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Optimum shape in brick masonry arches under dynamic loads by cellular automata

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Abstract

The main goal of this research is determination of optimum shape brick masonry arches under dynamic loads by cellular automata. In this paper, samples of semi-circular, obtuse angel, four-centered pointed, Tudor, ogee, equilateral, catenaries, lancet and four-centered arches are modeled. Then they are analyzed and optimized under acceleration–time components of Elcentro earthquake. For arch response optimization, the results were used in cellular automata computational model. Then using provided rules for modeling, the mentioned arches are analyzed and optimized. The results of error range and time of analysis in automata cellular model and FEM software compared. Finally comparing the results of CA (Cellular Automata) method and FEM (Finite Element Method) method, shows that although precision is less in CA method, but the time of analysis and optimization is so much smaller in it.

Keywords: optimum shape, arch, brick masonry, dynamic load, tensile stress, cellular automata.

1. Introduction

Traditionally, an arch is defined as a part of circle or bow, but our particular definition of arch is as follows: it is a curve surface for covering that its span is higher than its depth (Heyman, 1982). Brick masonry arches have been used to span covering of considerable length in many different applications. Structural efficiency is attributed to the curvature of the arch, which transfers vertical loads laterally along the arch to the abutments at each end (Blasi and Foraboschi ,1990). Transferring of vertical forces gives a rise to both horizontal and vertical reactions at the abutments. The curvature of the arch and its restraint by the abutments cause a combination of flexural stress and axial compression in it. The depth of arch also its rise and configuration can be manipulated to keep stresses primarily compressive and because the brick masonry is very strong in compression, so brick masonry arches can support considerable load (Brickwork, 1989).

Regarding the importance and application of arches in traditional structures, arches optimization has been considered (Huerta, 2001). There has been some research on brick masonry under dynamic loads (Kumarci et al, 2008). Dynamic or time history analysis is an analytical method for determining reflections during the earthquake in structures. Through this analysis, response of structure under loadings which are related to time has been studied (Hughes, 1987). Dynamic analysis and optimization of arches need to consume a long time; it is necessary to use a proper computational model such as cellular automata to analyze and optimize the arches in less time and also for more acceptable results .Cellular automata is a decentralized computation model. It is a good method for computation and simulation of complicated behaviors by local data (Wolfram, 2002). The present research goals are modeling, analyzing and optimizing complicated behaviors of semi-circular, obtuse angel, four- centered pointed, Tudor, ogee, equilateral, catenaries, lancet and four-centered arches, under dynamic load using cellular automata. The main importance of this research is showing the ability of analyzing and optimizing of every arch after one time of modeling in a so much shorter time.

2. Modeling, analyzing and optimizing arch shape using FEM software

At the first step arch modeling has been conducted by FEM software. Furthermore, dynamic analysis has been conducted applying north-south horizontal accelerations of Elcentro earthquake in which the time, maximum acceleration, maximum velocity and maximum displacement are 31.98(s), 0.31(g), 33 (cm/sec) and 21.4 (cm), respectively (Fig.1) and SOLID65 is used for analysis in this stage. Arch shape optimization emphasized on the minimizing of arch weight. In FEM software, the base and top thickness, maximum tensile stress and weight of structure have been defined as design variable, state variable and objective function, respectively. For example, optimum shape of semicircular arch in FEM software has been shown (Fig.2). Regarding the extra time for analysis and optimization, the optimization has been conducted in design optimum processor by means of Sub problem approximation method. This is an estimating method for variable designing, state and objective function via curve fitting tool. It is a general method for solving many engineering problems (Crisfield, 1985).

2.1 Geometrical modeling

According to shape optimization design variables, such as base thickness (t_0) and top thickness (t_1) as parameters, all the key points are defined as follows (Fig.3):
Point 1: (0, 0) Point (2): (R, 0) Point3: (-R, 0) Pint4: (0, R)
Point 5(R+t0, 0) Point6: (-R-t_0, 0) Point 7: (0, R+t_1)

