

# An experimental study on settling velocity of regular shaped elements for underwater erosion protection

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## Abstract

Underwater erosion protection works had always been a difficult task, particularly in flowing water. Knowledge of settling behavior of protective elements is important for any underwater construction with protective elements. Experiments have been conducted to investigate the settling behavior of underwater protective elements. Three different regular geometric shapes (cubic, square box-shaped and rectangular box-shaped) made of two different materials (sand-cement blocks and geobags with loose sand) have been considered. Prior to experimentation, an expression for estimating the settling velocity has been derived using dimensional analysis of salient variables involved in the process. The functional arrangement obtained from the analysis takes advantage of the fact that the particle Reynolds number is a function of only one variable, the dimensionless particle parameter. Experimentation is conducted in a 130 cm Plexiglas settling column of size 30 cm X 30 cm fabricated at the Hydraulics and River Engineering Laboratory of Water Resources Engineering Department, BUET. A total of 240 numbers of settling velocity measurements for sixteen different elements have been conducted and their falling behavior have been observed. The experimental results have been analyzed to develop a relationship for predicting the settling velocities. Comparisons have been made with the relevant settling velocity formulas available in literature. It is found that the proposed formula has reasonable prediction accuracy with an error of 2.38% for geobag and 3.91% for sand-cement block. It is hoped that the outcome of this study will be helpful for determining the underwater placement of protective elements.

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## 1. Introduction

In recent decades, there has been a consistent trend of increased use of the settling (or fall) velocity for sediment as it explains the underwater behavior well. The settling velocity of

regular shaped element is of great importance since they are extensively used in construction of various underwater structures. An accurate description of elements settling velocity is of particular interest, as other hydro-dynamic properties of the elements under moving water may be inferred from it.

Numerous investigators derive settling velocity of sediment particles. Cheng (1997) proposed an explicit formula for evaluating the settling velocity of individual natural sediment particles. Ahrens (2000, 2003) and Chang and Liou (2001) proposed formulas working with the quartz data subsets of Hallermeier (1981). The limiting value of particles Reynolds number is around 1000. Jiménez and Madsen (2003) proposed a relationship which affords the ability to predict settling velocities that accounts for the influence of particle shape and roundness, if these are known. However, his formula covers the sand range (nominal diameter up to 2 mm) only. Smith and Cheung (2003) studied the settling characteristics of calcareous sand. Göğüş et al. (2001) worked extensively to develop a technique to predict shape factor and settling velocity of both regular and irregular shaped particles.

Cosiderable research has also been performed by Mehta et al. (1980), Boillat and Graf (1981), Swamee and Ojha (1991), Chang (1998) and Camenen, B. (2007). However, very few researches had been conducted for relatively large particles which can be used in other laboratory experiments.

When an element falls through a fluid, the velocity at which the drag and gravity forces acting on the element are in balance is defined as its settling velocity ( $w$ ). For single elements, the fall velocity can be predicted from the equilibrium between the gravity and drag forces. In addition to the gravitational force on the element, element motion depends on the magnitude of forces caused by local flow patterns that develop around a freely falling element. These patterns are as follows:

1. Separation: When the Reynolds number increases, the pattern of flow separation changes. Flow separation affects the shear and pressure distribution on the surface. If the separation point is well forward on the body there is a reduction in shear, an increase in pressure, and an increase in drag. The reverse effects are observed if the separation point is well downstream from the point of stagnation.
2. Vortex formation: As the separation zone develops, vortices are formed at the trailing edge of the particle, and they create fluctuations in pressure and alternating transverse thrust and torque on the particle.
3. Circulation is defined as the line integral of the tangential velocity component about any closed contour in the flow field. If a submerged particle rotates, the additional motion gives rise to circulation that causes a lift force acting on the particle perpendicular to the motion of the particle (Alger and Simons, 1968 and Mehta et al., 1980).

Because of the fluctuating forces, the fall of an element in a liquid may be subjected to three classes of motion: sliding, tipping, and rotation. These forms of motion may occur separately or in combination. They affect the fall velocity of the element, and are generally related to a Reynolds number and physical properties of the element.

In this paper, the settling velocities of elements having three different regular geometric shapes (cubic, square box-shaped and rectangular box-shaped) made of two different materials (sand-cement and geobags with loose sand) and of different sizes falling axially through water at rest in a settling column have been experimentally investigated. The available formulae for predicting fall velocity are not satisfactory for those elements. The

purpose of the experimental study was to determine a relationship between the Reynolds number and elements physical properties.

