

Modification of Webster's delay formula under non-lane based heterogeneous road traffic condition

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Abstract

Delay is one of the most important performance measures of signalised intersections. Various models including Webster's classical delay formula have been developed in countries with car dominated traffic stream to estimate average delay per vehicle at signalised intersections. Webster's classical delay formula has been formulated under UK situation where the road traffic condition is homogeneous as well as lane based and consequently the formula may not estimate delays accurately under heterogeneous road traffic condition. As a result, it is necessary to modify Webster's delay formula to make it usable under non-lane based mixed road traffic condition. In this study, the Webster's delay formula has been modified to suit the road traffic situation of Bangladesh. For this purpose, data have been collected using video camera at different signalised intersections of Dhaka city to measure average delay per vehicle at each cycle. Based on these data, a model in the form of multiple linear regression has been developed, which attempts to keep the first and second terms of Webster's delay formula as it is but to modify the adjustment term. The model has been calibrated to form a 'Modified Webster's Delay Formula', which is subsequently validated by comparing the expected delays with observed delays. The model provides a coefficient of correlation of 0.68 and all the independent variables are found to be statistically significant. In the comparison between expected and observed delays, although somewhat scatter has been found, the agreement between the two was found to be satisfactory so that the model has the potential to provide sufficiently accurate values of delay under Bangladesh condition.

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1. Introduction

The growth of traffic congestion in the road network of large cities in developing countries like Bangladesh is a serious concern from the point of view of traffic engineers. The congestion at the junctions is mostly crucial because the performance of junctions affects the performance and productivity of the whole road network significantly. To reduce conflicts and ensure orderly movement of traffic at the urban intersections, it is a common practice to introduce fixed time traffic signals at uncontrolled or priority controlled or traffic police controlled intersections if the conditions warrant their choice. There are a number of criteria to assess the performance and level of service of signalised intersections among which delay is used most widely. The reason is that the meaning of delay is generally well understood and drivers can experience it directly.

There are a number of widely used delay models to determine the average delay per vehicle at a signalised intersection, all of which are developed in countries with car dominated traffic stream, where the road traffic condition is homogeneous and lane based (Webster 1958, Miller 1963, Akcelik 1981 and Teply et al. 1995). Among these models, the oldest and the most popular one is the Webster's classical delay formula, which has been originated in UK. The urban road traffic situation of Bangladesh is quite different from that of UK. The traffic is mixed with a wide variation in the operating and performance characteristics of vehicles. The traffic includes both motorised (e.g. bus, car, auto rickshaw) and non-motorised vehicles (e.g. rickshaw, bicycle, push cart). The vehicles which travel in the same right of way also vary in size, manoeuvrability, control and dynamic characteristics. Another striking feature of the road traffic operating condition is that, despite having lane markings, most of the time lane discipline is not followed no matter whether non-motorised vehicles are present or not. At intersections, there is notable lateral movement and vehicles tend to use lateral gaps to reach the front of the queue. Under this condition, the available delay models including the one presented by Webster are certainly expected to produce erroneous results if directly applied. Consequently, need arises to modify the Webster's delay formula to cope with Bangladesh conditions and hence to produce accurate measure of delay.

The specific objective of this study is to develop a delay model for non lane based heterogeneous road traffic condition by means of field data collection, model formulation and calibration so that delay based performance evaluation can be more accurately done for signalised intersections of Bangladesh.

2. Literature review

Using deterministic queuing analysis, Webster, F.V. (1958) developed a model for estimating the delay incurred by motorists at undersaturated signalised intersections that became the basis for all subsequent delay models. The mathematical form of the model is

$$d = \frac{c(1-\lambda)^2}{2(1-\lambda x)} + \frac{x^2}{2q(1-x)} - 0.65 \left(\frac{c}{q^2} \right)^{1/3} x^{2+5\lambda} \quad (1)$$

where

d = Average delay per vehicle on the particular approach of the intersection

c = Cycle length

q = Flow

λ = Proportion of the cycle which is effectively green for the phase under consideration,
and

x = Degree of saturation (volume to capacity ratio)

In Equation (1), the first term represents the average delay to the vehicles assuming uniform arrivals. The second term represents the additional delay due to the randomness of vehicle arrivals. This additional delay is attributed to the probability that sudden surges in vehicle arrivals may cause the temporary oversaturation of the signal operation. The third term is a semi-empirical adjustment term that was introduced in the model to account for specific field conditions. The model is applicable for isolated intersections and the permissible upper limit of traffic flow for the model to provide satisfactory results is slightly less than full saturation.

