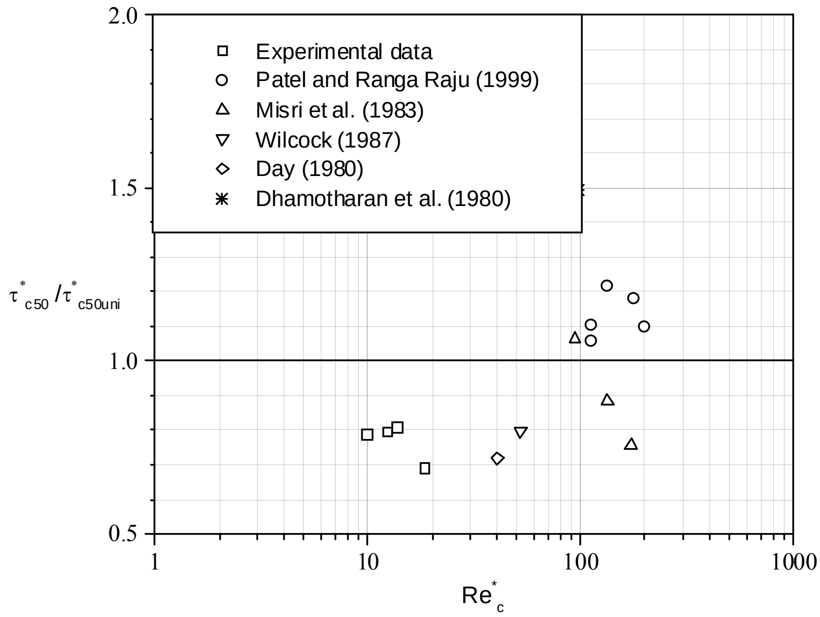

Fig. 5. Variations of $\frac{\tau^*_{c50}}{\tau^*_{c50((uni)}}$ with Re^*_c for experimental value and others.

From above discussion it is clear that the nonuniformity of sediment affects τ^*_{c50} and the measure of nonuniformity of sediment is the geometric standard deviation, σ_g
By taking these two parameters, it can be written,

$$\tau^*_{c50} = F(\sigma_g) \tag{5}$$

from the definition of grain Reynolds number, R^*_c

$R^*_c = \frac{u_{*c} d}{\upsilon}$, where u_{*c} is the shear velocity

$$= \sqrt{\frac{\tau'_c}{\rho}}\frac{d}{\upsilon} \qquad = \sqrt{\frac{\tau^*_{c50}(\rho_s - \rho)gd}{\rho}}\frac{d}{\upsilon}$$

$$= \sqrt{\tau^*_{c50}(s-1)g}\,\frac{d^{3/2}}{\upsilon} \tag{6}$$

or, $$\tau^*_{c50} = \frac{R^2_{*c}\upsilon^2}{(s-1)gd^3} \tag{7}$$

So, equation (6) may be written as

$$\frac{R^2_{*c}\upsilon^2}{(s-1)gd^3} = F(\sigma_g)$$

or, $$R_{*c} = F\left[\left(\sigma_g^{1/2} d^{3/2}\right)\sqrt{\frac{(s-1)g}{\upsilon^2}}\right] \tag{8}$$

A new nondimensional parameter is found in the right hand side of equation 8. If R^*_c is replaced by eq. 6, the equation 8 transforms to the form of

$$\sqrt{\tau^*_{c50}(s-1)g}\,\frac{d^{3/2}}{\upsilon} = F\left[\frac{\sigma_g d^3 (s-1)g}{\upsilon^2}\right]^{1/2}$$

or, $$\tau^*_{c50} d^3 = F(\sigma_g d^3) \tag{9}$$

Regression with the available data sets as shown in Fig. 6 and Fig. 7, equations 10 and 11 are found as

$$R^*_{c50} = 0.09\left(\sigma_g^{1/2} d_{50}^{3/2}\frac{\sqrt{(s-1)g}}{\upsilon}\right)^{1.05} \tag{10}$$

and $$\tau^*_{c50} d^3_{50} = 0.015\left(\sigma_g d^3_{50}\right)^{1.08} \tag{11}$$

Either of the two equations can be conveniently used as a tool for calculating the critical shear stress of median-sized sediment of nonuniform sediment mixtures.