## 2. Review of literatures

In 1851, Stokes obtained the solution for the drag resistance of flow past a sphere by expressing the simplified Navier-Stokes equation together with the continuity equation in polar coordinates. Using his solution, the following expression for settling velocity of spherical particles can be derived as:

$$R^* = \frac{\Delta g d^3}{18\nu^2} \quad (1)$$

$$\text{or, } R^* = \frac{A}{18} \quad (2)$$

where  $\Delta = (\rho_s - \rho)/\rho$ ;  $\rho_s$  and  $\rho$  = density of the particle and the density of the fluid, respectively;  $g$  = acceleration due to gravity;  $d$  = characteristics diameter of the particle;  $\nu$  = kinematic viscosity of water;  $A = \Delta g d^3 / \nu^2$  = Archimedes buoyancy index. Equation (1) is only valid for  $R^* < 1$ .

Rubey (1933) developed a simple equation to predict fall velocity based on equating the buoyant weight of a particle to the sum of viscous and turbulent flow resistance. Rubey's (1933) equation is:

$$R^* = \sqrt{\left\{ 36 + \frac{2g(\rho_s - \rho)d^3}{3\rho\nu^2} \right\}} - 6 \quad (3)$$

Hallermeier (1981) made an extensive study on settling velocity of particles over a wide range of Reynolds number. For turbulent flow the equation is:

$$R^* = 1.05\sqrt{A} \quad (4)$$

Van Rijn (1993) proposed a very simple equation to predict settling velocity as:

$$w = 1.1\sqrt{\Delta g d} \quad \text{for } d > 0.1 \text{ cm} \quad (5)$$

Cheng (1997) shows two general relationships for drag coefficients for sediment particles falling in a fluid; they may be written as:

$$C_D = \frac{4\Delta g d}{3w^2} \quad (6)$$

and

$$C_D = \left\{ (a/R^*)^{1/c} + b^{1/c} \right\}^c \quad (7)$$

where  $C_D$  = drag coefficient. Coefficients  $a$ ,  $b$ , and  $c$  are dimensionless numbers which have approximately the following role: coefficient  $a$  is important at low Reynolds numbers, laminar flow; coefficient  $b$  is important at high Reynolds numbers, turbulent flow; and coefficient  $c$  was determined by fitting to data, with Reynolds numbers in the range  $1 < R^* < 1000$ . Cheng (1997) gives the following values for these constants:  $a = 32$ ,  $b = 1.0$ , and  $c = 1.5$ , that are appropriate for his data set. Combining Equations (6) and (7) and solving for the positive root of the quadratic equation gives Cheng's (1997) settling velocity equation as:

$$R^* = \left( \sqrt{25 + 1.2d_*^2} - 5 \right)^{1.5} \quad (8)$$

where  $d_* = d \left( \frac{\Delta g}{\nu^2} \right)^{1/3}$  is the dimensionless particle parameter.

Chang and Liou (2001) suggested a formula for computation of settling velocity in a fractional form as:

$$R^* = \frac{aA^n}{18(1 + aA^{n-1})} \quad (9)$$

When the value of  $A$  is small and the value of  $n$  is less than one, (9) turns out to be  $R^* = A/18$ , because the value of  $aA^{n-1}$  is much larger than one. When the value of  $A$  is large, (9) is converted into  $R^* = aA^n/18$ , because the value of  $aA^{n-1}$  is smaller than one. Chang and Liou (2001) suggested the following values for the coefficients  $a = 30.22$  and  $n = 0.463$ .

Göğüş et al. (2001) developed an iterative technique in order to find settling velocity of regularly shaped angular particles. This technique consists of the following equations:

$$\Psi = \left( \frac{a_1 + b_1}{c_1} \right) \left( \frac{a_1 + b_1 + c_1}{\nabla} \right) \quad (10)$$

$$R_* = w\rho\sqrt{(a_1 + b_1)} / \mu \quad (11)$$

$$C_D^* = \alpha(R_*)^\beta / \Psi \quad (12)$$

where  $\Psi$  = shape factor;  $a_1$ ,  $b_1$  and  $c_1$  = maximum, intermediate and minimum dimension of a particle respectively;  $\nabla$  = volume of original particle;  $R_*$  = modified Reynolds number;  $\rho$  = density of the fluid;  $\mu$  = dynamic viscosity of the fluid;  $C_D^*$  = modified drag coefficient;  $\alpha$ ,  $\beta$  = empirical constants related with shape factor,  $\Psi$ .