Following this classical work, numerous studies were conducted in the field of estimating delays at signalised intersections. As a result of these studies, a number of delay models based on deterministic queuing theory were proposed to suite different field conditions. Among these, the most notable are the models developed by Miller (1963) and Akcelik (1981) in Australia, the models developed for use in HCM 1985 (TRB 1985), HCM 1994 (TRB 1994) and HCM 2000 (TRB 2000) in United States and the model developed by Teply et al. (1995) in Canada. However, these models are analytically superior to Webster's classical model in the sense that they can successfully deal with oversaturated conditions and the effect of progression and platooning.

3. Methodology

The expression for delay, given by Equation (1) was not derived entirely theoretically. The first and second terms have a theoretical meaning, but the last term is purely empirical. It is a correction term to give a closer fit for all values of flow. However, this empirical correction term was derived based on UK situation, where the traffic is car dominant and lane based. So, the most crucial aspect of modification of the whole formula is to keep the first and second terms as it is and completely omitting the third term of Equation (1), while introducing some other adjustment term with the first and second term, calibrated from local road traffic condition.

In its simplest form, the modification of Webster's delay formula under non lane based mixed road traffic condition can be accomplished by adding an empirical adjustment term with the sum of first and second terms, which has been calibrated based on field observations of delays. Hence, the general pattern of the modified Webster's delay formula will be as follows:

$$d = \frac{c(1-\lambda)^2}{2(1-\lambda x)} + \frac{x^2}{2q(1-x)} + adj \quad (2)$$

In the above equation, adj is the adjustment term, which is to be calculated from delays observed in field. To make use of this term in the calibration, it must be based on the field value of delay and calculated from the following equation:

$$adj = d - \frac{c(1-\lambda)^2}{2(1-\lambda x)} - \frac{x^2}{2q(1-x)} \quad (3)$$

In the above equation, d is the actual delay observed in field. If the left hand side of the above equation is taken as the dependent variable, it needs to be regressed against a set of independent variables.

It is convenient to take the same variables as the independent variables present in the adjustment term of Webster's classical delay formula. An additional variable unique to non lane based mixed road traffic condition, which is the percentage NMV present in the traffic stream, is worthy to be used.

4. Data collection

4.1 Site selection

There are a large number of signalised intersections in Dhaka city, which vary with respect to geometric design, phasing pattern, operating conditions and degree of enforcement. It is practically impossible to cover all these intersections to modify Webster's delay formula. First of all, it is necessary to carry out a reconnaissance survey to assess the suitability of different sites to collect data pertaining to delay and also to make selection of suitable sites to place video camera and record traffic flow. However, a set of criteria were adapted on the basis of which the selection of site was finalised. Because the delays are mainly associated with a particular approach of an intersection, it is necessary to adapt the criteria in an approach specific manner, not just in an intersection specific manner. The criteria are listed as follows:

- Non motorised vehicles are present in significant proportion.
- Discipline of starting and stopping is as strict as possible.
- Traffic volume is not very high at least during pre peak period.
- Vehicle arrival pattern is random and effect of platooning is as small as possible.
- Right turning movement either shared or right turn is banned.
- Neither the right turners are blocked by opposing straight ahead vehicles nor any exclusive phase provided for right turning.
- Approach is as far as possible free from the disturbance of pedestrians, bus stops and parked vehicles.
- Proportion of large vehicles is not very high.
- No central roundabout is present at the middle of the intersection.
- Availability of elevated place is there to enable data collection by video.

Based on the criteria listed, a total of three intersections were selected and at each of these three intersections, two approaches were selected for data collection. The intersections and approaches are Elephant Road Intersection (North and South approaches), Kataban Intersection (North and South approaches) and Shantinagar Intersection (North and East approaches). The basic characteristics of these approaches are given in Table 1. Elaborate description can be found in Imran (2006).

In this study, data were collected from video recording. The advantage of this method is that from the same video footage different parameters can be extracted. Another advantage is that it is possible to run the video footage as many time as desired.

First of all, it is necessary to select a suitable vantage point from where video recording can be done. The video camera should be situated at a place from where the beginning to the end of the queue is visible as well as the stop line is visible if possible. It is better to

choose the vantage point such that the drivers of vehicles can not see the video camera readily, otherwise, after seeing the video camera their behaviours may be changed from natural behaviours.

