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A linear cost minimization model for water supply systems with constrained sources

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Abstract

Municipal water supply systems primarily depend on surface and groundwater sources to meet the demand. Water quantity and quality limitations of the sources often impose economic constraints on system operation requiring additional treatment cost including more expensive alternative sources in the system. It increases the cost of water production. Selection of the appropriate water-sources to minimize production cost is a challenging task when the system depends on multiple sources having different attributes and cost coefficients. This paper presents a linear cost minimization model for such a multiple-source groundwater-based water supply system. The model decides on the optimum production amount from each source with the objective of cost minimization for a specified set of demand and source constraints. The model would be useful for system analysis, planning and management purposes such as, analyzing water production at various levels of system loss or unaccounted-for water (UFW), or determining optimal production schedule under different system operation scenarios. The model is applied to simulate a groundwater-based sub-network of Dhaka city water supply system where seasonal demand from the system is the highest, when the groundwater level is relatively low, and water production cost varies with the operation mode of the deep tube wells (DTWs). Model results show that significant cost reduction would be possible in different operational scenarios through optimal production scheduling at various UFW levels while ensuring a minimum supply to the local coverage areas of the DTWs.

Keywords: Linear programming, cost minimization model, unaccounted-for water, water supply, Dhaka city.

1. Introduction

Management of urban water supply systems becomes difficult if there is a shortfall in water availability from the sources constrained by water quantity or quality limitations.

In groundwater-based systems, the production wells cannot operate, especially during the dry season, when the groundwater level falls below a certain limit. This makes supply management more challenging since domestic water demand from the system is the highest during the dry season. To meet the demand in these constrained situations, operable production wells are run for longer hours at an increased cost of production. Since the unit operation cost is different for each well, contribution of each well to the overall cost increase is different. Therefore the increase in production cost can be minimized by operating the wells having lower unit cost of production more than those with higher unit cost. Cost optimized water production and supply systems have been developed in many cities such as Memphis (Pezeshk et al., 1994) and Pittsburgh (Nitivattananom et al., 1996). A similar approach using an iterative dynamic programming method has been followed to design the water supply system in other cities of USA (Zessler and Shamir, 1989). This paper presents a linear optimization model that determines the optimum combination of deep tube wells (DTWs) and their operating schedule to minimize the cost of water production. The model can be also used to reschedule well operation in case of breakdown or new installation of DTWs, thereby helping in planning and decision making for system management.

The present model has been applied to a selected groundwater-based sub-network of the water supply system of Dhaka city, Bangladesh. The overall loss from the system as 'unaccounted-for water' (UFW) is estimated to be approximately 50%. Dhaka Water Supply and Sewerage Authority (DWASA) has adopted strategies to reduce the UFW gradually to 28% from its present level (Haq, 2006). The model determines cost-minimized combinations of DTWs at different levels of UFW or system loss. The model is also used to analyze optimal conditions in two scenarios of system maintenance and expansion.

2. Model description

The aim of the model is to specify the production amount of each DTW to fully satisfy the demand by operating the wells with lower unit cost as much as possible. However, a minimum amount of water is produced by each well for its local coverage area irrespective of the unit production cost. Therefore, the model is formulated for a cost minimization problem rather than a benefit maximization problem. The main features of the model are described below.

(a) Decision variable: The monthly water production from each DTW is the decision variable in the model. Billing records and other data regarding the water supply system are usually compiled on a monthly basis. A computational time step of one month is assumed to be reasonable for system planning and management.

(b) Cost coefficient: The monthly water production cost consists of fixed cost and variable cost. Fixed cost includes salary of well operators, and cost of operation and maintenance. Variable cost includes cost of direct electricity consumption, fuel used for electricity generators, and bleaching powder and chlorine gas used for water purification. Since the fixed cost does not vary with water production amount, only variable cost is considered for calculating the cost coefficient.

(c) Objective function: Since meeting the water demand is a priority in the model, the economic objective is cost minimization rather than profit maximization or minimizing the hours of operation. Thus the objective function minimizes the total cost of water production, i.e.

$$\text{Minimize} \sum_{i=1}^{m} C_i X_i \qquad (1)$$

where, C_i = cost coefficient (Taka/m^3) of DTW_i which produces X_i amount of water (m^3/month), and m = total number of DTWs in the system.