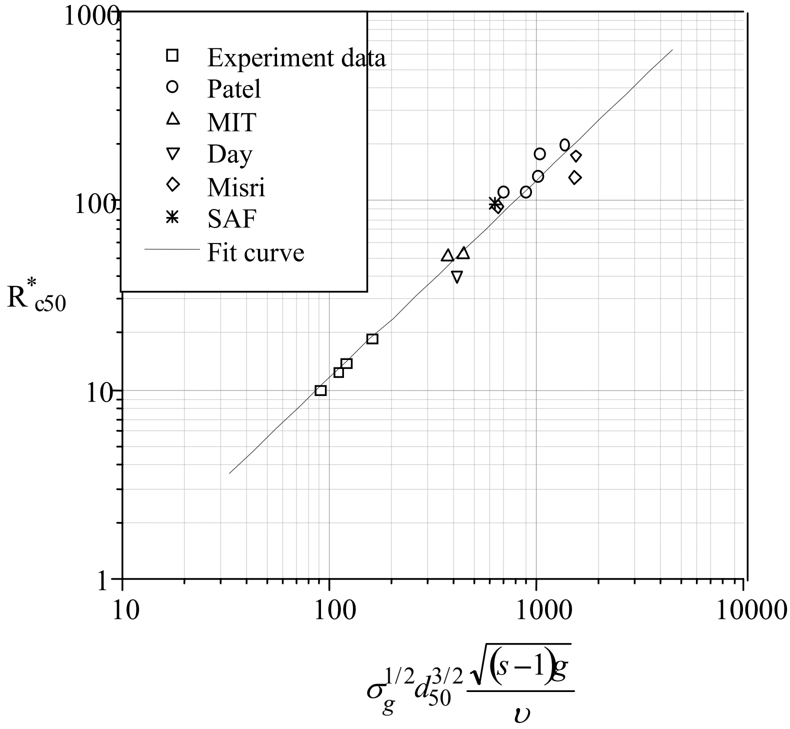

Fig. 6. Variation of R^*_{c50} with $\sigma_g^{1/2} d_{50}^{3/2} \frac{\sqrt{(s-1)g}}{\upsilon}$

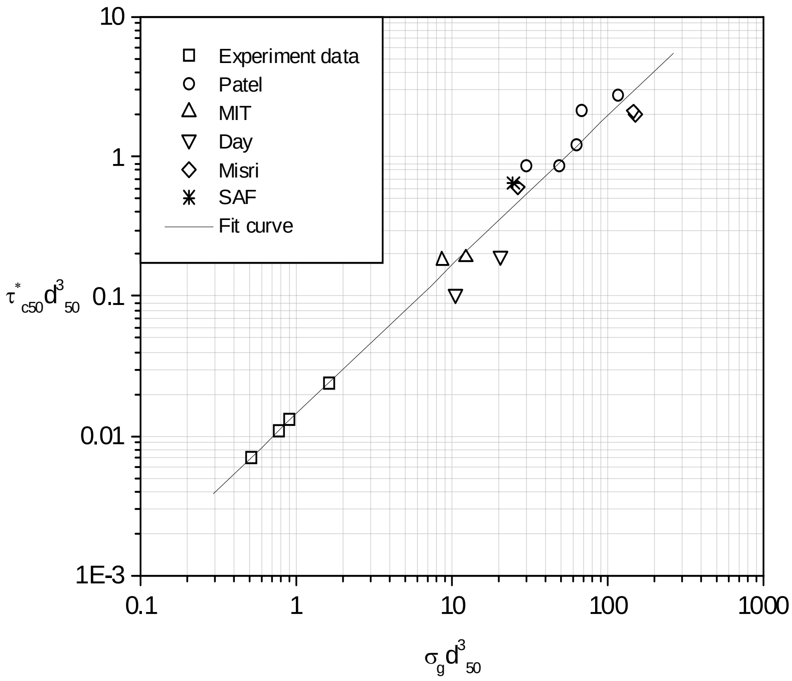

Fig. 7. Variation of $\tau^*_{c50} d^3_{50}$ with $\sigma_g d^3_{50}$.

6. Conclusions

The critical shear stress of median sized sediment in nonuniform sediment mixes is different from that of uniform sediment of the same size. Present analysis shows inadequacy of using Shields' threshold value of median sized sediment of nonuniform sediment mixtures. An analytical method is presented here for the computation of τ^*_{c50} given due consideration to the nonuniformity of sediment mixtures. Regression with the available data sets equations have been developed for the calculation of τ^*_{c50} in case of nonuniform sediment mixes. The equations developed here can conveniently be used as a tool for calculating the critical shear stress of median sized sediment of nonuniform sediment mixtures.

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Notations

D^*_{50} dimensionless grain diameter for incipient motion

d_{gr} Ackers and White's dimensionless grain diameter

d_a arithmetic mean size of sediment mixture;

d_i size of a particular fraction in a sediment mixture and estimated as geometric mean of two sieve sizes d_{s1} and d_{s2} as $=\sqrt{(d_{s1}.d_{s2})}$;

d_{16} size in the mixture, such that 16% material by weight is finer than this size;

d_{50} size in the mixture, such that 50% material by weight is finer than this size;

d_{84} size in the mixture, such that 84% material by weight is finer than this size;

d_g geometric mean size of the mixture;

d_σ representative size = $d_g.\sigma_g$;

g acceleration due to gravity;

i_B proportion of a particular size fraction, d_i, in transport;

i_b proportion of a particular size fraction, d_i, in sediment bed;

q_B bed load discharge per unit width of channel by weight;

R_b hydraulic radius corresponding to grain resistance;

R'_b corrected hydraulic radius corresponding to grain resistance;

R^*_{c50} grain Reynolds number of median-sized sediment

S longitudinal slope of the channel bed;

s specific weight of sediment = $\frac{\gamma_s}{\gamma_f}$

W^*_i dimensionless bed load parameter for a particular size fraction, d_i;

γ_f unit weight of water;

γ_s unit weight of sediment;

$\Delta\gamma_s$ $\gamma_s - \gamma_f$;

ν kinematic viscosity of water;

σ_g geometric standard deviation of sediment mixture, $\sqrt{\left(d_{84}/d_{16}\right)}$;

τ_c critical shear stress

τ'_0 bed shear stress corresponding to grain resistance;

τ'_{*i} dimensionless grain shear stress for size fraction d_i

τ^*_{ci} dimensionless critical shear stress for size fraction d_i;

τ^*_{c50} dimensionless critical shear stress of median size d_{50};

τ^*_{c50uni} Shields' threshold value of d_{50} sizes

τ^*_{ca} dimensionless critical shear stress of mean size da

$\tau^*_{c\sigma}$ dimensionless critical shear stress of dσ