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Macro and micro mechanical responses of granular material under varying interparticle friction

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Abstract

Numerical study is carried out to investigate the macro and micro mechanical responses of granular materials due to the variation of interparticle friction in the assembly with different initial porosity using distinct element method (DEM). Assembly with different interparticle friction is prepared isotropically and then subjected to biaxial compression test under strain controlled condition maintaining a constant confining pressure. The data obtained from biaxial compression test are analyzed and the results are reported focusing the effect of interparticle friction on the macro as well as micro mechanical responses of granular system.

Keywords: Interparticle friction, distinct element method, dilatancy

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1. Introduction

It is well known that particle to particle friction has important role in a particulate system to resist shear. But experimentally, it is difficult to explore the role of interparticle (particle to particle) friction in a particulate system due to the limitation of experimental facilities at present time. For this reason, there are very few experimental studies available in the literature focusing the effect of interparticle friction in the particulate system. One of such study was performed experimentally by Skinner (1969) by shearing a random assembly of spherical particles and reported that both the effective angle of shearing resistance at constant volume and at peak for a given initial porosity do not increase monotonically with increase in the interparticle friction. However, micro mechanical responses with the variation of interparticle friction were not studied in this experimental work since it is quiet difficult to explore micro features with the laboratory experiment. Alternatively, numerical technique like DEM can be used to investigate the effect of interparticle friction on the mechanical responses specially the micro responses of granular media. Earlier studies regarding the effect of interparticle friction using DEM mainly focus the macro mechanical responses in dense system (c.f. Oger et al. 1998, Thornton 2000). However, these macro responses by numerical studies differ from authors to authors. For example, Hu and Molinari (2004) reported that when the coefficient of friction is smaller than 0.50, there is no obvious variation in the stress ratio but when the coefficient of friction is equal to and larger than 0.55, there is a sudden increase of the stress ratio in its decline stage while the results of Powrie et al. (2005) showed that the shear strength gradually increases with the increase of interparticle friction. This suggests that more study is necessary. However, the micro mechanical responses are not studied in details in most of the early studies. But a comprehensive study is necessary to understand the micro mechanical responses of a granular system due to the variation of interparticle friction in the assembly with different initial porosity. So the major aim of this contribution is to study the micro mechanical features of granular system due to the variation of interparticle friction using DEM in the assembly with different initial porosity. In addition, macro mechanical responses specially the effect of interparticle friction on the mobilized angle of friction at residual state for assembly with different initial porosity is investigated and compared using the DEM.

2. Distinct element method

DEM was first introduced by P.A. Cundall (1971) for rock mass problem and later extended for granular materials (Cundall and Strack, 1979). The detail methodology is available in the literature (Cundall and Strack 1979, Rothenburg and Bathurst 1989). The method is based on the assumption that time step taken should be chosen very small so that during a single time step disturbances can not propagate disc further than its immediate neighbour. Newton's second law of motion is used to calculate the motion of the disc resulting from the forces acting on it. The acceleration is integrated over small time step to calculate velocity and the velocity is again integrated over small time step to get the displacement. The force-displacement law is used to find the contact force from displacement and thus the calculation cycle continues.

3. Numerical model

In the numerical model, a linear spring-dashpot model is used to describe the contact force between particles. At each contact point, a set of normal and shear springs, dashpots and no-extension joints as well as a shear slider are incorporated. The spring and dashpot give resistance to equilibrate the normal and tangential forces while the shear slider start working when the tangential force is equal to or greater than $\mu f_{n,}$ where μ=coefficient of interparticle friction, f_n=normal contact force. A flexible boundary as used by Iwashita and Oda (2000) is employed in left and right boundaries. The particles in the flexible boundary is linked with each other like chain and overlapped. The boundary behaves like a membrane and is capable to stretch or shrink. The vertical movement of each particle in the linked flexible boundary is restricted to move together with the top and bottom boundaries. But the horizontal movement is not restricted but free to move. 9800 particles with nine different sizes are used to generate the assembly in a rectangular frame following a fairly smooth gradation curve as shown in Figure 1.