(d) Constraints:

(i) Demand constraint: Since water demand varies with time, monthly water production requirement is guided by the demand. To satisfy the total demand at any time, total water production must be more than or equal to the total water demand, i.e.

$$\sum_{i=1}^{m} X_i \geq D \qquad (2)$$

where D = total water demand in a given month (m^3/month).

(ii) DTW capacity constraint: Each DTW cannot produce more than its specified capacity. Also, the DTW cannot operate if the static water level falls below a certain level. However, DWASA does not monitor the static water level at the DTW locations. The general practice of DWASA operators is to close a DTW when production falls below 1,000 liter/minute in response to the lowering of the static water level. This constraint is indirectly considered by setting a minimum production requirement equal to 1,000 liter/minute. Thus,

$$X_i \leq Y_i\, Cp_i \qquad (3)$$

and DTW_i is stopped if production from the well is less than a specified amount, i.e.

$$X_L Y_i - X_i \leq 0 \qquad (4)$$

where, Cp_i = production capacity of DTW_i, Y_i = binary logical variable = 0 or 1, and X_L = 1,000 liter/minute. Y_i = 0 means DTW_i does not operate and 1 means otherwise. If water production falls below a specified amount (X_i < 1,000 liter/minute), Eq. (4) forces Y_i to take up a value of 0 (DTW_i does not operate). Additionally, $X_i \geq 0$, $\sum Y_i \leq m$, $Y_i \geq 0$, $Y_i \leq 1$, and Y_i = integer. Eq. (3) limits the maximum water production from DTW_i within the production capacity (Cp_i) if the well operates ($Y_i = 1$). If DTW_i does not operate ($Y_i = 0$), Eq. (3) forces the corresponding production (X_i) to be equal to 0.

To ensure a minimum supply from each DTW to its local coverage area, the well should not remain inoperative for a long time. So each DTW, even if its unit production cost is relatively high, should produce a minimum amount of water to meet the local demand. Considering the normal practice of the DWASA operators and assuming 8 to 10 hours of operation per day, the minimum monthly water production from each DTW is equivalent to 40% of its water production capacity. Thus,

$$X_i \geq X_{min} \qquad (5)$$

where, $X_{min} = 0.4\, Cp_i$. Eq. (4) may become redundant to Eq. (5) if Cp_i is relatively high.

It is assumed that pressure within the system is uniform and no flow variation occurs due to pressure difference. Since observed water pressure in the DWASA network is unavailable, the effect of head loss was not included in the model.

3. Case study: Dhaka city

The water supply system in Dhaka city managed by DWASA depends mostly on the DTWs in 6 'Maintenance, Operation, Distribution and Service' (MODS) zones of the city (Haq, 2002). The water supply system in Uttara thana, part of MODS zone 5, was selected for model application (Fig. 1). At an estimated growth rate of 2.01% (in 1991), total population in Uttara thana was 131,875 in 2001 (BBS, 2005). The study area is relatively small and less complex in terms of population characteristics and land use pattern. The selected water supply sub-network runs on 15 DTWs and serves an area of 36.92 km^2. The sub-network is partially isolated from the main DWASA network having only one connection with the main network through a booster pump and a valve, which are operated only in case of severe water scarcity to allow limited flow to the Uttara sub-network. For this study, it is assumed that exchange of water between the selected sub-network and the main DWASA network through this connection is insignificant.

Cost coefficients and constraints were determined based on primary data from the field and secondary data from DWASA records and documents. The production costs for optimal pumping schedule at different levels of system loss have been determined by the model. The model was also applied to analyze two probable scenarios: one representing temporary DTW breakdown and another for additional DTW installation. The model was run using the LINDO software (Schrage, 1991). Details on the methodology and model operation are given by Shah (2006).