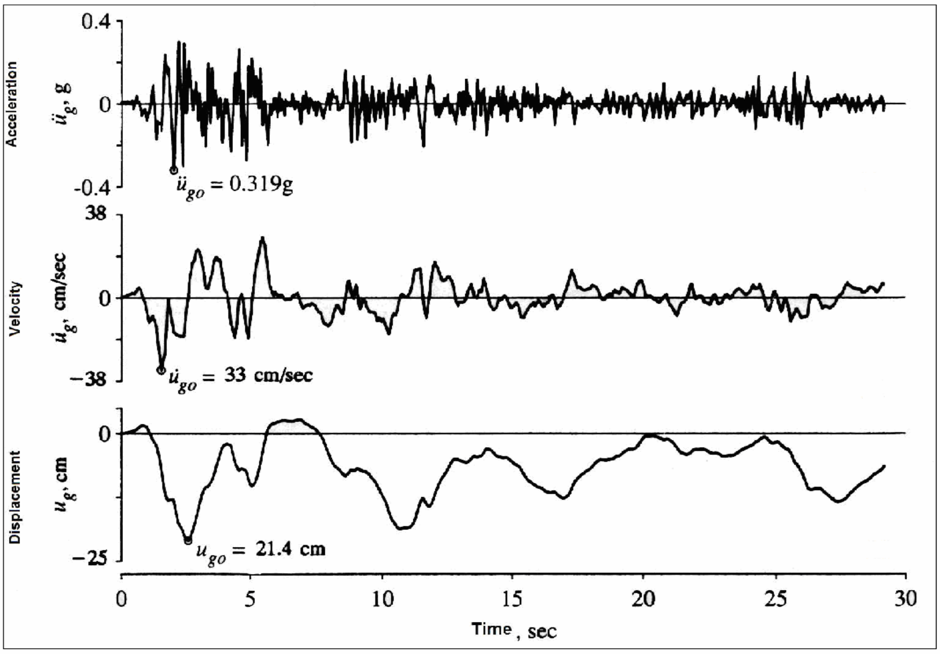


Fig. 1. North-south horizontal component of Elcentro earthquake

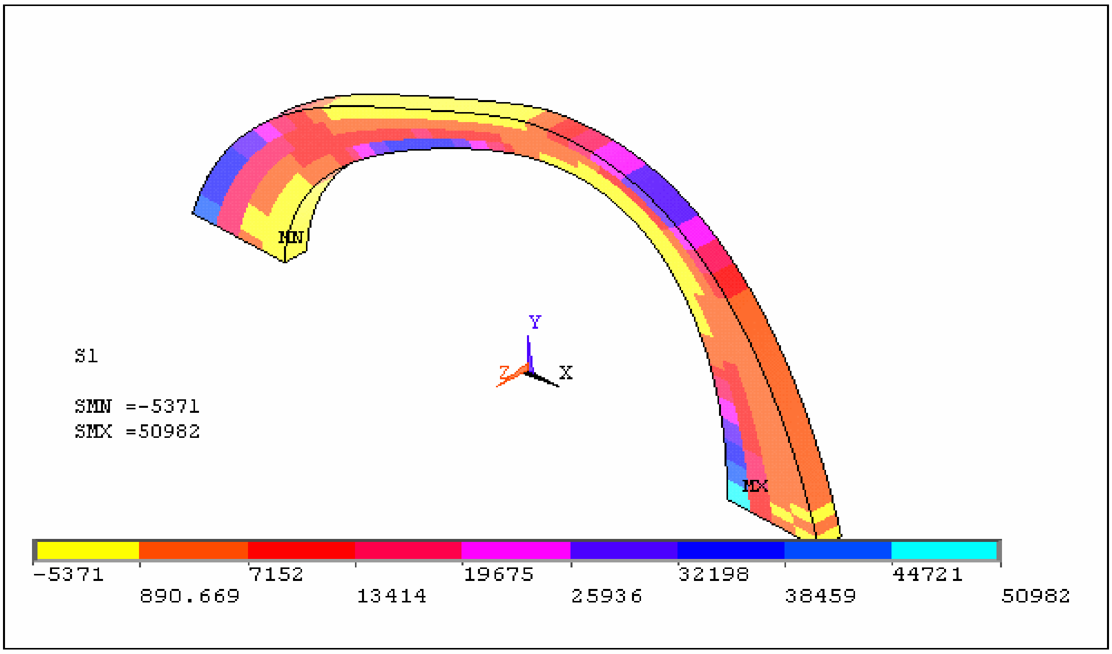


Fig. 2. Optimum shape of semicircular arch using FEM software

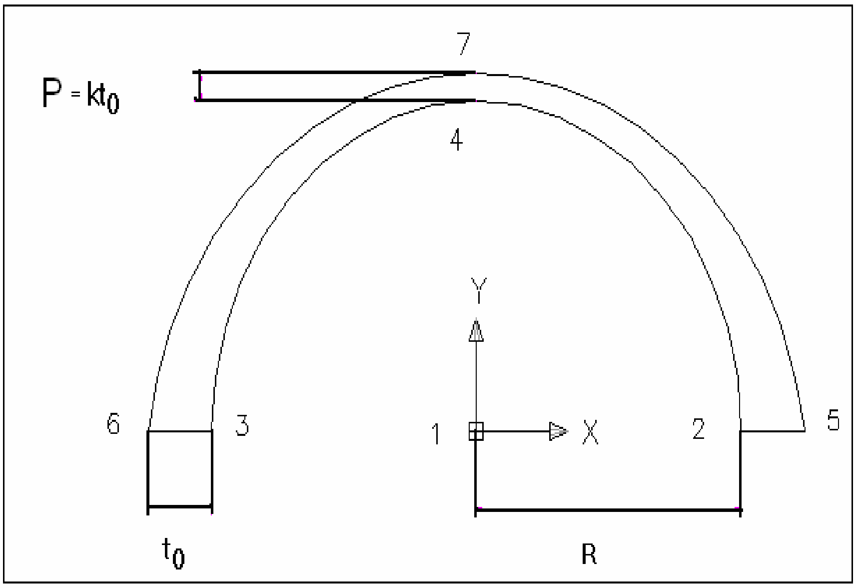


Fig. 3. Geometrical model of semicircular arch

In arch modeling, the tolerance increases because the thickness decreases from base to top (Abruzzese et al, 1995). It should mention that in modeled arch, the thickness decreases from base (t_0) to top (t_1) linearly and also arch thickness of axis is 20 (cm) in the length direction. The motion of support nodes is zero and dynamic force has no effect on them. In addition, brick masonry is made by brick and mortar as homogenous material (table1). The efficient factors in the inelastic nonlinear analysis have been shown in table 2. In the present paper, arch radius limit (R), maximum tensile stress, base and top thickness in optimum state are considered as 4-8 (m), 49000-5100 (KN/m^3), 0.8- 1.44 (m) and 0.2-0.35(m) respectively for all modeled arch.

Table1
Brick masonry characteristics (Bsthe, 1996)

density(ρ) (Kg/m^2)	Elastic modulus (N/m^2)	Allowable tension stress(f_t)	Poisson ratio (υ)
1460	5×10^8	0.5×10^5	0.17

Table 2
Effective coefficient in non elastic and nonlinear analysis (Baggio and Trovalusci, 2000)

motion coefficient for open crack	motion coefficient for close crack	allowable tension stress N/m^2	allowable compressive stress N/m^2
0.1	0.9	5×10^4	5×10^5

3. Cellular automata

At the beginning of 1950, cellular automata (CA) have been proposed by Von Neumann. He was interested to male relation between new computational device - automata theory

-and biology. His mind was preoccupied with generating property in natural events (Neumann, 1993).

He proved that CA can be general. According to his findings, CA is a collection of cells with reversible states and ability of computation for every thing. Although Van rules were complicated and didn't strictly satisfy computer program, but he continues his research in two parts: for decentralizing machine which is designed for simulation of desirable function and designing of a machine which is made by simulation of complicated function by CA (Neumann,1996).

Wolfram has conducted some research on problem modeling by the simplest and most practicable method of CA architecture too. In 1970,"The Game of Life" introduced by Conway and became very widely known soon. At the beginning of 1980, Wolfram studied one-dimension CA rules and demonstrated that these simple CAs can be used in modeling of complicated behaviors (Wolfram, 1983, 1984).

3.1 Definitions

CA is characterized by (a) cellular space (b) transfer rule (Moore, 2003).

For CA , cell, the state of cell in time t, sum of neighbors state at time t and neighborhood radius are denoted by i, S_i^t , η_i^t and r, respectively. Also, the rule is function of $\phi(\eta_i^t)$.

3.2 Change state rules

Each cell changes its state, spontaneously. The primary quality of cells depends on primary situation of problem. By these primary situations, CA is a system which has certain behavior by local rules. The cells which are not neighbors, have no effect on each other.CA has no memory, so present state defines the next state (Wolfram, 2002).

Quadruple CA is as CA= (Q, d, V and Φ), where Q, d, V and Φ are collection of possible state, CA dimension, CA neighborhood structure and local transferring rule, respectively.

For 1-d CA, amount of i cell (1≤i≤n) at t is shown by $a_i(t)$ and is calculated by this formula:

$$a_i(t+1)= \Phi\ [a_{i-1}(t),\ a_i(t),\ a_{i+1}(t)]$$

In this formula, if Φ is affected by the neighbors, it is general. If Φ is a function of neighbor's cell collection and central cell, it is totalistic.

$$a_i(t+1)= \Phi\ [a_{i-1}(t),\ a_i(t),\ a_{i+1}(t)]$$

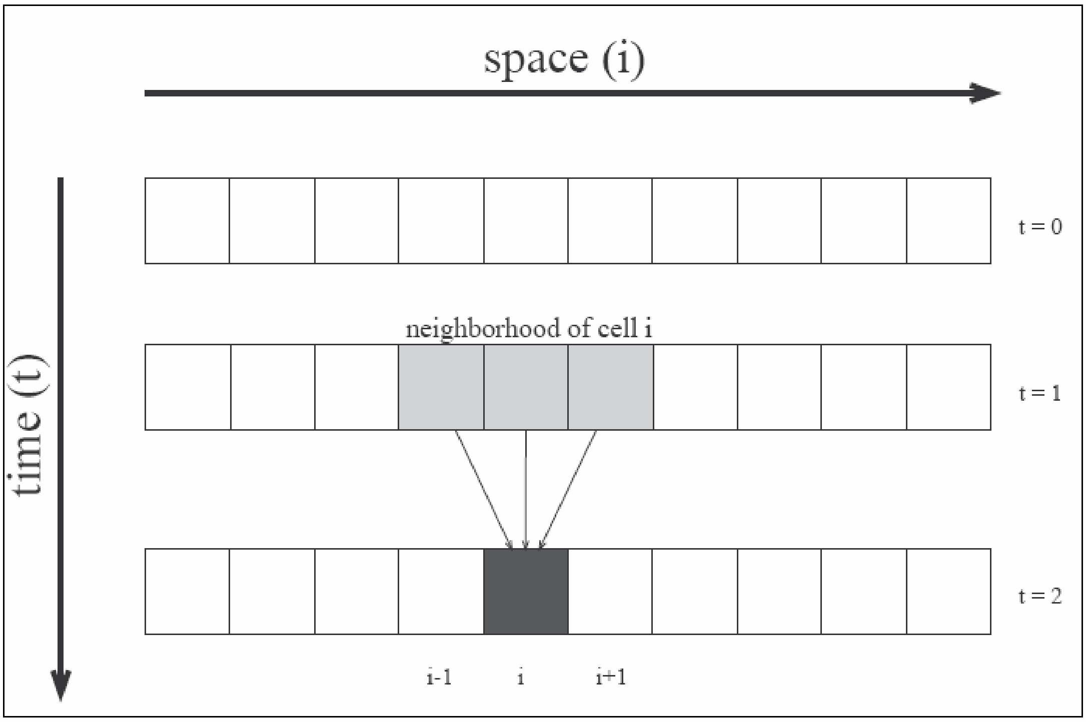


Fig. 4. Neighborhood space of Von Numann in 1-D CA

4. Arch modeling using CA

In this stage, regarding the definition of neighborhood radius and state reversal rule in three state 1-d CA, the data for each arch will be analyzed to find the rules of simulation of arch behavior. To achieve this aim, 100 samples of each arch radius, base and top thickness and maximum tensile stress were chosen and analyzed by two and three state algorithm (figure 5 defines two state algorithm completely). After one billion accomplishments, for 256 two-state rules and one million three-state rules, some models were provided for each arch. For example, figures 6 and 7 define semicircular rules and tensile stress efficiency, respectively.

5. Test of cellular automata models

Maximum tensile stress for 50 samples (according to algorithm in figure 8) has been provided. The error percent has been compared with another analyzed model in FEM software.

5.1 Test of CA model for semicircular arch

Maximum tensile stress was achieved for 50 samples of semicircular arches by CA. Figure 9 define comparison between maximum tensile stress in FEM and CA model**.** The mean of error percent in semicircular arch is 13.365%. Figure 10 represent error percent of each sample. Moreover, Fig. 11 illustrates the diagram of comparison between time of maximum tensile stress computation using CA and FEM software, respectively.