4. Assembly preparation and shear deformation

Two groups of assemblies are prepared varying their initial porosity. The first group of assemblies is designated as dense assemblies and the second group of assemblies is designated as relatively loose assemblies. In the first group six assemblies are prepared varying their interparticle friction but maintaining the same porosity. The porosity is kept constant for all the six assemblies after the end of the isotropic compression so that the effect of interparticle friction *(μ)* can be analyzed. To prepare the first assembly of this group, at the first stage, interparticle friction coefficient and friction coefficient between particles in the assembly and the loading platens are set zero so that particles in the assembly can come closer to their best. Then the assembly is subjected to uniform confining pressure of 100 kPa from all the directions with rigid boundaries (a boundary condition where the whole boundary moves together and not flexible) until stress oscillation becomes negligible. In the second stage, the assembly is compressed isotropically using the desired interparticle friction coefficient of 0.10, particle to boundary (loading platens) friction coefficient of 0.36 and flexible boundary with a confining pressure of 100 kPa until stress oscillation is negligible. To prepare the other five assemblies of this group, the same procedure is followed except that in the second stage of compression the interparticle friction is assigned 0.30, 0.50, 0.70, 1.00 and 1.50 respectively. At the end of the isotropic compression, the porosity of the six assemblies of this group with μ=0.10, μ=0.30, μ=0.50, μ=0.70, μ=1.00 and μ=1.50 becomes 0.1545, 0.1547, 0.1547, 0.1547, 0.1547 and 0.1547 respectively. In the shear deformation phase, interparticle friction coefficient is assigned 0.10, 0.30, 0.50, 0.70, 1.00 and 1.50 for assembly with μ=0.10, μ=0.30, μ=0.50, μ=0.70, μ=1.00 and μ=1.50 respectively. The boundary friction is 0.36 for all the assemblies.

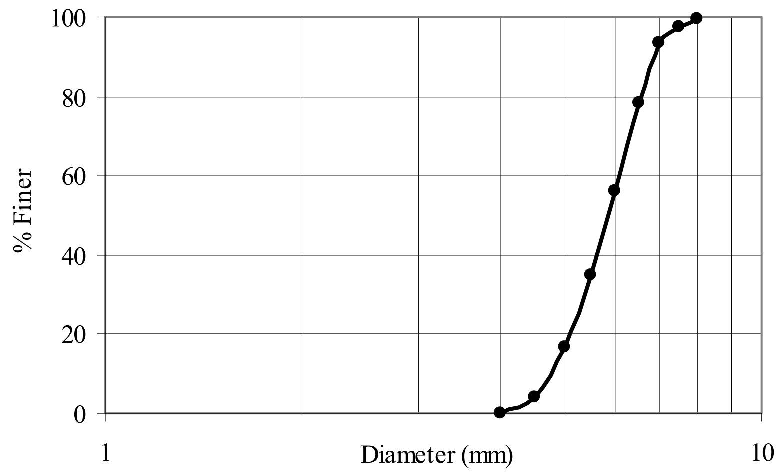


Fig. 1. Grain size distribution of circular particles used in the simulation

To prepare relatively looser assembly, a numerical technique is adopted. At the first stage of isotropic compression, the interparticle friction coefficient is assigned 0.50 tentatively and boundary friction is assigned zero so that the particles have restriction to come closer to each other and thus the assembly have more porosity than the assemblies of first group. The assembly is then subjected to uniform confining pressure of 100 kPa from all the directions with rigid boundaries Then in the second stage of isotropic compression, the assembly is compressed isotropically with the desired interparticle friction coefficient of 0.50, particle to wall (loading platens) friction coefficient of 0.36 and a confining pressure of 100 kPa until the stress oscillation becomes negligible using flexible boundary. The porosity at the end of the isotropic compression becomes 0.1919. In the shear deformation phase, the interparticle friction coefficients are varied assigning interparticle friction coefficient 0.10, 0.50, 1.00 and 1.50. Figure 2 shows a typical granular assembly used in the simulation.

The numerically prepared assemblies are subjected to deviatoric shear deformation through biaxial compression test under strain controlled loading. The top and bottom boundaries move vertically step by step keeping the lateral pressure constant (100 kPa). At each step, cyclic calculation is repeated until convergence is reached. Particles are allowed to rotate freely starting from the assembly preparation to the shear deformation. Digital data like stress, strain, particle position, boundary position, contact details etc. are recorded while the test is in process. The calculation condition and physical constants used in the numerical study are given in Table 1.