Table 1
Basic characteristics of selected intersections and approaches

Intersection	Approach	Traffic Volume	Non Motorised Vehicles	Parking and Bus Stop	Pedestrian Movement	Platooning
Elephant Road	South	Very Low	Moderately High	None	Moderate	None
	North	Low	Moderately High	None	Moderate	None
Kataban	South	Moderate	Moderately High	Moderate	Moderate	Small
	North	Moderate	High	None	None	None
Shantinagar	North	Moderately High	Moderate	Moderate	Heavy	Moderate
	East	Low	High	None	Heavy	None

Vehicular movements were captured by using an 8 mm portable video camera system for all the approaches separately, which has the advantage of providing a permanent and comprehensive record of traffic movement, which can readily be used to obtain the repeated field picture as many times as required during analysis. All video recordings were carried out in February 2006 - March 2006 during pre peak hours. In all, more than 2.5 hours of traffic data were recorded on video tapes for this study.

The video recordings were first examined in the laboratory to screen out the cases that were not suitable for this study. Only undersaturated cycles were considered for delay calculation, where the degree of saturation is preferably below 0.9 and no residual queue (i.e. the queue, which could not discharge in the previous cycle) exists at the beginning of cycle. A total of 35 cycles were extracted as observations for six approaches of the three intersections selected. Table 2 shows the number of observations for selected approaches of the selected intersections.

Table 2
Number of observations for different approaches under study

Name of Intersection	Approach	Number of Cycles
Elephant Road	South	6
	North	7
Kataban	South	3
	North	9
Shantinagar	North	5
	East	5

4.2 Delay

The field determination of delay by using the stopped vehicle count method is not expected to provide accurate results under non lane based mixed road traffic condition. The reason is that the speed and acceleration-deceleration characteristics are different for different vehicle types and hence the acceleration-deceleration component will be highly variable. Another reason is that many vehicles infiltrate through the gaps to reach the

head of the queue and makes the concept of 'First In First Out' (FIFO) inapplicable. Also because discipline is not very strict, it is better to go for a cycle by cycle determination of delay rather than calculating delay as a whole.

In this study, average delays for a particular cycle were calculated by averaging for a particular cycle the delays experienced by all the vehicles. The delay experienced by a particular vehicle is taken as the stopped delay plus the delay corresponding to acceleration-deceleration.

To determine the stopped delay, the arrival time (the instant when the vehicle comes to stop) and the departure time (the instant when the vehicle starts moving) for each and every vehicle were noted. The stopped delay is simply the time difference between the arrival time and the departure time of the vehicle. On many occasions, some vehicles arrive when the queue has dissipated and the signal is still green, which allows the vehicle to clear the intersection without stopping. In this case, the stopped delay is zero.

To determine the acceleration-deceleration delay, it is necessary to measure the speed and acceleration-deceleration characteristics of different types of vehicles directly from the field. However, in this study, these parameters were taken from one of the previous studies (Arasan et al. 2003) pertaining to non motorised vehicles. For simplicity and based on Akcelik et al. (2001) and Maini (2002), the acceleration and deceleration are assumed to be equal and the acceleration-deceleration delay is taken simply as the speed divided by the acceleration to the nearest integer. Table 3 shows the values of these parameters.

Table 3
Acceleration-deceleration parameters of different vehicle types

Vehicle Type	Mean Free Speed (km/hr)	Acceleration (m/s ²)	Acceleration Deceleration Delay (sec)
Bicycle	12	0.5	7
Tricycle (Rickshaw)	10	0.4	7
Motorised Two Wheeler	39.58	1.35	8
Auto Rickshaw	31.13	0.7	12
Car	37.89	1.1	10
Light Commercial Vehicle	37.8	0.9	12
Heavy Vehicle	37.2	0.7	15

On many occasions, some vehicles arrive when the queue has just started to dissipate and they only have to slow down or hardly to stop. In these cases, the stopped delays were taken as zero and the full or a fraction of the acceleration-deceleration delays were used whichever applicable depending on arbitrary judgement. However, these cases constitute only a small proportion of the vehicles which arrive during the cycle and the arbitrary judgement does not lead to considerable errors.

The average delay per vehicle for a particular cycle was taken as the sum of stopped delays and acceleration-deceleration delays for all the vehicles, which arrive during the cycle from start of red to next start of red, divided by the total number of vehicles which arrive during the cycle from start of red to next start of red. However, a small proportion of vehicles (especially some of the bicycles and motorcycles) infiltrate through the queue

and violate the signal whenever they get opportunity. The effect of these vehicles upon average delay was ignored.