### 3. Theoretical analysis

To estimate the settling velocity of elements, two different approaches can be followed: (1) an idealized one in which the element is assumed to be a sphere; and (2) a more realistic one in which the natural shape is considered. In general, the first approach is used extensively (for instance, sediment grain size is calculated by assuming it to be spherical), although some methods take into account the sediment shape.

For single elements, the settling velocity can be predicted from the equilibrium between the gravity and drag forces, the drag coefficient  $C_D$  being the main unknown. Stokes (1851) found

that  $C_D$  is inversely proportional to the particle Reynolds number  $R^*$  ( $R^* = wd / \nu$  where  $d$  = diameter of the particle) when  $R^* < 1$ . On the other hand, under the condition of high Reynolds number ( $R^* > 10^5$ ), the drag coefficient was found to be a constant (Dallavalle, 1948 and Schlichting, 1979).

The settling velocity of a sphere in water can be estimated by solving the balance between the gravitational force or submerged weight force and the drag resistance:

$$\text{Gravitational force, } F_g = (\rho_s - \rho)g \frac{\pi}{6} d_n^3 \quad (13)$$

where  $d_n$  is the nominal diameter of the element defined as the diameter of a sphere having the same volume and mass as the measured element. It can be calculated as:

$$d_n = \left( \frac{6V}{\pi} \right)^{\frac{1}{3}} = (d_l d_w d_t)^{\frac{1}{3}} \quad (14)$$

where  $V$  = original volume of the element;  $d_l$ ,  $d_w$ , and  $d_t$  are the respective length, width and thickness of the element.

$$\text{Drag force, } F_D = C_D \frac{\pi}{4} d_n^2 \frac{\rho w^2}{2} \quad (15)$$

where  $C_D$  = drag coefficient.

At terminal velocity, the drag force on the element is equal to the element's submerged weight. From equation (13) and (15), drag coefficient can be expressed as:

$$C_D = \frac{4\Delta g d_n}{3w^2} \quad (16)$$

Using the dimensionless particle parameter,  $d_*$  defined as

$$d_* = d_n \left( \frac{\Delta g}{\nu^2} \right)^{\frac{1}{3}} \quad (17)$$

From equation (16) and (17) another relationship for  $C_D$  is found to be

$$C_D = \frac{4d_*^3}{3R^{*2}} \quad (18)$$

Drag coefficient can be determined from equation (16) or equation (18) if the settling velocity is known.

### 3.1 Development of non-dimensional relationship for settling velocity

Settling velocity of an angular element through a fluid can be expressed as a function of relevant variables as follows:

$$w = f(W', \rho, L, \mu, D, \Psi) \quad (19)$$

where  $W'$  = submerged weight of the element;  $L$  = characteristics length of the element;  $\mu$  = dynamic viscosity of water;  $D$  = diameter of the settling column; and  $\Psi$  = non-dimensional factor describing the shape of the element.

In non-dimensional form equation (19) can be written as:

$$f\left(\frac{W' \rho}{\mu^2}, \frac{wL\rho}{\mu}, \frac{D}{L}, \Psi\right) = 0 \quad (20)$$

The submerged weight,  $W'$  can be replaced by  $g(\rho_s - \rho)L^3$ . The characteristics length of the element is selected as the nominal diameter ( $d_n$ ) defined by equation (14). Following this assumption the characteristics length,  $L$ , used in the expressions of the first, second and third non-dimensional term of equation (20), is replaced by  $d_n$ . After all these changes, equation (20) becomes:

$$f\left(\left(\frac{\Delta g}{v^2}\right)^{\frac{1}{3}} d_n, \frac{wd_n}{v}, \frac{D}{d_n}, \Psi\right) = 0 \quad (21)$$

The first and second non-dimensional term of equation (21) can be replaced by dimensionless particle parameter ( $d_*$ ) defined in equation (17), and  $R^*$  respectively. Inserting  $d_*$  and  $R^*$  into equation (21) results in:

$$f\left(d_*, R^*, \frac{D}{d_n}, \Psi\right) = 0 \quad (22)$$

The effect of the settling column wall on the fall velocity predictions is considered in the term  $D/d_n$  of equation (22). However, the ratio of settling column diameter to maximum dimension of the element used in the experiments showed that this effect is negligible in this study (McNown et al., 1948; McNown and Newlin, 1951).