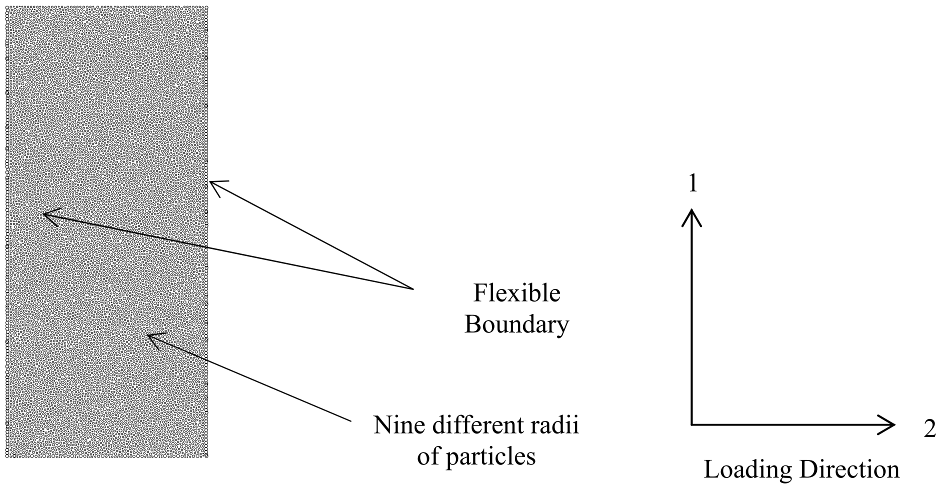


Fig. 2. A typical granular assembly

5. Effect of interparticle friction

5.1 Stress-strain-dilative response

Figure 3 shows the effect of interparticle friction on the stress-strain and dilative response of dense granular assemblies. From the Fig. 3a it is clear that stress ratio increases with the increase of interparticle friction coefficient especially at the peak stress level. However the stress-strain curves have a tendency to merge each other at the residual state. With the increase of interparticle friction the assembly becomes stronger more and more which results in the increase of stress ratio. This numerical result is little bit different from the numerical result by Hu and Molinari (2004) in which they reported that when the coefficient of friction is smaller than 0.50, there is no obvious variation in the stress ratio but when the coefficient of friction is equal to and larger than 0.55, there is a sudden increase of the stress ratio in its decline stage. But the result reported in this study is quite similar with the result obtained by Powrie et al. (2005) and seems to be persuasive. Figure 3b shows the dilative behavior of the dense assembly under various interparticle friction coefficients. The volumetric strain increases with the increases of interparticle friction coefficients. This is probably due to the reason that the increase in interparticle friction makes more strong assembly which shows higher dilative behavior when undergoes shear.

Table 1
Calculation condition and physical constants used in the numerical study

Calculation condition and physical constants	Value
Number of particles	9800
Radii of particles	2 to 4 mm
Number of particle size	9 different size
Incremental time step (Δt)	1.00×10^{-6} s
Particle density	2650 kg/ m^3
Confining pressure	100 Kpa
Interparticle friction coefficient	0.10, 0.30, 0.50 and 0.70
Loading platen friction coefficient	0.00 and 0.36
Normal spring constant (k_n)	1.00×10^8 N/m
Tangential spring constant (k_s)	1.00×10^7 N/m
Cohesion between particles	0.00
Damping constants at contact	0.05

5.2 Mobilized angle of friction

Figure 4(a) and Figure 4(b) shows the effect of the variation of interparticle friction on the mobilized angle of friction for dense (η=0.1547) and relatively loose (η=0.1919) granular assemblies respectively. Average value of mobilized angle of friction is considered at residual state (average value from 7.5% to 8% axial strain).The mobilized angle of friction is calculated based on the following equation $\varphi_{mob} = sin^{\ 1}(\frac{\sigma_1 \quad \sigma_2}{\sigma_1 + \sigma_2})$ where, φ_{mob} is the mobilized angle of friction, σ_1 is the major principal stress and σ_2 is the minor principal stress. In the Figure 4(a), it is observed that the mobilized angle of friction is almost constant for bigger values of interparticle friction coefficient at the residual state. For example, mobilized angle of friction is almost constant for interparticle friction coefficient 0.70 to 1.00 and even for interparticle friction coefficient 0.50; mobilized angle of friction is not so different. But mobilized angle of friction is significantly small for lower value of interparticle friction coefficient. For example, for interparticle friction coefficient 0.10, mobilized angle of friction is lower compare to interparticle friction coefficient 1.00 or 1.50. At peak stress level, the value of mobilized angle of friction is gradually increasing with the increase of interparticle friction coefficient. The rate of increase is small for higher values of interparticle friction coefficient and large for lower values of interparticle friction coefficient. However, the difference in the mobilized angle of friction at peak and residual state for the same interparticle friction increases with the increases of interparticle friction. For example, the difference is 2° when interparticle friction coefficient is 0.1 but 15° when interparticle friction coefficient is 1.5. This indicates that the increase in interparticle friction increases the macroscopic strength of a granular assembly. Thus the numerical investigation shows that interparticle friction has influence on developing the strong assembly in resisting shear and consequently, resulting in the increase in macroscopic strength at peak stress level.