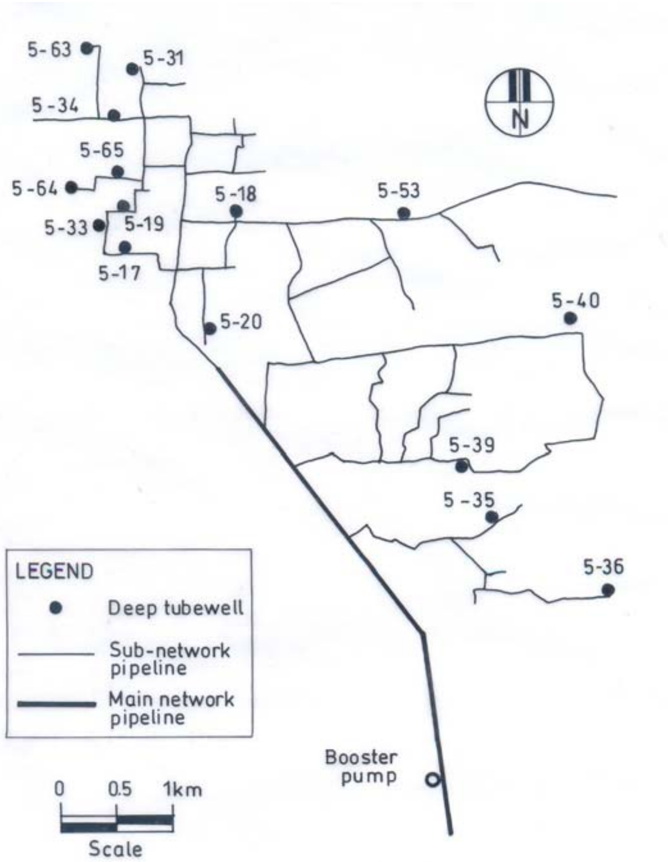

Fig. 1. Uttara water supply sub-network and deep tube well locations

3.1 Estimation of consumption, cost coefficient and capacity

The model was applied with a complete set of data that could be gathered for one financial year: July 2001 to June 2002. Monthly water consumptions at 50%, 35%, 31% and 28% system loss levels were estimated from the water billing records of Uttara (see Table 1). These system loss levels represent the intermediate targets of the long-term plan of DWASA to gradually reduce the UFW. Unit water production cost of the DTWs varies with the mode of operation (electricity or diesel) and water purification process. The unit production cost (cost coefficient) for each DTW was estimated from the records of related cost items. The coefficient varies from 0.57 to 2.41 Taka/m^3 for the 15 DTWs. From available records, the average production capacities of the DTWs were found to vary between 1.20×10^5 and 1.64×10^5 m^3/month.

Table 1
Monthly water consumption in Uttara estimated from billing records of DWASA

Year	Month	Total water consumption (m^3/month)	Water consumption (m^3 /month) at a given level of system loss (%)			
			50%	35%	31%	28%
2001	July	450,652	901,304	693,311	653,119	625,906
	August	569,227	1,138,455	875,735	824,968	790,594
	September	681,625	1,363,250	1,048,654	987,863	946,702
	October	694,021	1,388,043	1,067,726	1,005,829	963,919
	November	735,283	1,470,566	1,131,205	1,065,628	1,021,227
	December	661,770	1,323,540	1,018,108	959,087	919,125
2002	January	696,424	1,392,848	1,071,422	1,009,311	967,256
	February	702,501	1,405,002	1,080,771	1,018,118	975,696
	March	741,297	1,482,594	1,140,457	1,074,344	1,029,579
	April	739,504	1,479,009	1,137,700	1,071,746	1,027,090
	May	783,934	1,567,869	1,206,053	1,136,137	1,088,798
	June	810,334	1,620,668	1,246,668	1,174,398	1,125,464

3.2 Optimum water production

Fig. 2 shows a comparison of monthly model-optimum results with actual production records where the dotted lines are the trend lines fitted to the data for each system loss level. The nearly-horizontal lines indicate that the optimum production requirement is highly independent of the actual production. Since DWASA does not have a specific guideline for DTW operation, all wells are operated for longer hours when the demand increases. This arbitrary operation of DTW causes this deviation from the optimum requirement. Fig. 3 shows that the recorded water production is generally higher than the optimum water production at different levels of system loss. Total water production capacity increased in August/01 and January/02 due to inclusion of new DTWs in the system. The recorded water production exceeded the total capacity in November/01 and June/02. This might have occurred because the capacity of a DTW was estimated from the present production rate whereas the actual capacity of the DTW could be higher in the past, allowing higher water production. Optimum water production amounts show a generally increasing trend because of increasing demand in successive months. However, in a given month, the optimum water production amount is lower at a lower level of loss.