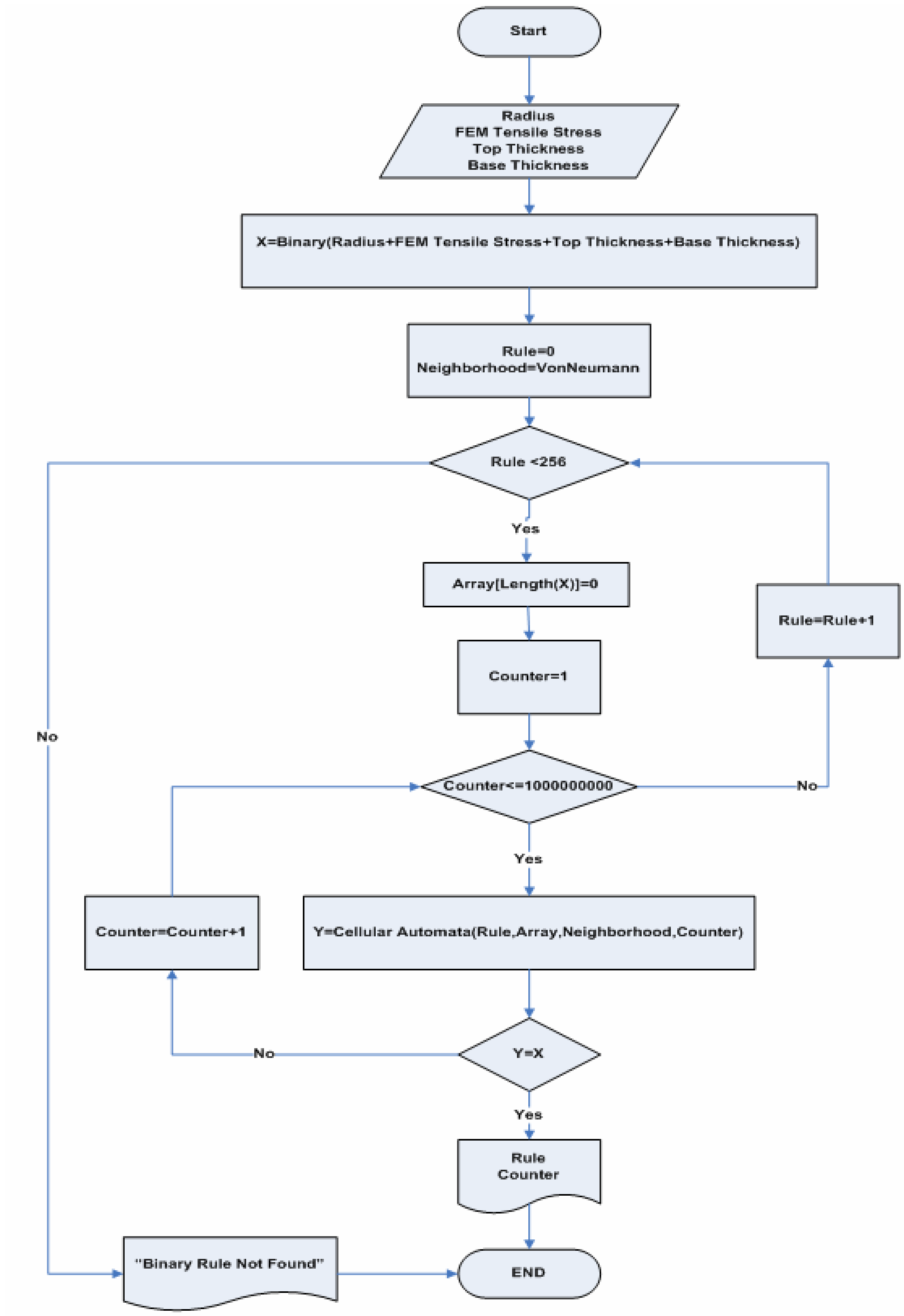


Fig. 5. An algorithm for finding two- state 1-D cellular automata model for arch behavior modeling

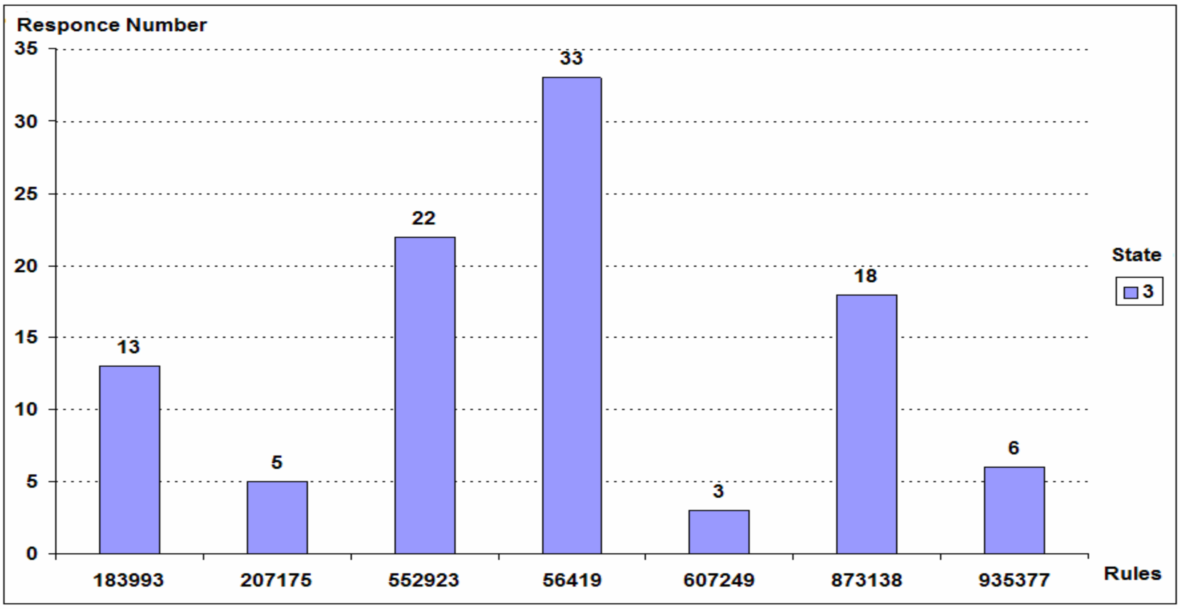
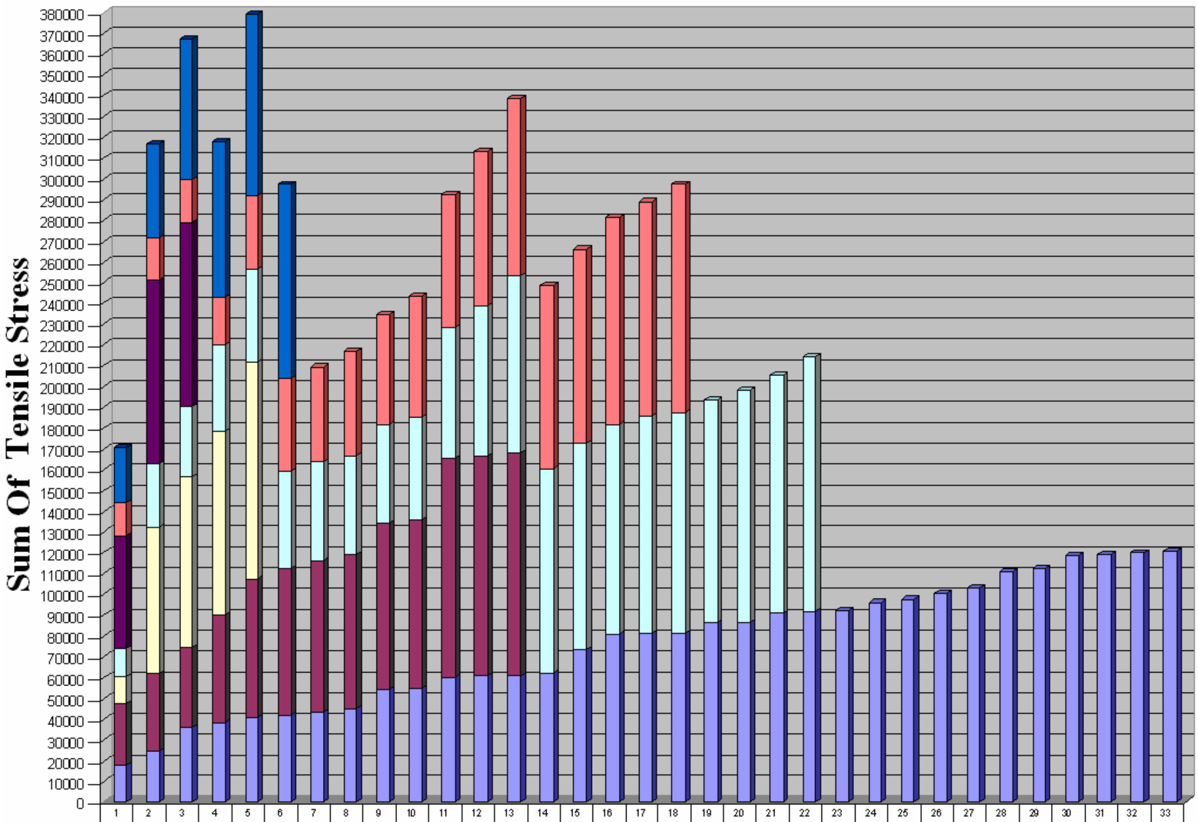


Fig. 6. Diagram of three-state rules of cellular automata and some of samples in semicircular arch



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Rule 935377	26358	45024	67286	74694	87335	93557											
Rule 873138	15917	20594	20812	23311	35033	44815	45805	49885	52532	58340	63607	74485	85167	88313	93348	99212	103151
Rule 607249	54371	88340	88522														
Rule 552923	13835	30494	33920	41455	44692	46525	47630	47918	48044	49222	62818	72230	85114	97882	99077	100922	104435
Rule 207175	12707	70411	82279	88213	104218												
Rule 183993	29252	37145	38085	51675	67103	70544	72917	74104	79835	81225	105734	105945	107581				
Rule 56419	18115	24687	36114	38408	40529	41898	43282	44871	54125	54630	59968	60623	60635	62468	73437	81086	81351