Figure 5 shows the comparative results of the mobilized angle of friction at residual state level due to the variation of initial porosity before shear deformation. Interesting points can be figured out from the Figure 5. Even though the initial porosity is different, the magnitude of the mobilized angle of friction is almost same at residual state. Moreover, the tendency of both the curves is same. The mobilized angle of friction is almost constant for bigger values of interparticle friction although the initial porosity is different. A careful examination of Figure 5 shows that the values of the mobilized angle of friction are small as compared to the given angle of interparticle friction. For example, when the assigned value of interparticle friction coefficient is 1.0 (angle≈45°), the corresponding mobilized angle of friction in dense system is nearly 19°.

The reason behind is that, in the present study, circular shape particle is used to model the particulate media. But the shape of natural granular material like sand is not circular but non-circular or angular. This noncircular shape of particles gives resistance to rotation inherently while in this model, the circular particles rotates freely. So, after peak the stress falls rapidly due to excessive rotation of circular particles. The use of non circular particles in modeling granular media may overcome this problem.

5.3 Sliding contact fraction

Figure 6a shows the effect of interparticle friction on the sliding contact fraction at residual state (6% axial strain) in dense system (first group). The sliding contact fraction, S_f is defined as the ratio of the number of sliding contact to the total number of contact at any given time step. For convenience the sliding contact fraction is expressed in percentage. From the figure, it is observed that with the increase of the interparticle friction, sliding contact fraction decreases. At zero interparticle friction, the sliding contact fraction is nearly 100% (Figure 6(a)). It indicates that when deviatoric shear is given to a dense system of discs with zero interparticle friction, the number of sliding contact is almost same as the number of total contact. But for interparticle friction coefficient 1.50, the sliding contact fraction is about 2% (Figure 6(a)). The decrease of sliding contact fraction is the indication of the increase of macroscopic strength of the system. This decrease of sliding contact fraction is rapid up to μ=0.50 and after μ=0.50, this difference is not so significant. This tendency indicates that the change in the development of macroscopic strength is significant for lower value of interparticle friction. Thus, when the interparticle friction is small, the assembly is also macroscopically less strong and this may probably be the reason why the mobilized angle of friction with lower value of interparticle friction is small as compared to that with higher value of interparticle friction. Figure 6(b) shows the variation in sliding contact fraction due to the variation of the interparticle friction in relatively loose assembly at residual state (6% axial strain). Similar behavior is observed in the relatively loose assembly.

5.4 Distribution of highly rotated particles

Influence of the variation of interparticle friction on the rotation of particles is investigated. Figure 7 shows the influence of interparticle friction on the number and distribution of highly rotated particles in clockwise and anticlockwise direction at residual state (6% axial strain). The particles which have rotation more than 20° both in clockwise and anticlockwise direction is considered and others are omitted. The non-filled black circles indicate particles having anticlockwise rotation while the filled black circles indicate particles having clockwise rotation. From the figure, it is observed that the number of highly rotated particles both in clockwise and anticlockwise direction increases with the increases of interparticle friction. With μ=0.10, highly rotated particles is not clearly concentrated across any line, with μ=0.50, highly rotated particles is clearly concentrated across cross line while with μ=1.50, highly rotated particles is clearly concentrated across a single line. So the change in interparticle friction has significant influence on the concentration of highly rotated particles and consequently in the formation of shear band (if shear band zone is defined on the basis of highly rotated particles, Iwashita and Oda, 1998; Powrie et al. 2005). The increase in the interparticle friction restricts the individual particles to slide which lead the particles to roll so as to balance the energy in the system.

In the relatively loose system (second group), highly rotated particles are not found to concentrate along any line or cross line as is shown in Figure 7 but discretely distributed in the whole assembly even though the interparticle friction is different.

