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Eliminating upside- down movements in suspension bridges

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Abstract

To eliminate upside-down movements generated by wind forces or during the passing of heavy trucks, suspension bridges require the application of downward uniformly distributed loads. Treated herein is the "Pulling down Cable" that makes use of the advantageous properties of arches and suspension cables. The mentioned cable takes the form of a convex parabolic shape so that it can apply a downward uniformly distributed load by pulling down equally spaced tension resisting strands attached to the bridge deck stiffening girders. The downward uniformly distributed load can balance the upward uniformly distributed load created by wind, seismic or traffic vibrations. These forces (either temporary or permanent) can be controlled by the amount of applied tension at both cable ends, which are anchored to the bridge towers. This paper presents the derivation of the related equations governing reactions, tensile forces, length and shape of the cable and the attached strands. An example of the use of the Pulling down Cable is considered to demonstrate its capability to enhance the stability of a suspension bridge against vibration movements. Finally, it may be concluded that an extra stiffening of suspension bridges can be achieved by adopting the Pulling down Cable system.

Keywords: Suspension bridge, pulling down cable, upside- down movements

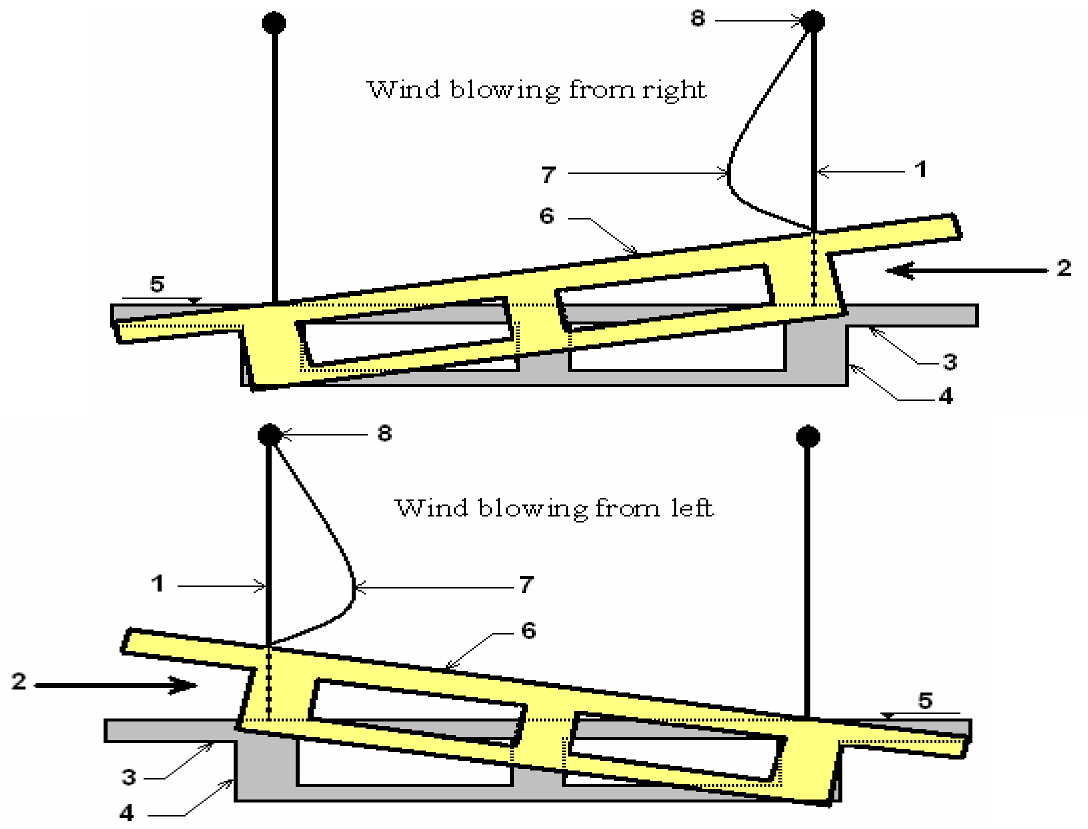
1. Introduction

Steel and/or concrete arches have been used to bridge intermediate spans in different ways. It might hold bridge deck slabs by high strength steel hangers, or by supporting it directly. According to its structural properties, arches can support the uniformly distributed loads, which are usually acting downwards and perpendicularly to the bridge profile, by the axially generated compressive stresses throughout its whole body (Wang, 1962).