The shape of the element influences its settling velocity. For the present study, the shape of the elements was fixed, only cubes and box-shaped prism. Moreover, the  $C_D$  value decreases rapidly outside the Stokes region ( $R^* < 1$ ) and becomes nearly constant for  $10^3 < R^* < 10^5$  (Van Rijn, 1993). When the element settling velocity increases ( $R^* > 10^4$ ), then the effect of shape on the drag decreases continuously (Göğüş et al., 2001). Therefore,  $\Psi$  may be assumed to be constant in this study. Thus, equation (22) can be reduced to:

$$R^* = f(d_*) \quad (23)$$

#### 4. Experimentation

Experiments have been carried out to verify equation (23). For this purpose, a square shaped Plexiglas settling column of 30 cm a side and 130 cm height was constructed at the Hydraulics and River Engineering Laboratory, BUET (Raju, 2011). The confining effect of the size of settling column on the fall velocity may be evaluated on the basis of the work of McNown et al. (1948) who related  $d/D$  with  $W_d/W_i$ , for a range of particle Reynolds number in which  $d$  = diameter of a falling sphere;  $D$  = diameter of the column;  $W_d$  = measured fall velocity of the sphere; and  $W_i$  = fall velocity of the same sphere in a fluid of infinite extent. The longest dimension of the elements used in the experiment was 7.2 cm, so that  $d/D = 0.24$ , if the particle is replaced by a sphere of 7.2 cm diameter. For this ratio and  $R^*$  in the range of  $9 \times 10^3$  to  $4 \times 10^4$  corresponding to the range of test data,  $W_d/W_i = 0.95$ , which indicates less than 5% measurement error caused by the column. The settling column is shown in Photograph 1.

In this study the sand cement blocks used are cubical and box-shaped prism of ten sizes. The cube shaped blocks are commonly used as erosion protection elements in Bangladesh context. Others are used to have a generalized and more precise correlation for fall velocity computation. Different blocks used for the present study is shown in Photograph 2. The dimensions of blocks are listed in Table 1. Geobags of six different sizes are used in this study as shown in Photograph 3. They are rectangular and square shaped. The length to width ratio ranges from 1.73 to 1.09. The dimensions of the bags are listed in Table 2.



Photograph 1: Plexiglas settling column in the Hydraulics and River Engineering Laboratory, BUET



Photograph 2: Various sizes of sand cement block used in the study

Table 1  
Dimension of sand cement blocks

Type of block	Length, $d_l$ (mm)	Width, $d_w$ (mm)	Thickness, $d_t$ (mm)
D1a	22.90	23.16	24.10
D2a	20.98	20.72	20.48
D3a	15.96	15.98	16.02
D4a	30.08	31.30	16.26
D5a	25.70	26.10	19.20
D6a	16.24	16.12	13.00
D7	31.30	30.88	16.26
D8	40.10	40.72	16.56
D9	40.68	40.60	20.60
D10	40.98	41.28	26.66



Photograph 3: Various sizes of geobag

Table 2  
Dimension of geobags used in the experiment

Type of geobag	Length, $d_l$ (mm)	Width, $d_w$ (mm)	Thickness, $d_t$ (mm)
A1	60.24	38.60	7.02
A2	51.94	47.70	7.02
B1	51.60	29.80	8.60
B2	42.90	38.00	9.04
C	42.06	26.20	8.14
E	71.20	40.90	10.9

## 5. Measurements and observations

Following stepwise procedure have been followed for settling velocity measurement in the laboratory:

- i) At first five numbers of elements is randomly taken from each type to measure its physical properties. They are immersed in water for one day before taking wet weight. After oven drying dry weight is measured. Also the dimension is measured with a slide calipers.
- ii) Clear, fresh water is poured in the column and waited for about five hours to attain uniform temperature and zero velocity. Trial test is done before final test to observe the performance of the column with water.
- iii) Elements are immersed in water for one day before conducting the experiment.
- iv) The elements are released with the help of a tweezer and observed them crossing the initial line of measurement. The time between the initial line and final line has been recorded with a stop watch.
- v) Elements were released in the water with zero departure velocity and without any rotation. The initial orientations of geobags and prism shaped blocks are with their maximum surface areas and cubes with one of the surfaces normal to the motion of the particle.
- vi) They were released 5 cm below the maximum water level with the help of tweezer. Their required time of fall over 90 cm vertical distances was timed by a stopwatch, which had 0.01 second accuracy.
- vii) The fall velocities of elements were low enough that there was no need to use photographic or any other sophisticated method of measurement; therefore the elements could be timed using a stopwatch over the chosen distance of 90 cm. The test conducted for each element was repeated three times under the same conditions to reduce the probable error of the average observed time interval. Therefore the ultimate fall velocity for each type of element is obtained by averaging fifteen measurements.
- viii) A thermometer was placed at the upper, middle and lower section of the column to measure the temperature of the water. The temperature gradient between the top and the bottom of the column was found negligible during all experiments. All elements were dropped in water at the temperatures ranging between  $28^{\circ}C$  and  $28.5^{\circ}C$  and the effect of viscosity on fall velocity is negligible.
- ix) The behavior of element while falling is monitored and documented over the period of experiment by taking snaps and videos.