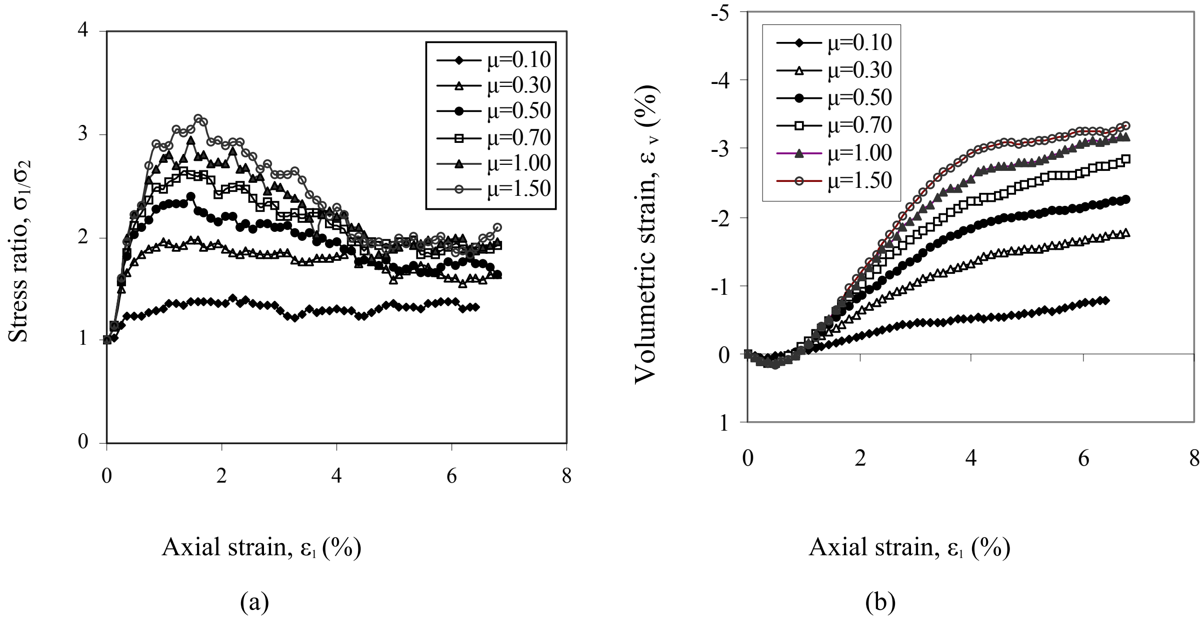


Fig. 3. (a) Stress-strain response; (b) volumetric strain response of dense granular assembly (first group) under various interparticle friction coefficients

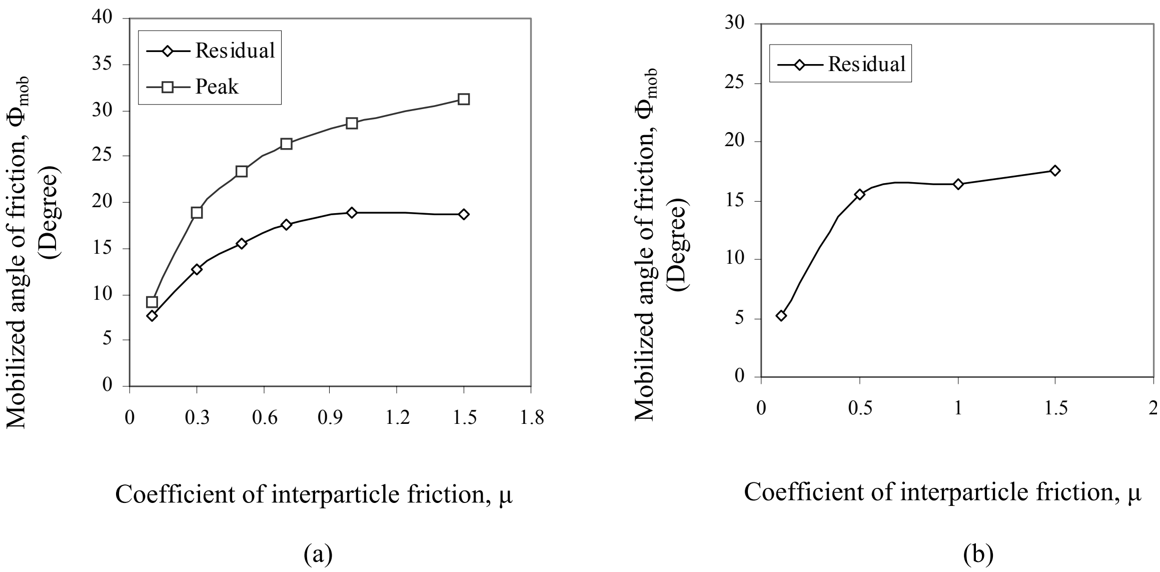


Fig. 4. Influence of the variation of interparticle friction on the mobilized angle of friction (a) For dense assemblies (first group); (b) For relatively loose assemblies (second group)