	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
Rule 935377																
Rule 873138	109767															
Rule 607249																
Rule 552923	106071	107179	111891	114300	122032											
Rule 207175																
Rule 183993																
Rule 56419	81465	86366	86492	90916	92159	92482	96169	97643	100202	102784	110875	112602	118327	119182	120046	120701

Fig. 7. Diagram of maximum tensile stress efficiency for each of cellular automata rules in semicircular arch

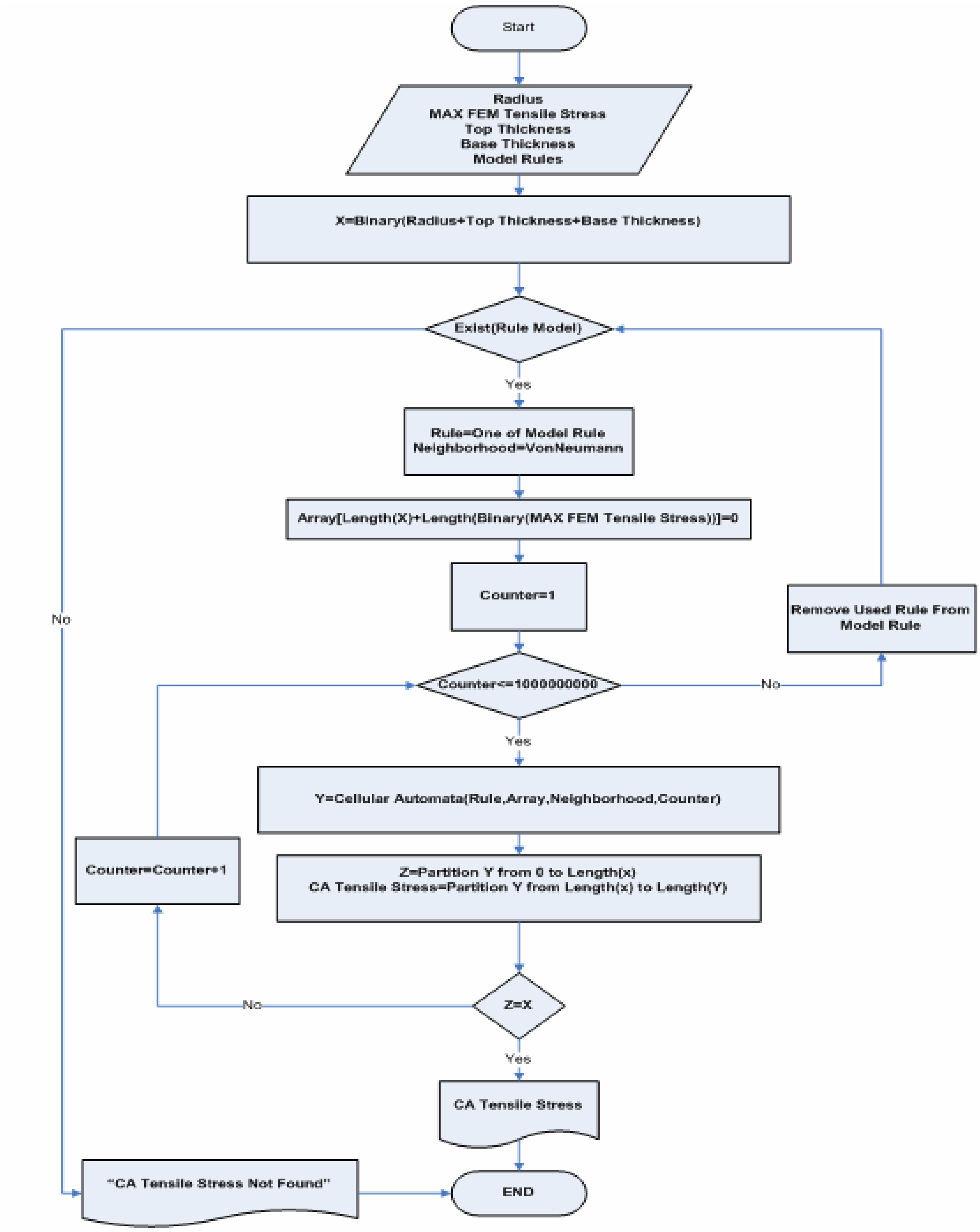


Fig. 8. An algorithm for analysis of arch behavior using two-state 1-D CA for maximum tensile stress

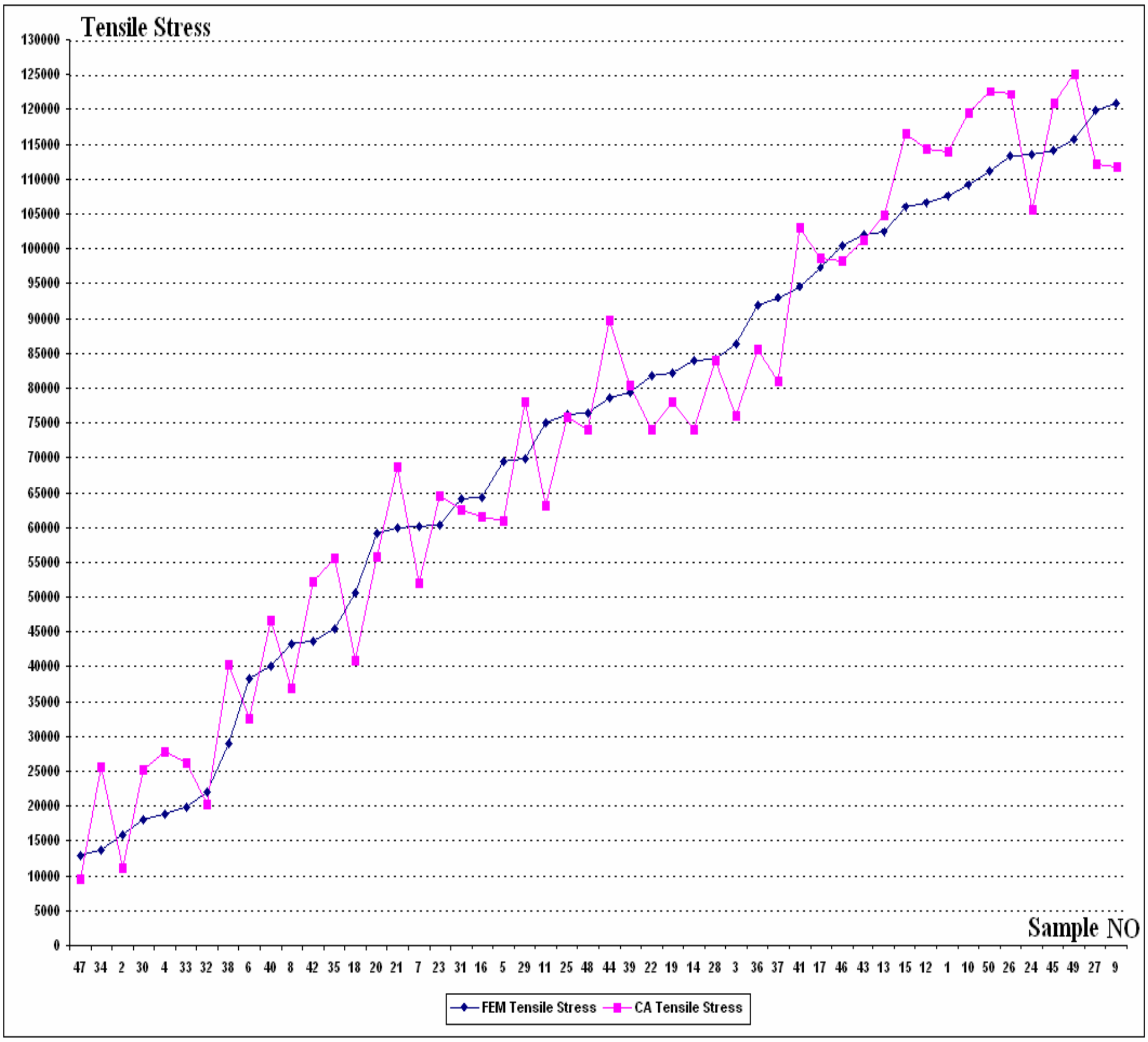


Fig.9. Comparison between maximum tensile stress using FEM software and CA model in semicircular arch

6. Arch optimization using CA

In this stage, by means of CA model for each arch top and base thickness were optimized. Considering optimized maximum tensile stress which is 51000(N/m^2), the range of radius, top thickness and maximum tensile stress in each arch are considered as input, so arch base thickness will be provided. In the next stage, size of arch radius, base thickness and maximum tensile strain are considered as input. So arch top thickness will be provided (arch base thickness optimization is defined in figure 12).