Suspension cables, on the other hand, are also used to construct long spans suspension bridges. It can restore its concave parabolic shape under the combined effects of its self weight, suspended external loads and the applied tension at each of its anchorages (Ramamrutham, 1988). It is usually designed to hold the bridge deck slab and the estimated traffic loads that are modeled as a uniformly distributed load. Moreover, suspension cables can carry the uniformly distributed loads, which are imposed perpendicularly downwards, by the axially generated tensile stresses throughout its whole length.

In spite of their different shape and structural action, arches and suspension cables, which can support downwards loads in an active manner, are very weak in resisting upwards imposed loads. While arches are not designed to resist tension throughout its own structure, suspension cables are completely unable to resist any applied upward force.

Suspension bridges are known as the longest constructed bridges all over the world. This type of bridges consists mainly of two high towers, two main suspension cables, many equally spaced hangers, and a thin deck slab stiffened by two stiffening girders. Owing to their slender and light construction, suspension bridges always suffer from vibrations caused by wind and seismic activities. These vibrations are trying repetitively, as shown in Figure 1, to move one side of the thin deck slab up and down with respect to the other side.



1. Hanger, 2. Wind force, 3. Original position of deck slab, 4. Stiffening Girder, 5. Original level, 6. Displaced deck slab due to wind force, 7. Bent hanger, 8. Suspension cable.

Fig. 1 Cross section showing the positions of a deck slab subjected to wind force.

While the downward movement can successfully be resisted by the pulling up hangers which are already designed to resist tension, the upward movement can be considered free because of the complete weakness of the hangers which cannot resist such compressive forces.

A thin deck slab of a suspension bridge, which is subjected repetitively to uplift forces, may suffer from top surface cracks. These cracks may lead to a fatigue failure with time. This was exactly what happened to the Tacoma Narrow Bridge in 1940.

The mentioned American bridge was 1810m in length. It collapsed under the effect of a storm when its velocity reached 68 km/h (Microsoft R Encarta R Reference Library, 2002). Just before failure, each side of the bridge deck slab was vibrating several meters up and down with respect to the other side.

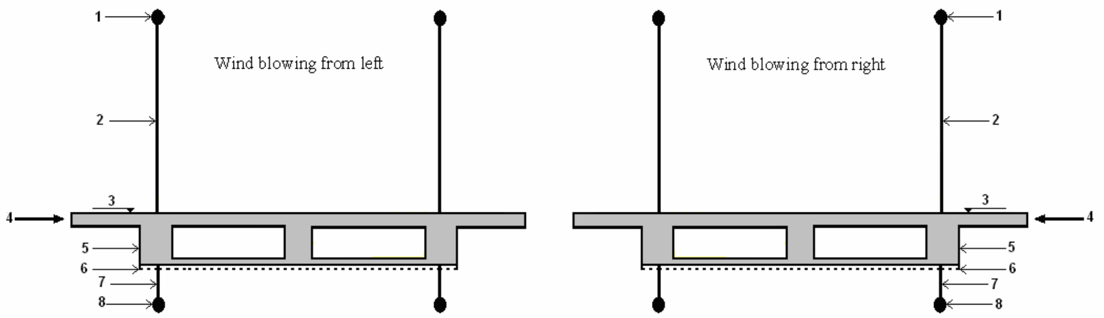
2. Definition of pull-down cables

As mentioned above, both of the structural systems (arches and suspension cables) are not good enough to support upward directed forces. To make use of some of the useful properties of these structural systems, the suggested Pull-down Cable system may now be introduced. It consists of a flexible high-tensile steel cable and a number of equally spaced tension resisting strands, see Figure 2.

1. Strand, 2. Pull-down Cable

Fig. 2 Longitudinal view of a suspension bridge stiffened by two Pull-down Cables.

The Pull-down Cable ends must be anchored properly to the suspension bridge towers. The strands are linking between the cable and the stiffening girder of the bridge. A couple of such system must be used one at each side of the served bridge in order to let it work. While strands are straight, the shape of the cable is convex, it looks like an arch but it is just a cable. Any upward force trying to lift any side of the suspension bridge deck slab, see Figure 3, can be resisted by the tension forces which will be generated in the strands. All the strands are well connected to the two cables, which will in turn be subjected to tensile forces along their lengths. These strands will be more than ready to resist such forces and transfer them to the end anchorages at the rigid bridge towers.