Fig. 3 shows that the actual water production can be reduced significantly if the DTWs are operated at an optimum condition at a given level of system loss. The reduction in water production would be 4-59% of the recorded water production during July/01-June/02. This optimal operation of DTWs would have saved DWASA approximately 3 to 57 % of total water production cost (Taka 0.46×10^5 to 9.30×10^5) per month for the selected sub-network in 2001-02.

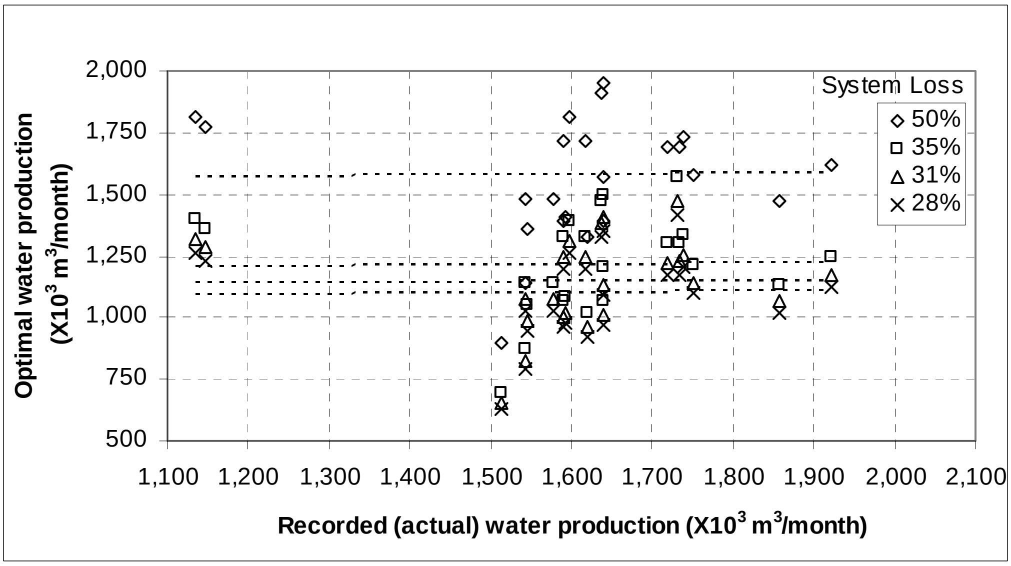

Fig. 2. Optimum production requirement and recorded production

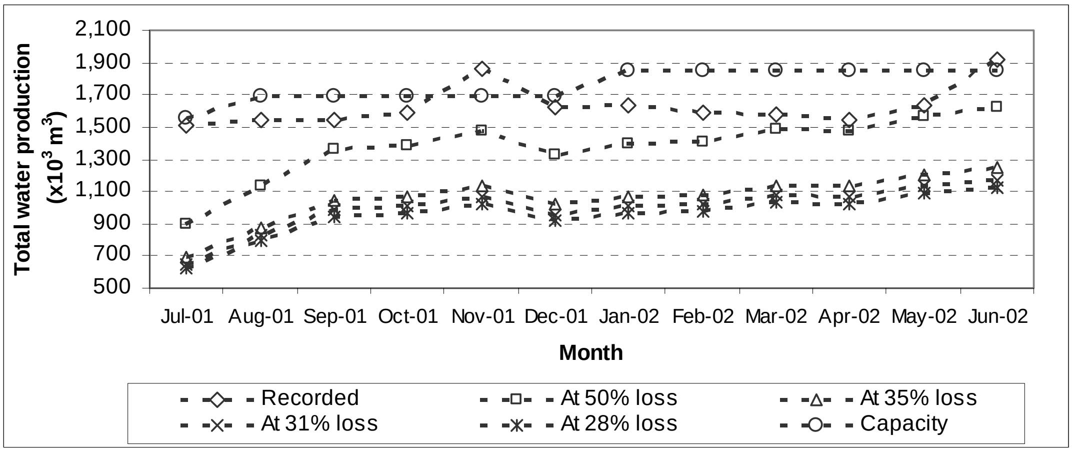

Fig. 3. Recorded and optimum water production during July/01-June/02

3.3 *Scheduling of DTWs for optimal operation*

In an ideal cost-minimized water production system DTWs with lower cost coefficients would be operated more to meet the specified demand. Consequently, DTWs with higher cost coefficients would be operated less, or may even remain idle for a period of time. In practical situations, however, all DTWs have to run everyday to ensure a minimum supply to their local coverage areas. Only operating hours may vary for cost minimization. If this constraint, given by Eq. (5), is not included in the model, optimal results show that some DTWs are not required to operate to meet the specified demand. Also, a lower number of DTWs are required to be operated at an optimum condition for a lower system loss. For example, in January/02, 10 DTWs would be required to operate at an optimum condition, whereas 13 DTWs are required in actual condition.

After including the minimum water production constraint, given by Eq. (5), in the model, optimal results show that the DTWs having higher cost coefficients produce the minimum amount of water (X_{min}) and the DTWs having lower cost coefficients meet the rest of the demand. However, the optimum water production from each DTW varies with the level of system loss and water demand. Table 