6.1 Top thickness optimization in semicircular arch using CA

In this stage, 50 semicircular arch samples were chosen for top thickness optimization. Their optimum maximum tensile stress range, arch radius and base thickness were 49000 to 51000 (KN/m^3), 4~8 meter and 0.8 to 1.44, respectively. After ward, the top thickness was calculated and compared with top thickness in FEM software (fig.15). The mean of error percent of top thickness calculation was 11.37%. Figure 13 and 14 show error percent of each sample in CA toward FEM software and comparison of optimization time of top thickness optimization in semi circular arch.

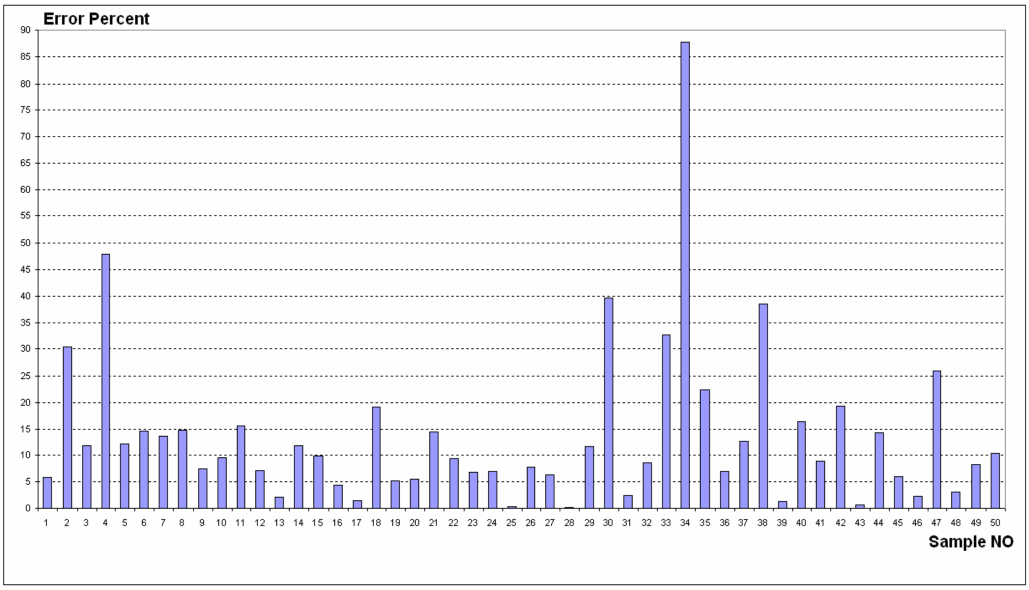


Fig. 10. Error percent of maximum tensile stress computation by CA to FEM software in semicircular arch

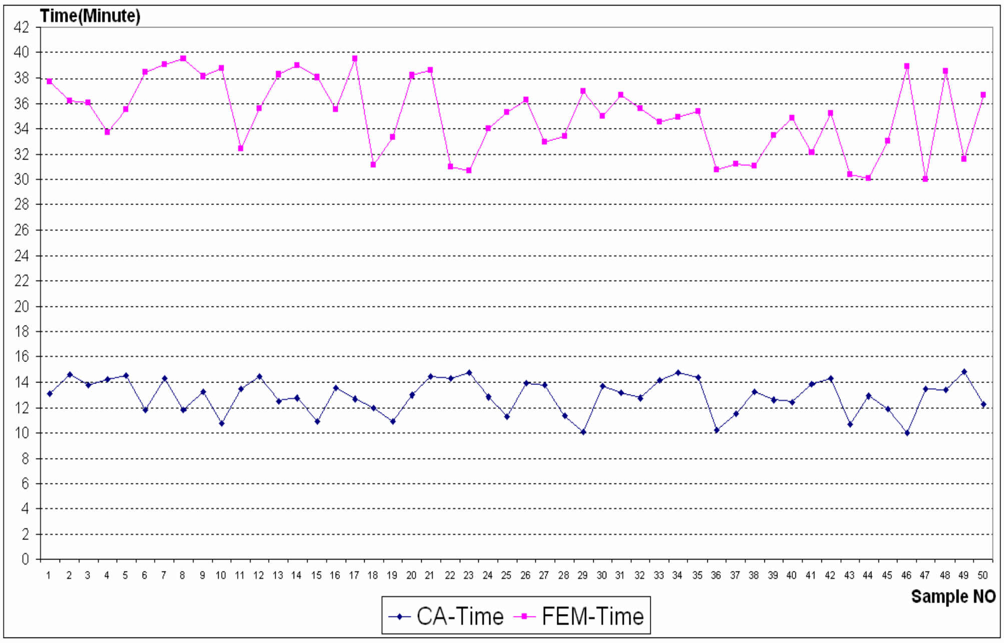


Fig. 11. Comparison between time of computation of maximum tensile stress by FEM software and CA

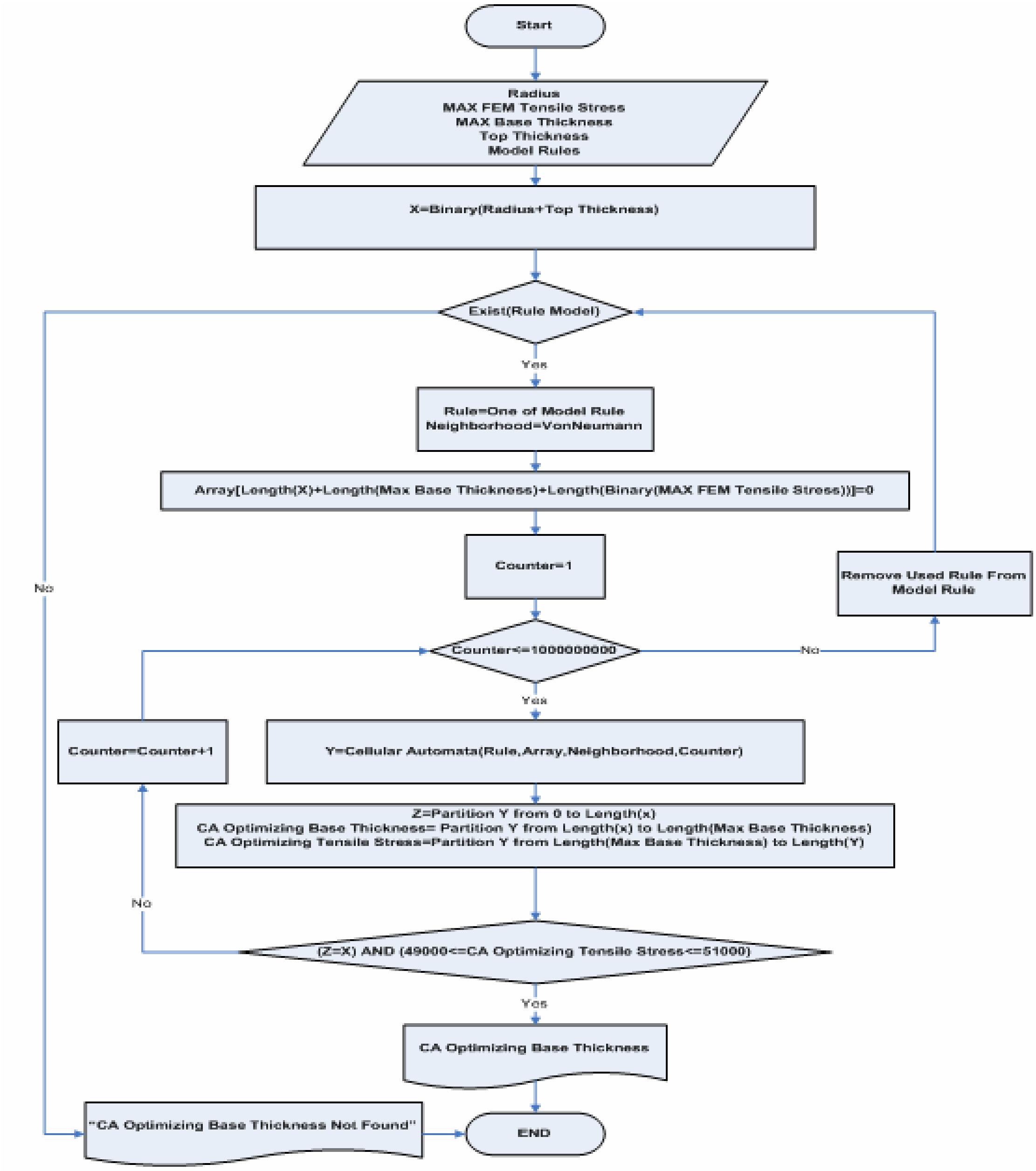


Fig. 12. Algorithm of arch thickness optimization using two -sate -1-D cellular automata