1. Suspension cable, 2. Hanger, 3. Original level, 4. Wind force, 5. Reduced Stiffening girder, 6. Original dimension, 7. Strand, 8. Pulling down cable.

Fig. 3. Fixation of a suspension bridge deck slab by a Pull-down Cable system.

By using a Pull-down cable system at each side of a suspension bridge, the upward movements of its deck slab can be eliminated and noting that the downward movements are well resisted by the suspension cable and hangers. This suggested solution results in stiffer deck slabs against vibrations leading to get more durable suspension bridges.

In the last half century, this problem was solved by constructing two huge stiffening girders, one on each side of any suspension bridge. These stiffening girders are designed to eliminate or to reduce to some extent the vibrations due to wind, seismic and traffic effects. But from the author's point of view, the self weight of these girders is so heavy and these girders are architecturally unjustified.

3. Derivation of governing equations of structural action of Pull-down cables

The governing equations of the geometry and the structural action of arches and suspension cables, the shape, length, reactions, tensile force and loading of Pull-down cables can be derived as follows:

3.1 Vertical reaction (V)

The Pull-down cable is not like other structural systems. It has no end reactions if the deck slab is not subjected to any uplifting pressure. Moreover, it has no end reactions even due to its self weight. But if it is activated by a wind uplifting pressure, subjected to the bridge deck slab, then there will be a noticeable tension along its length and a large end reactions.

Figure 4 shows a Pull-down cable ab. This cable is subjected to a uniformly distributed uplifting force w per unit run on the horizontal span ℓ. So it is tensioned by a force T. Point c which is situated along the cable path represents the highest point of the cable with respect to its end anchorages.

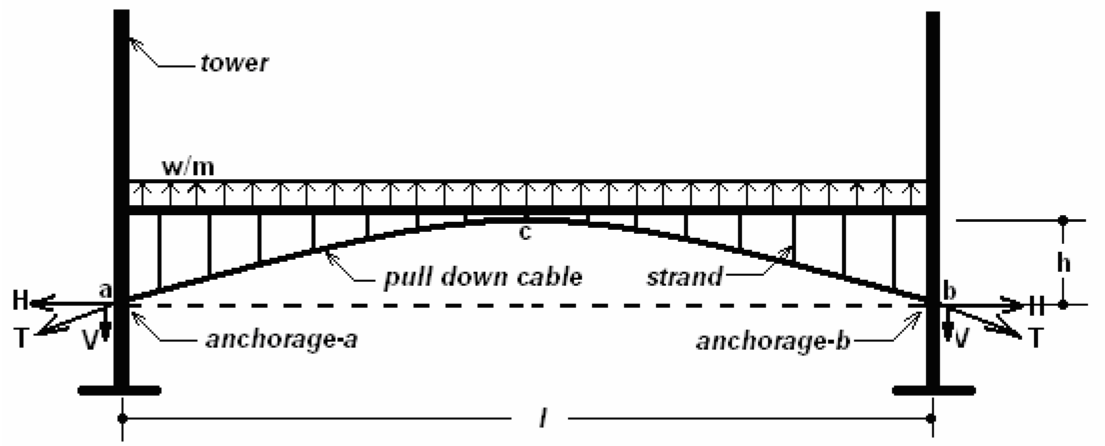


Fig. 4 Symbols, components and uniform loading of a Pull-down cable system.

This maximum height is denoted by the symbol h. The vertical end reaction V at each anchorage is calculated according to the following equation:

$$V = Va = Vb = \frac{wl}{2} \tag{1}$$

3.2 Horizontal end reaction (H)

By taking moments about point c of the forces on the right hand side of point c, one gets:

$$V \times \frac{l}{2} - \frac{wl}{2} \times \frac{l}{4} - Hh = 0 \tag{2}$$

By substituting the value of Vb from Eq. (1), one obtains

$$H = \frac{wl^2}{8h} \tag{3}$$

3.3 Maximum tension ($T_{\max}$)

The maximum tension in the cable $T_{\max}$ is the resultant reaction at points a and b and it is given by:

$$T_{\max} = \frac{w\ell}{2}\sqrt{1 + \frac{\ell^2}{16h}} \tag{4}$$

3.4 Shape of the Pull-down cable

Let the coordinates of any point p of the cable be (x, y) with respect to point c, as shown in Figure 5. Consider the equilibrium of the portion pc.