During experiments, the following observations were made:

- i) The cubic blocks, in general, did not follow the centerline of the column while they were falling, and in addition to tipping and sliding, they rotated all the time following a helical path.
- ii) The box shaped blocks followed almost the centerline of the settling column having the largest surface area perpendicular to the motion of the particle. Oscillation about the shortest axis and little sliding was observed.
- iii) Sand cement block falls faster than geobags as shown in Photograph 4.
- iv) As geobags have voids, their travel trail is not fixed.
- v) The path followed by geobag was nonvertical and approximately helical with significant sliding.
- vi) Initial orientation of falling of an element has no effect on settling velocity.



Photograph 4: Sand cement block is falling faster than geobag in the settling column

## 6. Results and discussions

The value of different parameters of settling velocity test for sand-cement block is presented in Table 3. The measured relative density and kinematic viscosity was 1.074 and  $8 \times 10^{-7} \text{ m}^2/\text{s}$  respectively. On the basis of expression of settling velocity shown in equation (23), a power regression analysis of the experimental data has been performed. The plot is shown in Figure 1. The coefficient of determination ( $R^2$ ) is found to be 0.96 indicating a good correlation. It is seen that as the dimensionless particle parameter increases the particle Reynolds number also increases. The final expression of settling velocity for block becomes:

$$R^* = 3.965 d_*^{1.33} \quad (24)$$

Equation (24) is valid within the experimental ranges of  $10^4 < R^* < 4 \times 10^4$  for particle Reynolds number and  $380 < d^* < 900$  for dimensionless particle parameter.

Table 3:  
Parameters of settling velocity test for different sand-cement block

Type of sand cement block	Characteristics diameter, $d$ (m)	Dimensionless particle parameter, $d^*$	Settling velocity, $w$ (m/s)	Particle Reynolds no., $R^*$
D1a	0.023	589.86	0.732	21124
D2a	0.020	522.78	0.647	16547
D3a	0.016	403.49	0.595	11745
D4a	0.024	626.18	0.631	19330
D5a	0.023	591.12	0.738	21342
D6a	0.015	379.33	0.574	10652
D7	0.025	631.68	0.636	19655
D8	0.030	756.93	0.66	24440
D9	0.032	817.16	0.729	29144
D10	0.035	897.65	0.85	37328

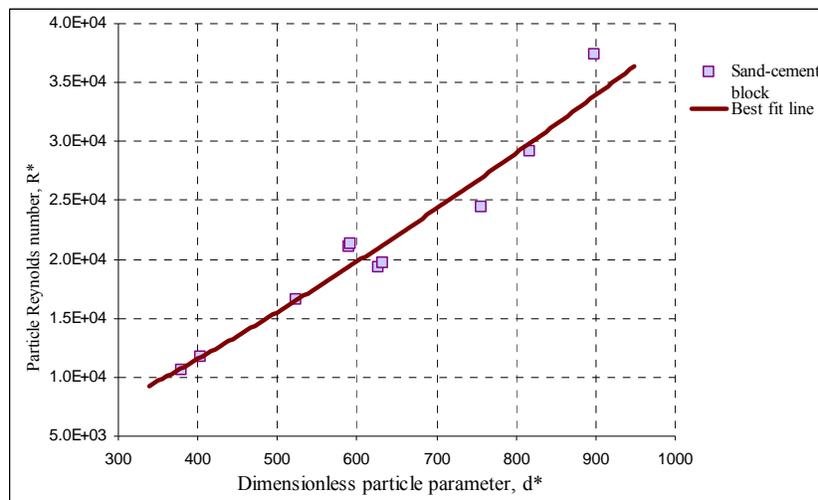


Fig. 1. Particle Reynolds number versus dimensionless particle parameter for sand-cement block

The value of different parameters of settling velocity test for geobag is presented in Table 4. The measured relative density was 0.5349. On the basis of expression of settling velocity shown in equation (23), a power regression analysis of the experimental data has been performed. The plot is shown in Figure 2. The coefficient of determination ( $R^2$ ) is found to be 0.95 indicating a good correlation. It is seen that as the dimensionless particle diameter increases the particle Reynolds number also increases which is consistent with literature. The two data point beside the best fit line is for square shaped geobag. The final expression of settling velocity for geobag becomes:

$$R^* = 6.124 d_*^{1.21} \quad (25)$$

Equation (25) is valid within the experimental ranges of  $9 \times 10^3 < R^* < 16 \times 10^3$  and  $410 < d_* < 640$ .