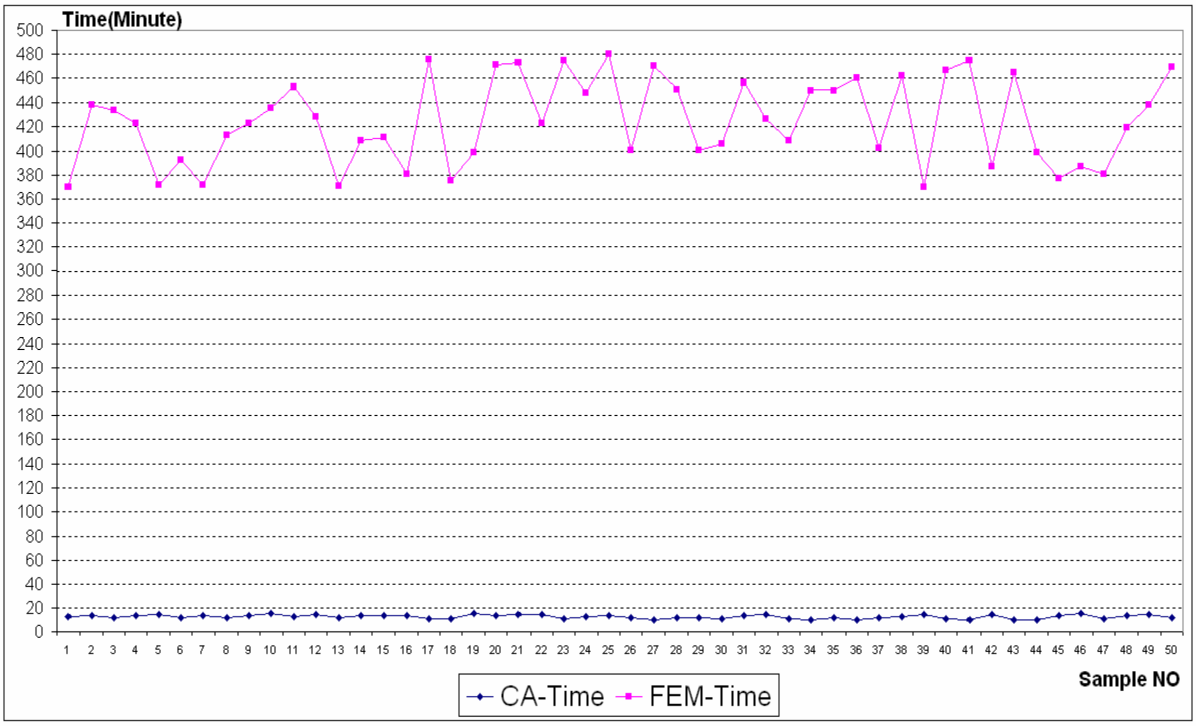


Fig. 13. Comparison between maximum tensile stress of semicircular arch using FEM software and CA model

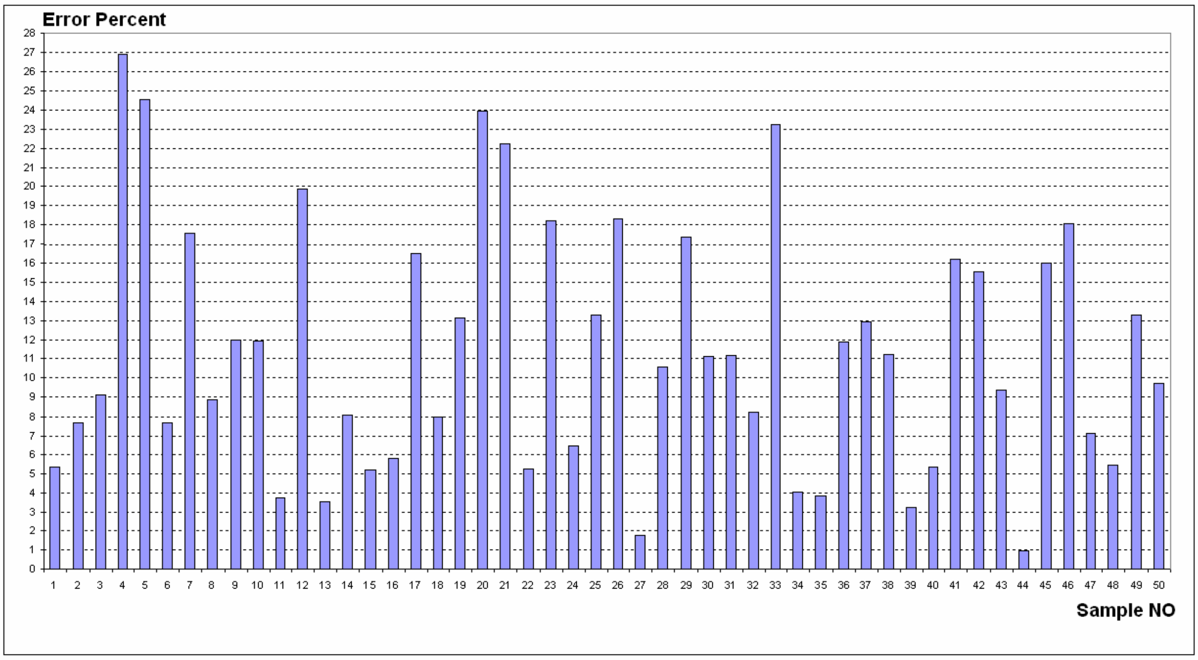


Fig. 14. Error percent of top thickness optimization in semicircular arch using CA model towards FEM software

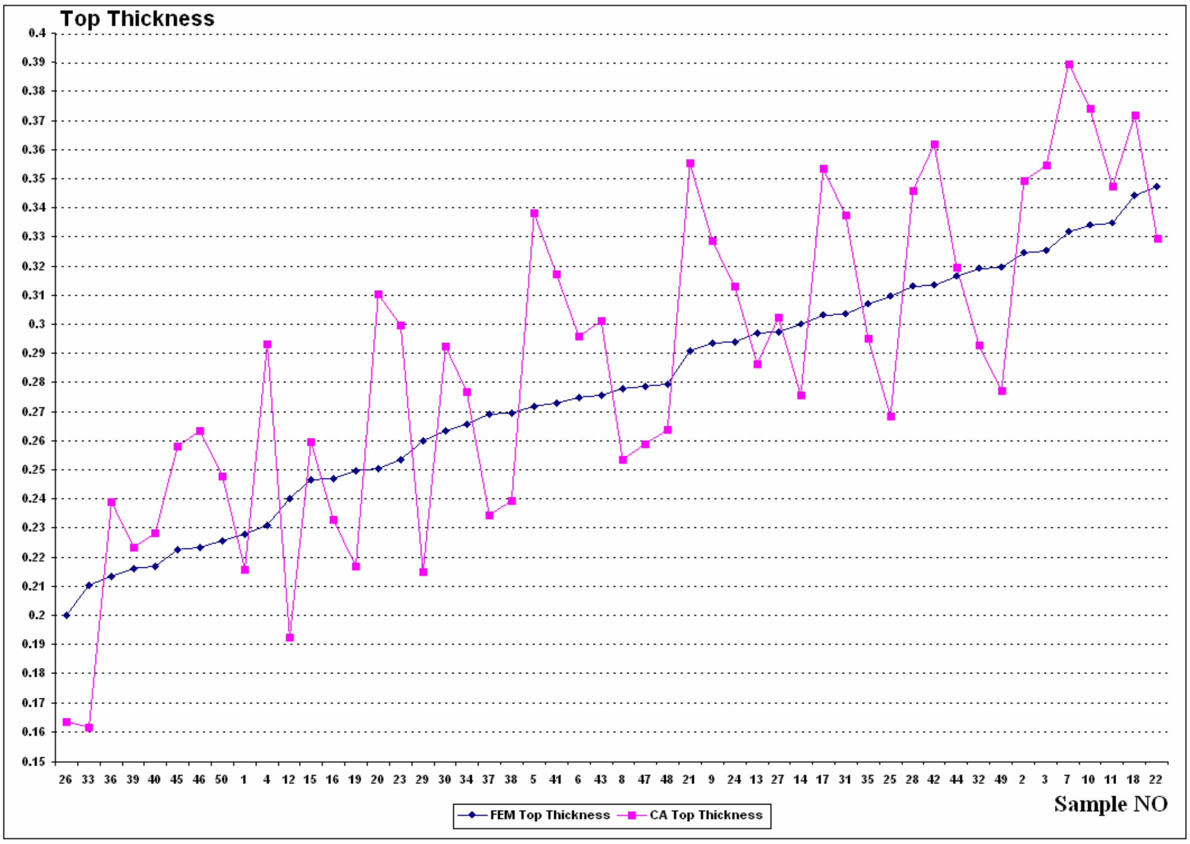


Fig. 15. Comparison of optimum range of arch top thickness using CA model and FEM software

6.2 *Base thickness optimization in semicircular arch using CA*

In this section, 50 semicircular arch samples were chosen for top thickness optimization. Their optimum maximum tensile stress range, arch radius and base thickness were 49000 to 51000(KN/ m^3), 4~8 meter and 0.2 to 0.35, respectively. After calculation of base thickness-according to algorithm in figure 12, the results were compared with base thickness in FEM software (Fig.18). The mean of error percent of base thickness calculation was 11.69%. Figure 16 and 17 show error percent of each sample in CA toward FEM software and comparison of optimization time of base thickness optimization in semi circular arch.