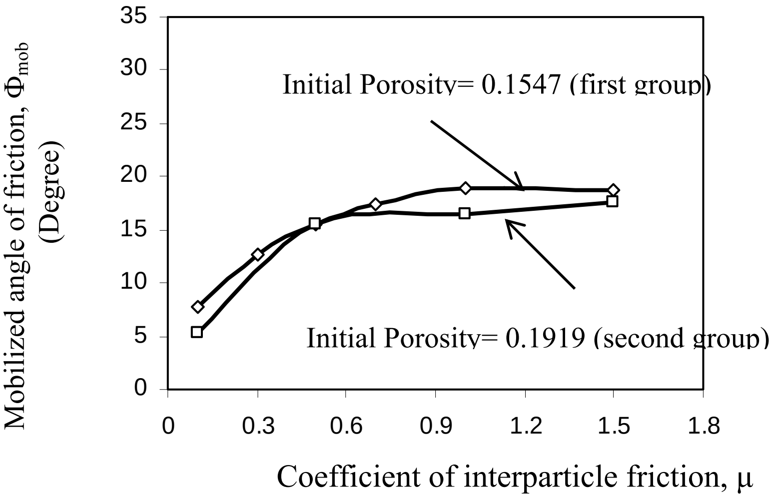
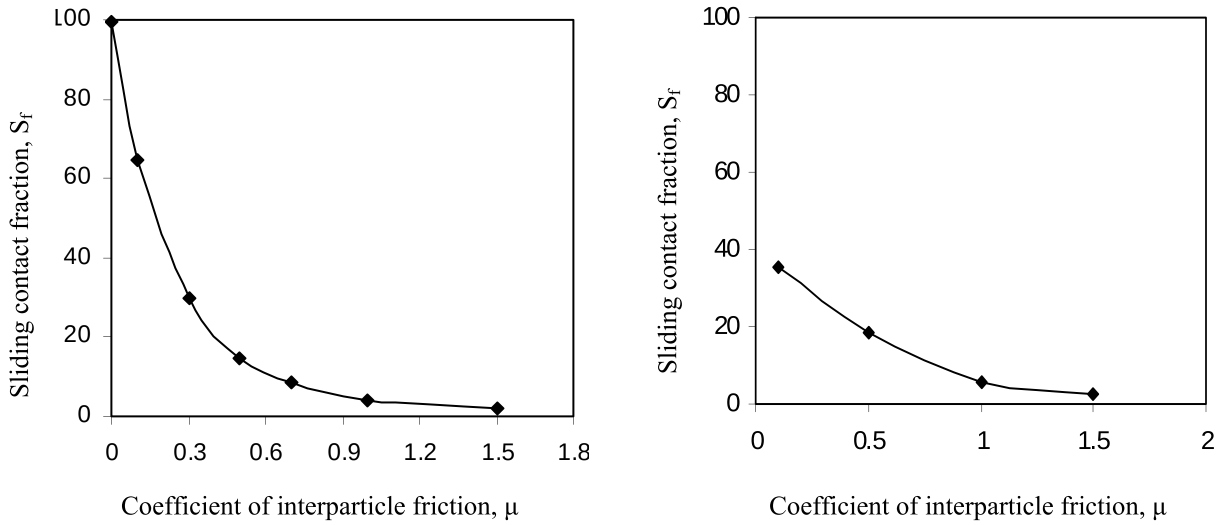


Fig. 5. Comparison of mobilized angle of friction at residual state due to the variation of initial porosity



(a) (b)

Fig. 6. Influence of the variation of interparticle friction on sliding contact fraction at residual state in (a) dense system; (b) relatively loose system

6. Conclusions

Numerical study is conducted on the assembly of disc shape particle under strain control loading to understand the influence of the variation of interparticle friction on the macro as well as micro mechanical responses in two group of assemblies (dense and relatively loose). The following points can be summarized as the important points of this numerical study:

- Mobilized angle of friction at residual state is almost constant for bigger values of interparticle friction but varies for smaller values of interparticle friction for granular assembly of disc. The change in the assembly density does not alter this behavior. However, the value of mobilized angle of friction is small as compared to granular materials like sand. Real shape particles with 3D analysis are suggested to increase the value of mobilized angle of friction.