4.3 *Signal timing parameters*

The cycle time was taken as the time interval between the start of red and the next start of red. During video recording, indications were given either visually by raising hand from a distant place or by giving audible warning from side whenever the signal changes to red or green. The cycle times were easily calculated from these indications.

The green times can also be calculated by these visual or audible indications. However, on several occasions, the queue starts to dissipate a few seconds before the commencement of green signal. In these cases, the green times were taken as the actual green time plus the number of seconds the queue start to discharge before the commencement of actual green. The effective green times for each cycle under observation were calculated by assuming a uniform number time of 3 seconds and a uniform lost time of 2 seconds for the phase gaining right of way. The effective green ratio (λ) was taken simply as the ratio of effective green time to cycle time.

4.4 *Traffic stream parameters*

In case of cycle by cycle analysis it is better to adapt the vehicle arrival rate as a parameter rather than using hourly traffic volume as the corresponding parameter. The reason is that the former can take into account the variability of vehicle arrival rate on different cycles and the corresponding effect on average delay, whereas the latter takes into account the arrival on hourly basis as a whole. Also, the former is more convenient because it can be incorporated into the delay formula directly on per second basis to obtain the average delay directly in seconds.

In this study, to determine the vehicle arrival rate, a classified count of vehicles which arrive between start of red to the next start of red was carried out and the count was converted to PCU by using the PCU factors given by Hoque, M.S. (1994). This PCU value was divided by the cycle length to get the vehicle arrival rate in PCU per second. Also from the classified vehicle count, the proportion of non motorised vehicles in percentage can be easily computed.

4.5 *Degree of saturation*

One of the most important parameters present in Webster's delay formula is the degree of saturation. This parameter requires the capacity, which in turn depends on the saturation flow. It is necessary to compute the saturation flow directly from field or from video footage rather than using empirical equations to determine the capacity and hence the degree of saturation.

In this study, the site specific saturation flow was calculated from the video footage of vehicle discharge for the selected sites. Because, the video did not focus on the stop line; it was practically impossible to make classified count of vehicles for every 6 second interval. Consequently, the exact measurement of saturation flow was not possible. However, the approximate saturation flow was computed by conducting a classified count of vehicles for a convenient time interval when the vehicles discharge. An equivalent hourly average was taken over several cycles in PCU per hour as the saturation flow. Table 4 shows the approximate saturation flows along with approach

geometry and traffic stream characteristics for different approaches under study for the selected intersections.

Table 4
Approximate saturation flows for different approaches under study

Name of Intersection	Approach	Approach Width (m)	Average Proportion of NMV (%)	Approximate Saturation Flow (PCU/hr)
Elephant Road	South	7.2	67.0	3146
	North	6.6	67.1	3396
Kataban	South	11.0	72.3	4378
	North	8.6	89.8	4657
Shantinagar	North	14.9	67.4	5986
	East	6.3	91.5	3247

Once the saturation flows are known, the capacity can be computed in PCU per hour. If the vehicle arrival rate for a particular cycle is taken as an equivalent hourly arrival rate, then the degree of saturation is simply the ratio of this equivalent hourly arrival rate to the hourly capacity.

5. Data analysis

First of all, it is necessary to prove statistically that the delays observed in the field are significantly different from the delays calculated by Webster's classical delay formula and hence to justify the modification of this formula for non lane based mixed road traffic condition. For this purpose, a 'statistical t test' was performed to assess whether the difference between observed and Webster's delay (observed minus Webster's) is non zero or not. Hence, the differences between actual field value of delay and the delay corresponding to Webster's classical delay formula were calculated for all 35 sets of observations and the mean and standard deviation of these differences were determined. If the difference is statistically found to be non zero, then the modification is justified. A null hypothesis is taken that the difference does not differ significantly from zero. If the t statistic exceeds the critical value, then the null hypothesis can be rejected. In Table 5, the statistical t test results along with relevant values are shown.

Table 5
Statistical t-test results

Mean of differences (sec)	Standard deviation of differences (sec)	Number of observations	t-statistics	Confidence level	Critical t	Remarks
-8.72	10.00	35	-5.159	95%	1.96	Significant

Based on the table, the null hypothesis can be rejected, which means the difference is statistically non zero. Hence the modification of Webster's delay formula under non lane based mixed road traffic condition is justified.