7. Conclusion

In the present paper, nine arches- semi-circular, obtuse angel, four- centered pointed; Tudor, ogee, equilateral, catenaries, lancet and four-centered arches- were modeled using FEM software and CA model. Figures 19, 20 and 21 show analysis and optimization time, the results which are provided by CA in arch modeling and the mean of error percent for arch analysis and its optimization, respectively.

Considering results, CA model can be used in simulation of all arches. Therefore, the time of calculation decreases. Also, it can be used in dynamic response, natural frequency and response of structure under different dynamic loads. To increase models precision, the rules which are larger than 1000000 and repeated more than 1000000000 times are needed.

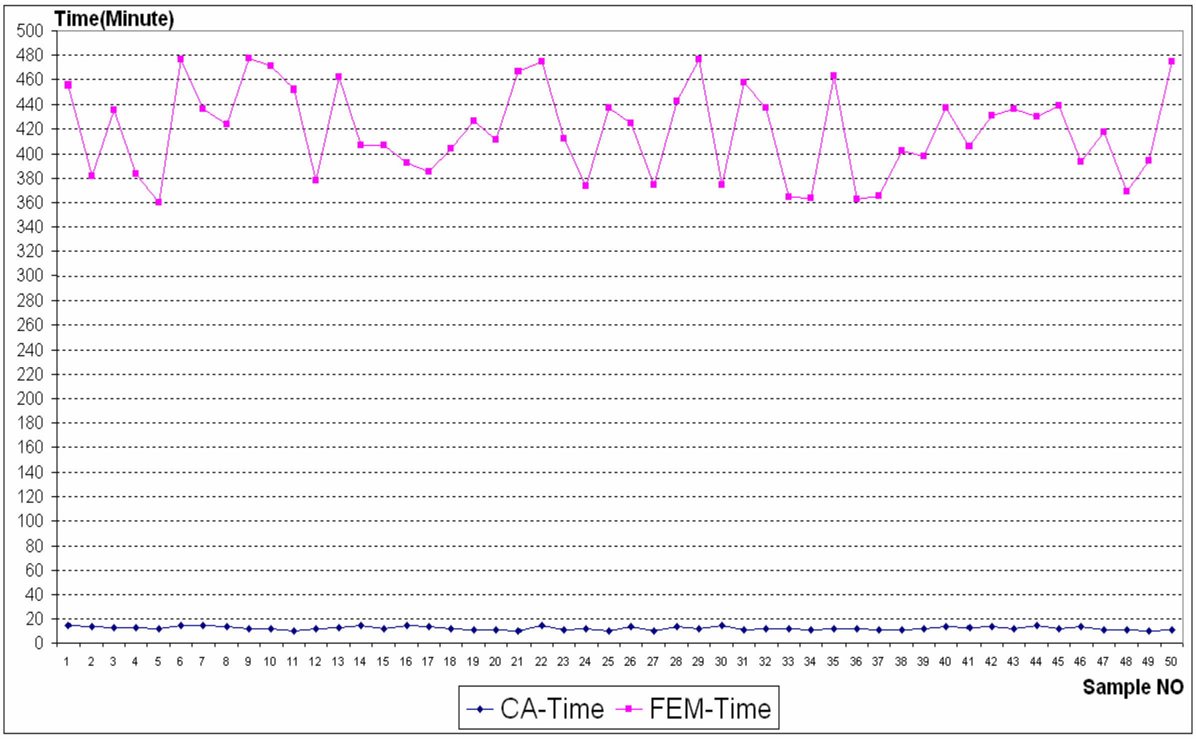


Fig. 16. Comparison between optimization time of base thickness in semicircular arch using FEM software and CA model

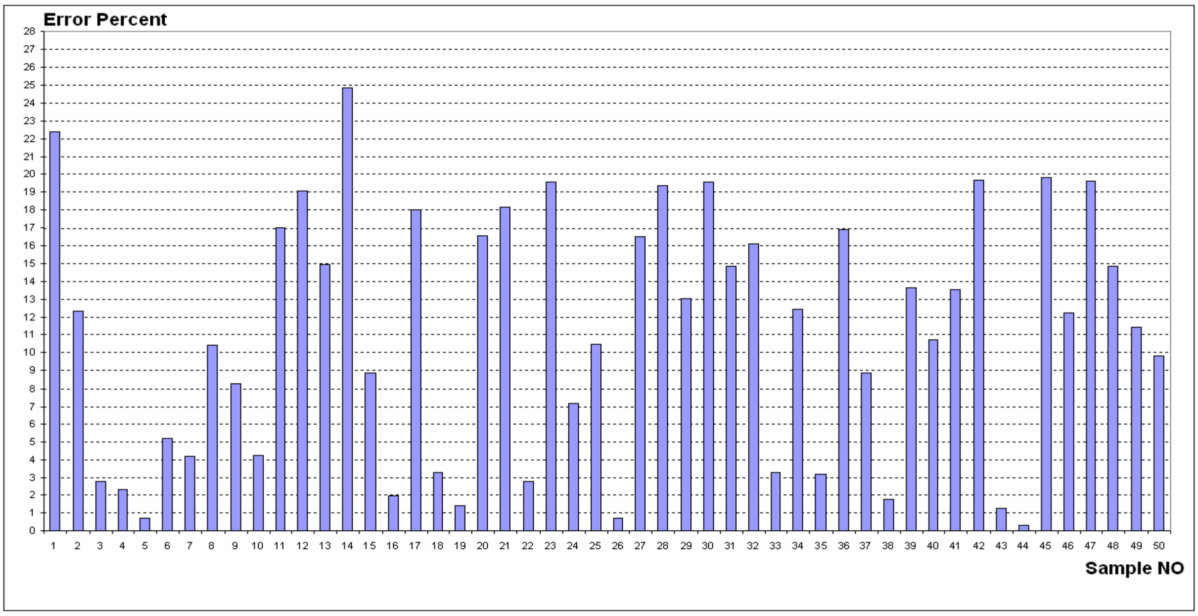


Fig. 17. Error percent of base thickness optimization in semicircular arch using CA model towards FEM software

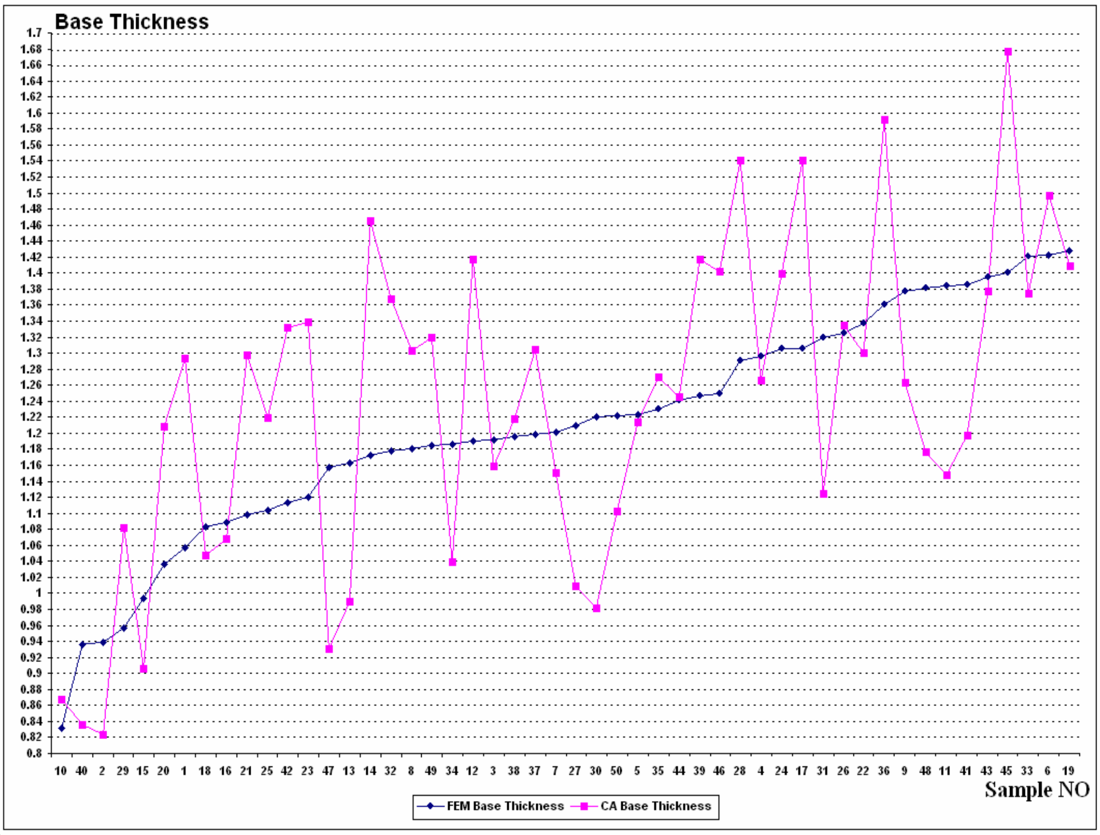


Fig. 18. Comparison of optimum range of arch base thickness using CA model and FEM software