Table 4  
Parameters of settling velocity tests for different geobag

Type of geobag	Characteristics diameter, $d$ (m)	Dimensionless particle parameter, $d_*$	Settling velocity, $w$ (m/s)	Particle Reynolds no., $R^*$
A1	0.02537	510.92	0.373	11807
A2	0.02591	521.83	0.357	11542
B1	0.02365	476.29	0.367	10829
B2	0.02452	493.80	0.397	12146
C	0.02078	418.48	0.357	9256
E	0.03166	637.71	0.396	15646

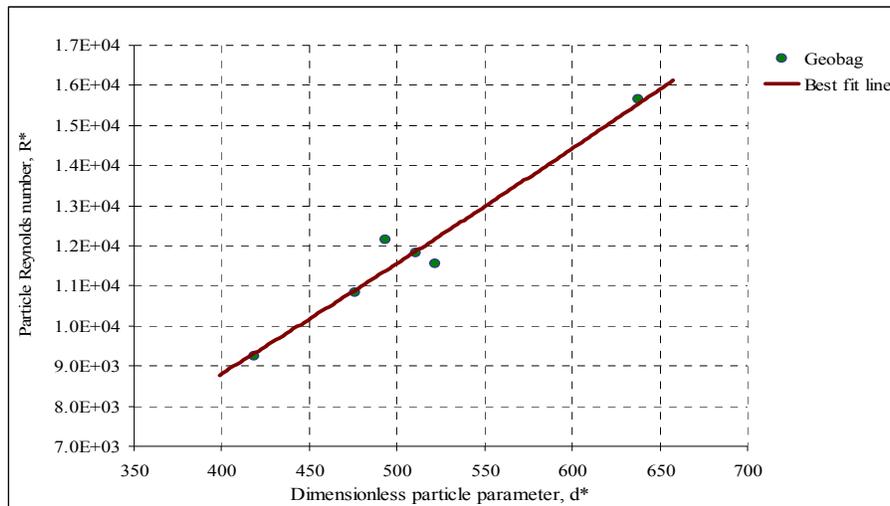


Fig. 2. Plot of particle Reynolds number against dimensionless particle parameter for geobag

The basic parameter used for the determination of accuracy of a formula is the average value of relative error where error is defined as

$$error = \frac{|predicted - observed|}{observed} \times 100 \quad (26)$$

### 6.1 Comparison of various settling velocity formulas

A comparison here can be made by predicted values using the equation of different investigators mentioned earlier and the measured values from the experiment.

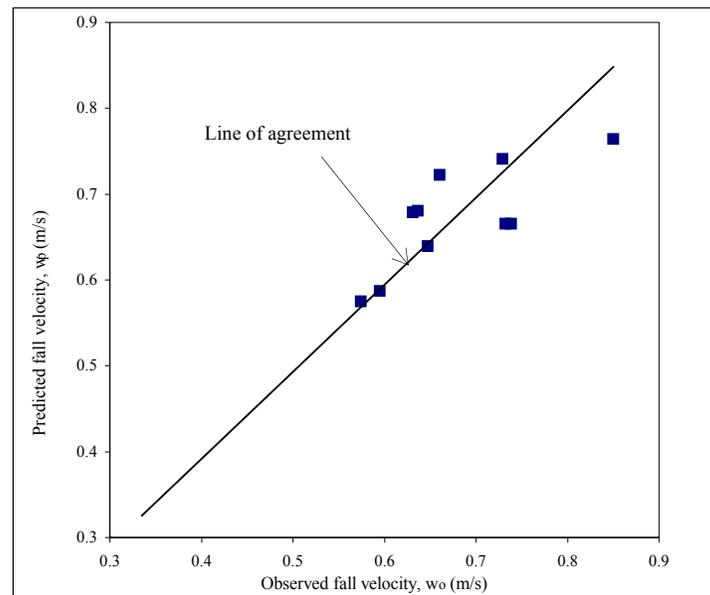


Fig. 3. Comparison of observed and predicted fall velocity using equation (24) for sand cement block

Figure 3 represents the predicted fall velocity by proposed empirical equation (equation 24) versus measured in this study. Three points are below the line of perfect agreement indicating predicted value is less than measured one. Three points are above the perfect line indicating predicted value is greater than measured one. Four points lie on the line of perfect agreement. The average relative error is 3.91%.