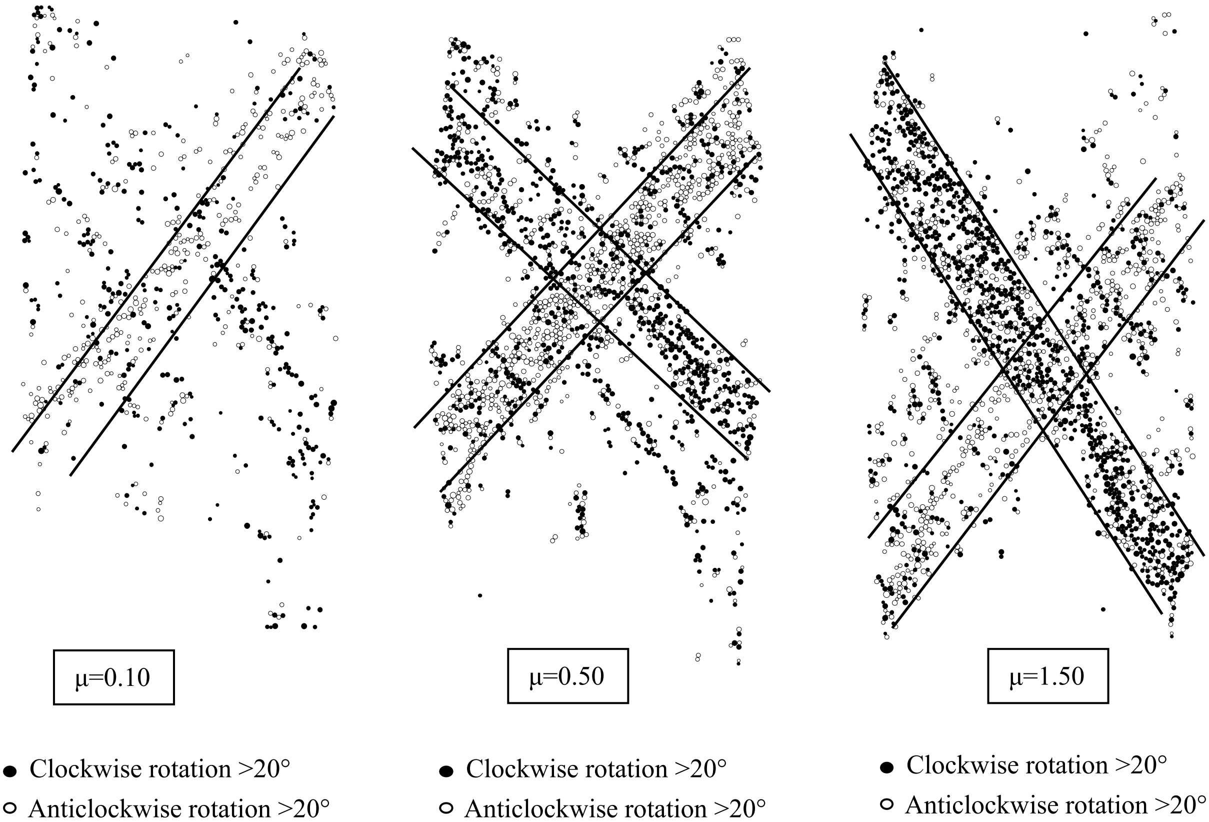


Fig. 7. Influence of the variation of interparticle friction on the distribution of highly rotated particles in dense granular system (first group).

- The magnitude of the mobilized angle of friction is almost same at residual state even though the initial porosity is different.
- With the increase of the interparticle friction, sliding contact fraction decreases although the initial porosity is different.
- The interparticle friction has significant influence on stress-strain-dilative response. Higher the value of interparticle friction, larger the macroscopic strength and dilation of the assembly becomes.
- Interparticle friction has significant role on the number and distribution of highly rotated particles in dense granular assemblies.

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