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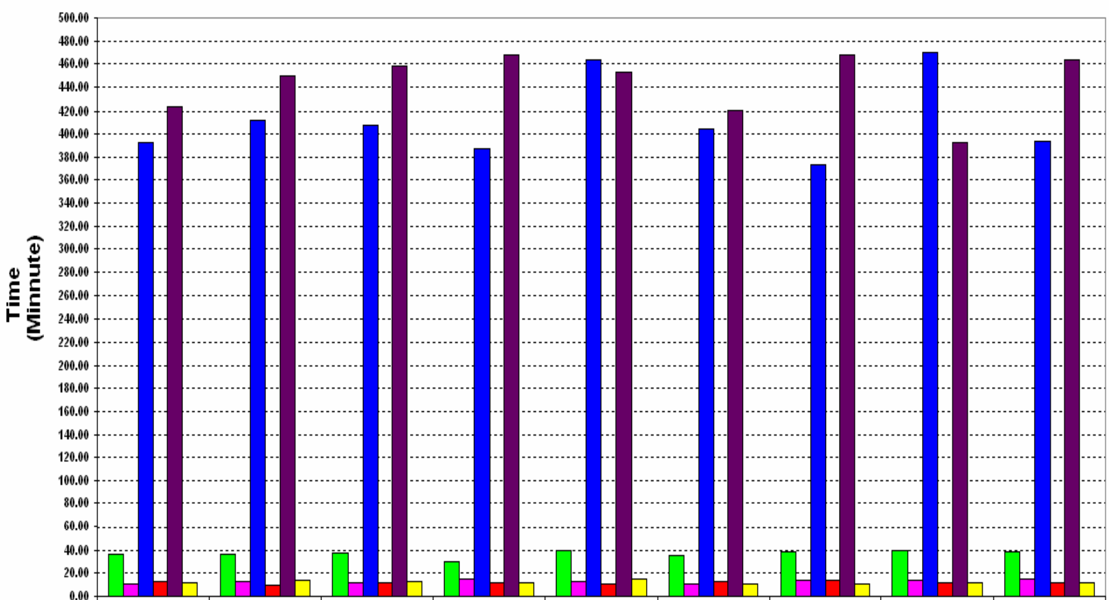
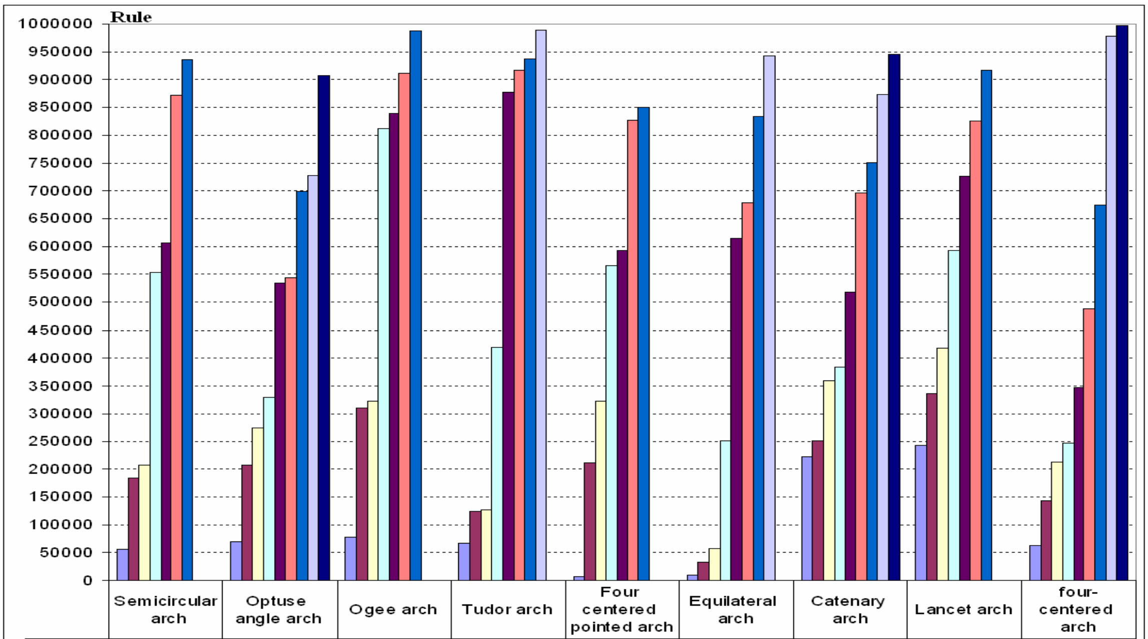


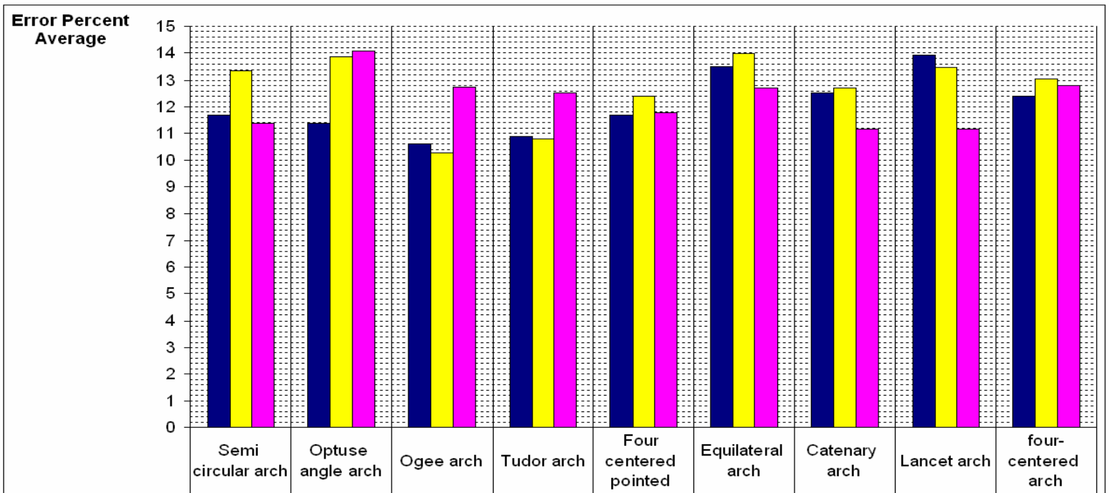
	Semicircular arch	Optuse angle arch	Ogee arch	Tudor arch	Four centered pointed arch	Equilateral arch	Catenary arch	Lancet arch	four-centered arch
FEM Analyzing Average Time	36.50	36.12	37.38	30.21	39.31	35.94	38.58	39.28	38.88
CA Analyzing Average Time	10.76	13.16	11.90	14.64	12.87	10.75	13.97	14.25	14.72
FEM Top Thickness Optimizing Average Time	393.11	411.52	407.31	387.41	463.49	404.40	373.67	470.07	393.57
CA Top Thickness Optimizing Average Time	13.37	10.09	12.28	11.86	11.26	13.02	13.56	11.63	11.37
FEM Base Thickness Optimizing Average Time	424.09	450.06	458.04	468.22	453.13	420.40	467.66	392.83	463.99
CA Base Thickness Optimizing Average Time	11.27	13.86	12.72	12.33	14.50	10.82	10.23	11.84	11.81

Fig.19. Comparison between mean of analysis and optimization time of all discussed arches using FEM software and CA model



	Semicircular arch	Optuse angle arch	Ogee arch	Tudor arch	Four centered pointed arch	Equilateral arch	Catenary arch	Lancet arch	four-centered arch
Rule 1	56419	69529	77288	67248	6406	9697	221489	242326	63451
Rule 2	183993	206031	309746	124076	211894	32920	250618	336930	143592
Rule 3	207175	273192	322007	126223	322673	58373	358815	418162	213445
Rule 4	552923	329800	811998	419541	566069	250451	384276	593481	246452
Rule 5	607249	534986	839370	877175	593895	614969	519602	726076	346562
Rule 6	873138	544091	911584	916598	827382	679062	696563	824867	488488
Rule 7	935377	699479	987632	936310	849684	834852	752276	917132	675673
Rule 8		727734		989632		943069	873276		977564
Rule 9		907377					945910		997074

Fig. 20. Comparison between provided rules for discussed arches using cellular automata



	Semi circular arch	Optuse angle arch	Ogee arch	Tudor arch	Four centered pointed	Equilateral arch	Catenary arch	Lancet arch	four-centered arch
Base Thickness Error Percent AVG	11.6918	11.3669	10.6008	10.8961	11.6958	13.527	12.5278	13.932	12.4059
Tensile Stress Error Percent AVG	13.365	13.8451	10.2532	10.7823	12.3959	13.9926	12.7088	13.4953	13.047
Top Thickness Error Percent AVG	11.3719	14.0841	12.7482	12.5271	11.7823	12.7226	11.1769	11.1753	12.8139

Fig. 21. Comparison between the mean of error percent of analysis of tensile stress and optimization of base and top thickness for discussed arches using CA model toward FEM software