Figure 4 represents the predicted fall velocity by proposed empirical equation (equation 24), Van Rijn (1989) and Cheng (1997) versus measured in this study. According to Van Rijn (1989) all data points are below the perfect line indicating predicted value is less than measured one. The average relative error is 12.97%. According to Cheng (1997) all data points except one are below the perfect line indicating predicted value is less than measured one. The average relative error is 10.95%.

Figure 5 represents the predicted fall velocity by proposed empirical equation (equation 24), Hallermeier (1981), Göğüş et al. (2001) versus measured in this study. According to Hallermeier (1981) all data points are below the perfect line indicating predicted value is less than measured one. The average relative error is 21.78%. According to Göğüş et al. (2001) four points are below the perfect line indicating predicted value is less than measured one. Four points are above the perfect line indicating predicted value is greater than measured one. Two points lie on the line of perfect agreement. The average relative error is 23.25%.

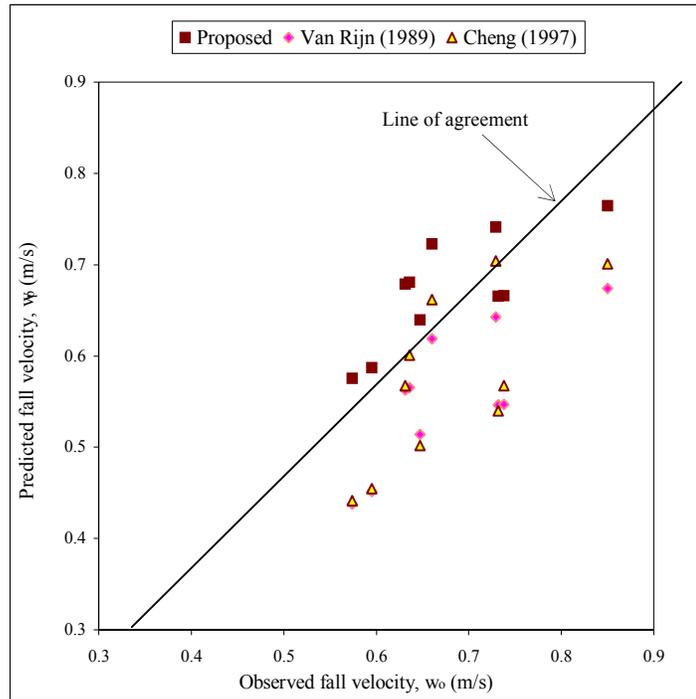


Fig. 4. Comparison of observed fall velocities with those calculated from formula proposed by Van Rijn (1989) and Cheng (1997) for sand cement block

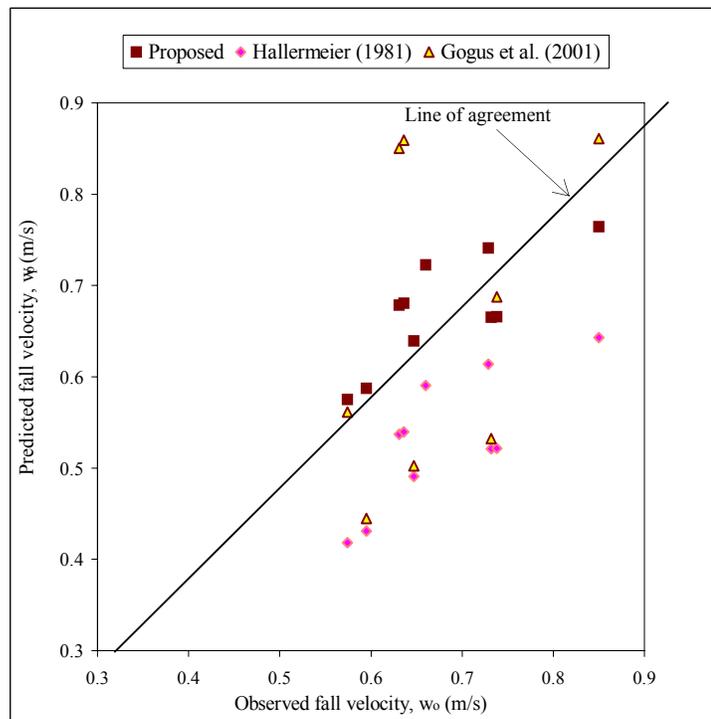


Fig. 5. Comparison of observed fall velocities with those calculated from formula proposed by Hallermeier (1981) and Gogus et al. (2001) for sand cement block

Figure 6 represents the predicted fall velocity by proposed empirical equation (equation 25) versus measured in this study. Four points lie on the line of perfect agreement. One point lie down in positive and one point on negative side. The average relative error is 2.38%; where error is obtained by equation (26).