2 shows the DTWs that produce the minimum amount of water at an optimum condition at 50% system loss during July/01-June/02. The shaded cells indicate that the corresponding DTWs would be operated for minimum water production during the specified month. A dash indicates that the corresponding DTW was not in service during the month, which is represented in the model by setting Xi = 0 for the well. Similar DTW operating schedules can be produced by the model for other levels of system loss.

3.4 Cost of water production

Model results indicate that the optimal cost of water production is lower than the recorded cost (Fig. 4). During actual operation of DWASA, all functioning DTWs are operated, whereas optimal operation requires that DTWs with higher cost coefficients produce the minimum amount of water resulting in a reduction in cost. Also, the total cost of water production would decrease if the system loss decreases. Therefore, the cost of water production by DWASA can be significantly reduced by an optimal operation of the DTWs. For example, up to 57% of the actual cost could be reduced at 28% loss level during July/01-June/02. The percent reduction in cost decreases with an increase in system loss.

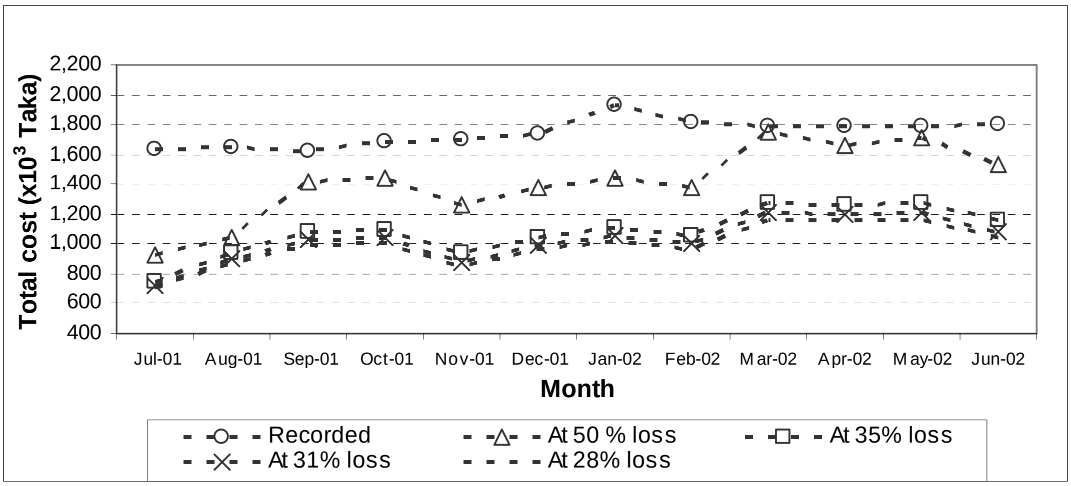

Fig. 4. Cost of water production in recorded and optimal conditions during July/01- June/02

3.5 Model application to probable scenarios

Two probable scenarios were analyzed with the model. Scenario 1 represents a situation where the system capacity is reduced because of inoperable (out-of-service) DTWs. Two DTWs: No. 5-18 (which had an electric generator run by diesel) and No. 5-19 (which runs on direct electric supply only) are excluded. Scenario 2 is a situation where two new DTWs were installed in the system. The scenarios were analyzed for the conditions in August/02. The results are presented as follows where the present optimum results are the 'existing' conditions.