5.1 Model formulation

In this study, it is intended to modify Webster's classical delay formula so that it can be used in the non-lane based heterogeneous road traffic condition of Bangladesh. To achieve this, the first and second terms of the formula will be kept as it is, because they represent the uniform and random component of delay and derived solely from queuing theory. The third term, which is only for adjustment and developed for UK condition has to be replaced with a quantity calibrated from field observations. In the modified formula, with the first and second terms, some additive adjustment term has to be introduced to best fit the local road traffic condition. In the model, this adjustment should be taken as dependent variable and regressed against independent variables, which were originally present in the adjustment term for Webster's classical delay formula. In addition, the percentage of NMV, which is unique to non lane based mixed road traffic condition, should also be taken as an independent variable. The model along with its mathematical form is as follows:

$$adj = \beta_0 + \beta_1 * q + \beta_2 * c + \beta_3 * x + \beta_4 * lamda + \beta_5 * pnmv \quad (4)$$

In the above equation:

- adj = Adjustment term for model.
- q = Vehicle arrival rate (PCU/sec).
- c = Cycle length (seconds).
- x = Degree of saturation.
- $lamda$ = Effective green ratio.
- $pnmv$ = Percentage NMV in traffic.
- β_0 = Constant term in the model.
- β_1 = Coefficient corresponding to q .
- β_2 = Coefficient corresponding to c .
- β_3 = Coefficient corresponding to x .
- β_4 = Coefficient corresponding to $lamda$.
- β_5 = Coefficient corresponding to $pnmv$.

5.2 Model calibration

First of all, it is necessary to examine the correlations between the dependent and all the independent variables. Table 6 shows the correlations of dependent variable and all independent variables. It was observed from the correlation matrix that some pairs of independent variables such as q - c , q - $lamda$, c - $pnmv$ and x - $lamda$ are highly correlated among them. As a result, they should not be combined in one equation simultaneously. For this reason, only the independent variables q , x and $pnmv$ were kept in the model and the remaining were excluded.

The data were analysed by using the statistical software SPSS. Table 7 shows the results of model calibration along with other statistical parameters.

From the results of model calibration, the regression model is seen to provide fair to good predictions with a satisfactory value of adjusted R square. The t-statistics came out to be high enough with low p -values indicating that the independent variables are fairly significant. The F-statistic also came out to be high enough to ensure overall significance.

Table 6
Correlation matrix of variables

	adj	q	c	x	lamda	pnmv
adj	1	-0.4045	-0.5837	-0.6985	0.3097	-0.4723
q	-0.4045	1	0.5747	0.2345	0.5016	-0.0209
c	-0.5837	0.5747	1	0.3613	0.0889	0.5955
x	-0.6985	0.2345	0.3613	1	-0.6687	0.1303
lamda	0.3097	0.5016	0.0889	-0.6687	1	-0.1181
pnmv	-0.4723	-0.0209	0.5955	0.1303	-0.1181	1

Table 7
Statistical results of model calibration

Model Summary						
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate		
1	0.841382	0.707924	0.679658	6.552829		
ANOVA						
Model		Sum of Squares	df	Mean Square	F-Statistic	p-Value
1	Regression	3226.332	3	1075.444	25.04553	2.01E-08
	Residual	1331.126	31	42.93956		
	Total	4557.459	34			
Coefficients						
Model		Unstandardised Coefficients		Standardised Coefficients	t-Statistic	p-Value
		B	Std. Error	Beta		
1	(Constant)	46.93263	7.856137		5.974008	1.32E-06
	q	-46.0372	16.64018	-0.27664	-2.76663	0.009463
	x	-37.3187	6.47407	-0.58122	-5.76433	2.4E-06
	pnmv	-0.36083	0.087927	-0.40233	-4.10368	0.000274

Once the coefficients are known, they can be substituted to Equation (4) to determine the adjustment term. Because the model is an additive one, the adjustment term must be added to the first and second terms to get the 'Modified Webster's Delay Formula' as shown in Equation (5) below.

$$d = \frac{c(1-\lambda)^2}{2(1-\lambda x)} + \frac{x^2}{2q(1-x)} + 46.93 - 46.04 * q - 37.32 * x - 0.3608 * pnmv \quad (5)$$

5.3 Model validation

Once the model has been calibrated, it is necessary to validate it, which is best achieved by comparing a completely external set delay observation with their expected counterpart using the calibrated model. However, due to scarcity of adequate number of observations in this study, the goodness of the model was checked by plotting the expected values of delay computed by using the calibrated model against the observed ones of the collected data used for model calibration. In Figure 1 the plot shows that the observed and expected delays are in good agreement although there is some scatter.