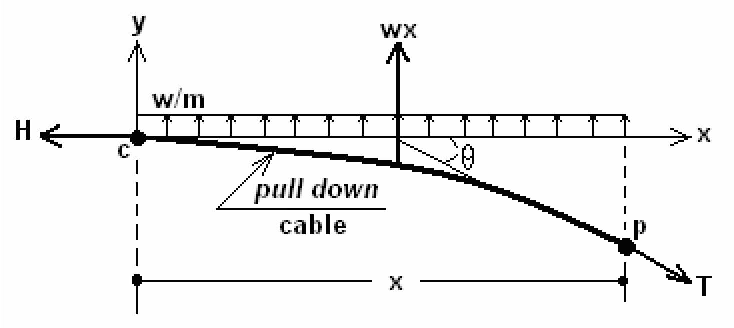


Fig. 5 Forces acting on a portion of a Pull-down cable.
P is any point along the cable.

Forces keeping point p in equilibrium are:

1. External load wx acting vertically upwards,
2. Tension $T_0 = H$ acting horizontally at point c, and
3. Tension T at point p acting along the tangent at p.

Let θ be the inclination of the tangent at point p with respect to the horizontal. By resolving the forces vertically and horizontally, one obtains

$$T\sin\theta = -wx \quad \text{or} \quad \tan\theta = -\frac{wx}{H} \tag{5}$$

But since $\tan\theta = \frac{dy}{dx}$, one gets, $\frac{dy}{dx} = -\frac{wx}{H} \Rightarrow \quad dy = \frac{-wx}{H}dx$ (6)

By integrating Eq. (6), one obtains, $y = \frac{-wx^2}{2H} + k$

(7)

k is a constant at point c , i.e. at $x = 0$ and $y = 0$, Eq. (7) furnishes $k = 0$ (8)

Therefore, $y = \frac{-wx^2}{2H}$ (9)

It is clear that this is an equation of a parabola.

Since $H = \frac{w\ell^2}{8h}$, Eq. (9) can be rewritten as:

$$y = \frac{-4hx^2}{\ell^2} \tag{10}$$

With either support treated as the origin, the equation for the cable is given by:

$$y = \frac{4hx(\ell - x)}{\ell^2} \tag{11}$$

3.5 Length of the Pull-down cable (L)

Equation to the cable with the point c (taken as the origin) is given by Eq. (10). Since $\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$, one can write

$$ds = \left[1 + \frac{32h^2x^2}{\ell^4}\right]dx \tag{12}$$

Therefore, the total length of the cable is given by

$$L = 2\int_0^{l/2}(1+\frac{32h^2x^2}{\ell^4})dx = \ell + \frac{8h^2}{3\ell} \quad (13)$$

4. Pull-down cable system for stiffening a suspension bridge: Example

The procedure of designing a suspension bridge is considered as one of the most complicated civil engineering problems. It depends upon the geometry, traffic intensity and the location of the bridge. It usually requires a prototype model to be tested, in side a wind tunnel, in order to estimate the bridge response to such simulated artificial loading.

In order to illustrate the use of the suggested Pull-down cable system, the bridge dimensions and wind forces will be estimated as follows, see Figure 6. The dynamic forces due to wind are assumed to act as uniformly distributed upward and downward linear forces applied to each side of the bridge deck slab. While the downward force is well resisted by the bridge structure, the upward free force (actual minus the dead load) will be taken as a uniformly distributed linear load of $w =$ 400 kg/m.