Table 2
DTWs that produce the minimum amount of water at 50% system loss

	Month	Well Number															No. of DTWs in minm. production	Total optimum water production ($x10^3$ m^3)
		5-17	5-18	5-19	5-20	5-33	5-31	5-34	5-35	5-36	5-39	5-40	5-53	5-63	5-64	5-65		
	July		▦	▦	▦	▦	▦		▦		▦		-	-	-	-	7	901
	August		▦	▦	▦	▦					▦		▦	-	-	-	6	1139
2001	September			▦							▦		▦	-	-	-	3	1363
	October			▦							▦		▦	-	-	-	3	1388
	November	▦									▦			-	-	-	2	1471
	December	▦					▦				▦	▦		-	-	-	4	1324
	January	▦			▦		▦				▦	▦			-	-	5	1393
	February		▦		▦	▦					▦		▦		-	-	5	1405
2002	March	▦					▦				▦		▦		-	-	4	1483
	April	▦					▦				▦		▦		-	-	4	1479
	May	▦					▦				▦				-	-	3	1568
	June	▦			▦										-	-	2	1621

▦ DTWs to be operated for minimum production

(a) Scenario 1: Production capacity reduced due to inoperable DTWs

In Scenario 1, the system is unable to supply the required amount of water if the system loss is 50%. Only about 92% of water demand can be satisfied in August/02 at 50% system loss. The total optimum cost would increase by 0.71% to 5.58% at different loss levels, and the cost would decrease with a decrease in loss level (Fig. 5). The optimum cost would increase since the excluded (inoperable) DTWs have relatively low cost coefficients, and the DTWs having higher cost coefficients would have to operate more to meet the water demand. The optimal DTW operation schedule in Scenario 1 would be different than the existing optimal schedule. Also, the total number of DTWs operated for minimum water production in Scenario 1 would be less than that in existing optimal condition.

(b) Scenario 2: Production capacity increased by additional DTWs

Although the new DTWs have relatively low cost coefficients, model results show that in Scenario 2 the optimum cost would decrease by 4% at 50% system loss only. At other loss levels, the optimum cost would either remain about the same or increase slightly (by about 0.4%) from the existing optimum cost (Fig. 6). Similar to Scenario 1, the optimal

DTW operation schedule would be slightly different in Scenario 2 than that in existing optimal condition. Also, the number of DTWs operated for minimum water production would increase at all system loss levels. Fig. 7 shows the % change in optimum cost in the two scenarios. Similar analyses can be performed for other scenarios for planning and system management.