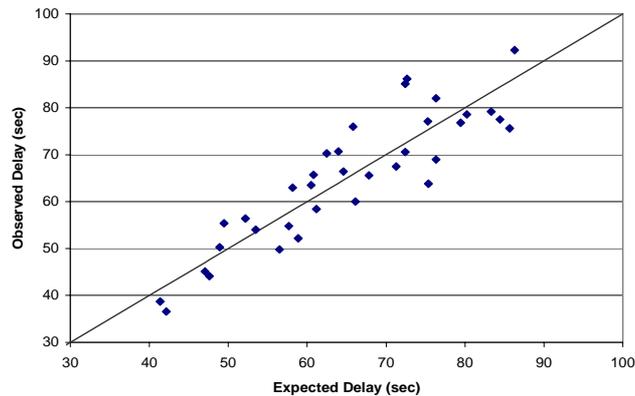


Fig. 1. Comparison between expected and observed delays.

6. Conclusions and recommendations

In this study, because the field calculated delays are significantly different from those calculated by Webster's classical delay formula, an attempt has been made to modify Webster's classical delay formula to cope with the local road traffic condition of Bangladesh, which is non lane based and mixed. From the calibration of the developed multiple linear regression model, almost all of the coefficients of independent variables were found to be statistically significant and the coefficient of correlation was found to be 0.68, which indicates acceptably good fit.

Due to some inherent limitations of the study and for its future betterment, it is recommended to use greater number of observational cycles including greater number of sites for model calibration. Also, the acceleration-deceleration delay should be measured by using field-determined values of speed, acceleration and deceleration under local conditions. In order to make the calculation of cycle-specific degree of saturation values more accurate, instead of using approximate values, systematic measurement of saturation flows and lost times should be performed in field. For consistency, completely different sets of PCU values should be used for vehicle arrival rate (PCU at mid-block) and saturation flow (PCU at intersection) calculation. The model developed in this study should be validated from a completely different set of external observations to understand the field applicability of the model. Besides Webster's delay model, other delay models should also be modified under the road traffic condition of Bangladesh to estimate delays for oversaturated conditions.

References

- Akcelik, R. (1981), "Traffic Signals: Capacity and Timing Analysis", Research Report ARR No. 123, Australian Road Research Board, Vermont South, Victoria, Australia.
- Akcelik, R.; Besley, M. (2001), "Acceleration and Deceleration Models", Proceedings, 23rd Conference of Australian Institutes of Transport Research, Monash University, Melbourne, Victoria, Australia.
- Arasan, V.T.; Kashani, S.H. (2003), "Modeling Platoon Dispersal Pattern of Heterogeneous Road Traffic", Transportation Research Record 1852, TRB, National Research Council, Washington DC, USA, pp. 175-182.
- Hoque, M.S. (1994), "The Modelling of Signalised Intersections in Developing Countries", Ph.D. Thesis, University of Southampton, UK.

- Imran, M.A. (2006), "Traffic Signal Design and Modification of Webster's Delay Formula under Non-Lane Based Heterogeneous Road Traffic Condition", B.Sc. Thesis, Bangladesh University of Engineering and Technology, Dhaka, Bangladesh.
- Maini, P., Khan, S. (2002), "Discharge Characteristics of Heterogeneous Traffic at Signalized Intersections", Proceedings, 4th International Symposium on Highway Capacity, pp. 258-270.
- Miller, A.J. (1963), Setting for Fixed-Cycle Traffic Signals, *Operational Research Quarterly*, 14, 373-386.
- Teply, S., Allingham, D. I.; Richardson, D. B.; Stephenson, B. W. (1995), "Canadian Capacity Guide for Signalized Intersections", Institute of Transportation Engineers, District 7, 2nd Edition, Canada.
- TRB (Transportation Research Board) (1985), "Highway Capacity Manual", Special Report 209, National Research Council, Washington, DC, USA.
- TRB (Transportation Research Board) (1994), "Highway Capacity Manual", Special Report 209, National Research Council, Washington, DC, USA.
- TRB (Transportation Research Board) (2000), "Highway Capacity Manual", Special Report 209, National Research Council, Washington, DC, USA.
- Webster, F.V. (1958), "Traffic Signal Settings", Department of Scientific and Industrial Research, Road Research Technical Paper No. 39, Her Majesty's Stationary Office, London, England.