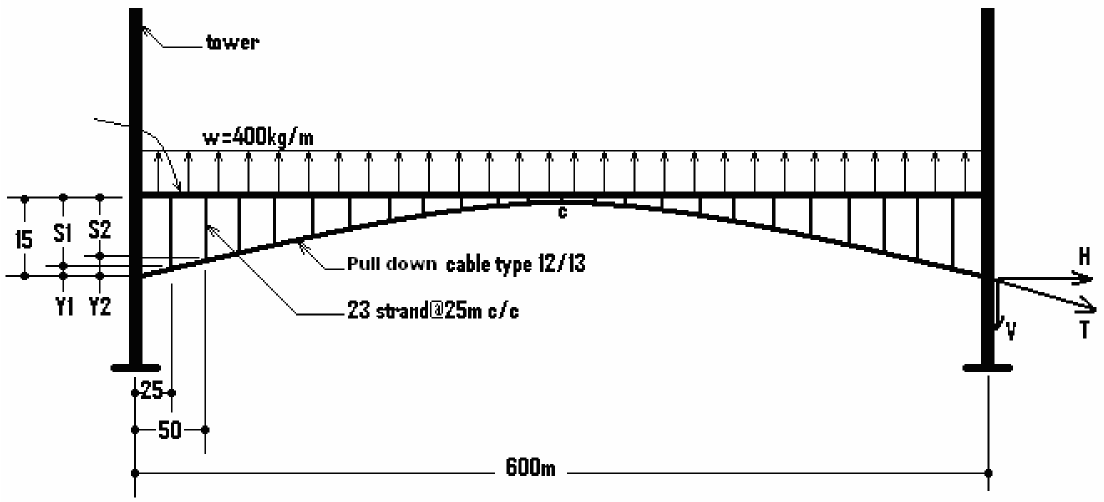


Fig. 6 Detail of a Pull-down cable system for stiffening a suspension bridge.

For the two Pull-down cables that are required to stiffen the aforementioned suspension bridge deck slab, we adopt a cable type 12/13 with the following properties (Westbrook, 1984):

- Strand diameter = 12.5 mm
- Cable diameter = 52 mm
- Strand cross-sectional area = 94.2 mm^2
- Cable cross-sectional area =1130 mm^2
- Ultimate load =1980 kN
- Initial force = 1380 kN = 138Ton

Based on the given data,

1. Vertical reaction $V = \frac{wl}{2} = \frac{0.4 \times 600}{2} = 120Ton$

2. Horizontal reaction $H = \frac{wl}{8h} = \frac{0.4 \times 600}{8 \times 15} = 2Ton$

3. Maximum tension $T_{max.} = \sqrt{V^2 + H^2} = \sqrt{120^2 + 2^2} = 120Ton$.

Note that; $T_{required} = 120\ Ton < T_{allowable} = 138\ Ton \Rightarrow O.K$

4. Length of the cable $L = \ell + \frac{8h^2}{3\ell} = 600 + \frac{8 \times 15^2}{3 \times 600} = 601m$

5. Parabolic equation of the Pull-down cable is given by

$$y = \frac{4hx(\ell - x)}{\ell^2} = \frac{4 \times 15x(600 - x)}{360000} = \frac{600x - x^2}{6000}$$

6. Tension force at each strand – In order to restore the parabolic shape of the Pull-down cable, under the effect of a uniformly distributed upward force, equally spaced strands are used. In this example, the spacing between center to center of strands will be taken as 25m. Each strand, under the effect of full loading, will pull down the amount of $25 \times 0.4 = 10Ton$

7. Required number of strands is $\frac{600}{25} - 1 = 23$

8. Length of each strand

Length of the first strand (25m away from the bridge tower) is:

$$S_1 = h - y_1 = h - \frac{600x - x^2}{6000} = 15 - \frac{600 \times 25 - 25^2}{6000} = 12.6m$$

Length of the second strand (50m away from the bridge tower) is:

$$S_2 = h - y_2 = h - \frac{600 \times 50 - 50^2}{6000} = 10.4m$$

Length of the central strand (300m away from the bridge tower) is:

$$S_{12} = h - y_{12} = 15 - \frac{600 \times 300 - 300^2}{6000} = 0$$

The above numerical example demonstrates the structural procedure of using the suggested new method of applying arched high tensile steel cables and equally spaced strands to stiffen a suspension bridge. It is clear that any dynamic forces (up lifting pressure in this case) induced by wind or seismic activities can be eliminated by the ability of the suggested system to resist tension forces. Maintaining the bridge deck slab in its original position under the effects of the mentioned external forces structurally means that the structure became stiffer than before.

5. Conclusions

Normally, any suspension bridge deck slab will be subjected to wind and seismic forces. These forces will cause vibrations i.e., repetitive upward and downward pressures. While the downward pressure can be resisted by the bridge structure, the upward pressure will be free to camber the deck slab or even break it. By adopting the suggested Pull-down cable system for both sides of any suspension bridge, this problem can be solved due to the ability of such system to prevent upward deck slab movements.

The expected advantages of applying the suggested Pull-down cable system to an existing suspension bridge is to stiffen its deck slab against wind and Seismic

Vibrations, while, by applying this method to a suspension bridge during its design stages can reduce the size of its stiffening girders.

Finally, more experimental work is required to transfer this theoretical approach from theory to the practical field. Plans are underway to build a prototype for testing in a wind tunnel.

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