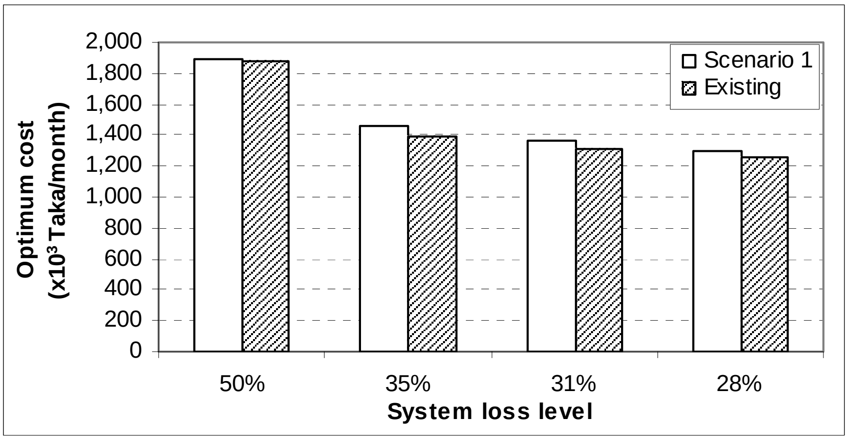

Fig. 5. Optimum cost in August/02 for a reduced number of DTWs

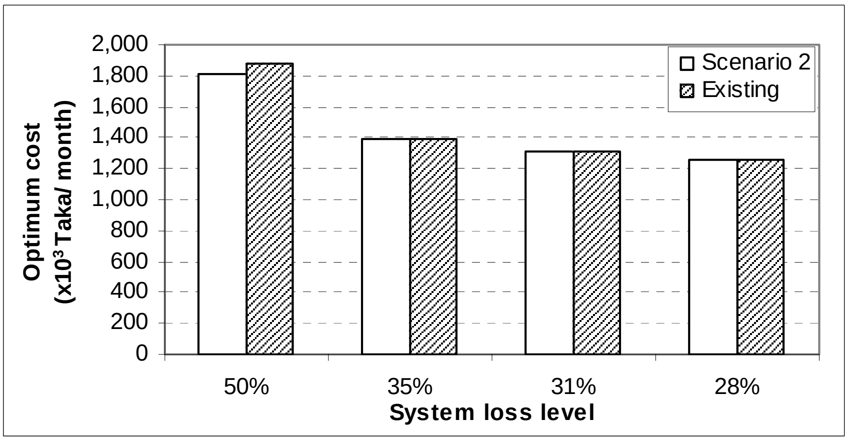

Fig. 6. Optimum cost in August/02 for an increased number of DTWs

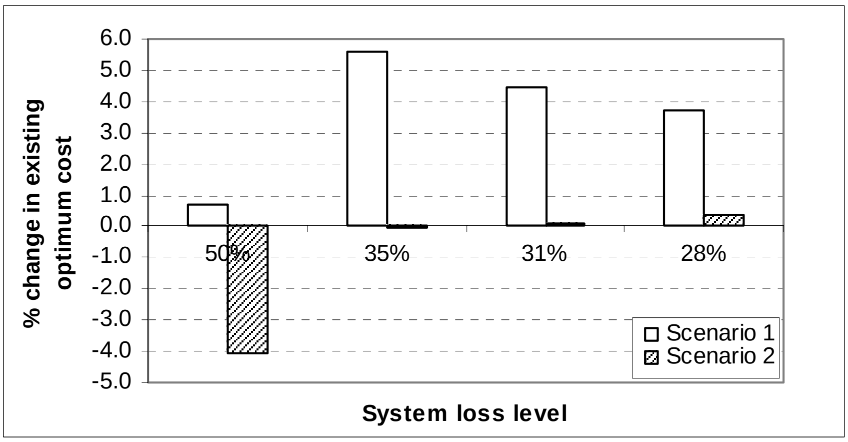

Fig. 7. Change in optimum cost in two scenarios in August/02

4. Conclusion

Management of groundwater-based water supply systems is challenging if the water availability is limited by constraints at the sources. A linear programming approach has been presented for minimization of water production cost, while ensuring a minimum water supply to the local coverage areas of DTWs. The present model was applied to a selected DTW-based sub-network of DWASA. Monthly water consumptions at four system loss levels are estimated from the billing records. Cost coefficients of water production are found to vary from 0.57 to 2.41 Taka/m^3. At optimal conditions, the water production requirements from the DTWs would be significantly reduced. For example, the reduction in water production would be 4-59% of the recorded water production during July/01-June/02 at system loss levels between 50% and 28%.

Two probable scenarios of reduction or extension of water production capacity of the sub-network were analyzed with the model. Scenario 1 represents a situation, where the system capacity is reduced because of inoperable DTWs. In Scenario 2, new DTWs were installed in the system. Model results show that the optimum cost would increase by 0.71% to 5.58% at various loss levels in Scenario 1. In Scenario 2, the optimum cost would decrease by 4% at 50% system loss, while at other loss levels the optimum cost would either remain about the same or increase slightly. In both scenarios the optimal DTW operation schedules differ from the existing optimal schedule.

The present optimization model can be used for decision-making in system operation and management of a groundwater-based water supply system. Primarily, the model can be used to decide on optimal operation schedule of DTWs, and analysis of technical crisis or future network expansion situations. However, before actual application, the present model would need improvement to incorporate the effect of head loss in the water distribution network.

Acknowledgement
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