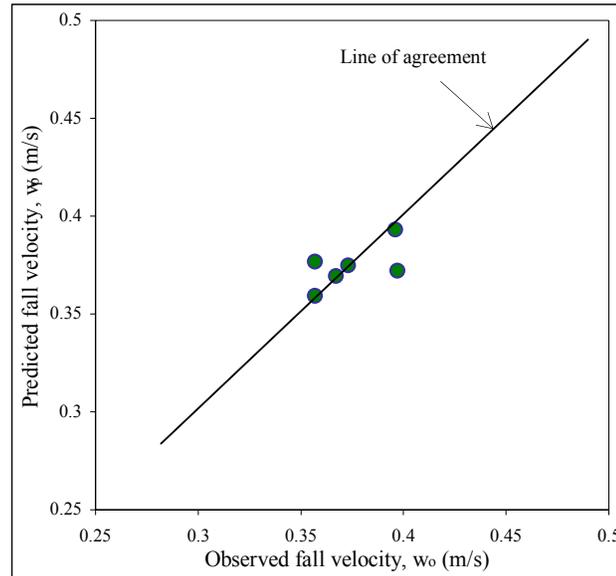


Fig. 6. Comparison of observed and predicted fall velocity using equation (25) for geobag

Figure 7 represents the predicted fall velocity according to Van Rijn (1989) and Cheng (1997) versus measured in this study. According to Van Rijn (1989) two points lie on the line of perfect agreement. Four points are above the perfect line indicating predicted value is greater than measured value. The average relative error is 7.06%. According to Cheng (1997) no data point lie on the line of perfect agreement. Four points are above the perfect line indicating predicted value is greater than measured value. Two points are below the perfect line indicating predicted value is less than measured value. The average relative error is 9.87%.

Figure 8 represents the predicted fall velocity according to Hallermeier (1981) and Chang and Liou (2001) versus measured in this study. According to Hallermeier (1981), no data point except one, lie on the line of perfect agreement. Three points are above the perfect line indicating predicted value is greater than measured value. Two points are below the perfect line indicating predicted value is less than measured value. The average relative error is 4.67%. According to Chang and Liou (2001) no data point lie on the line of perfect agreement. All data points lie below the line of perfect agreement indicating predicted value is less than measured value. The average relative error is 15.67%.

## 6.2 Summary of comparison

Comparison among fall velocity formula mentioned previously is performed on the basis of measured fall velocities of sixteen different sized particles. The summary of the comparison is presented in Table 5. From the table it is seen that the proposed empirical equations have the lowest error. Equation (24) has an average error of 3.91% where Cheng (1997) is the next with 10.95% error. Equation (25) has an error of 2.38% where Hallermeier (1981) equation performs better than the rest with 4.67% error.

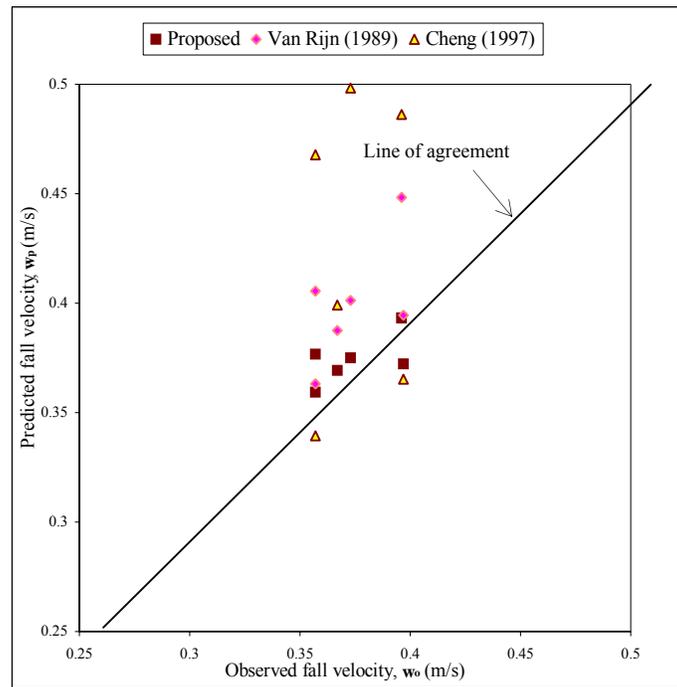


Fig. 7. Comparison of observed fall velocities with those calculated from formula proposed by Van Rijn (1989) and Cheng (1997) for geobag

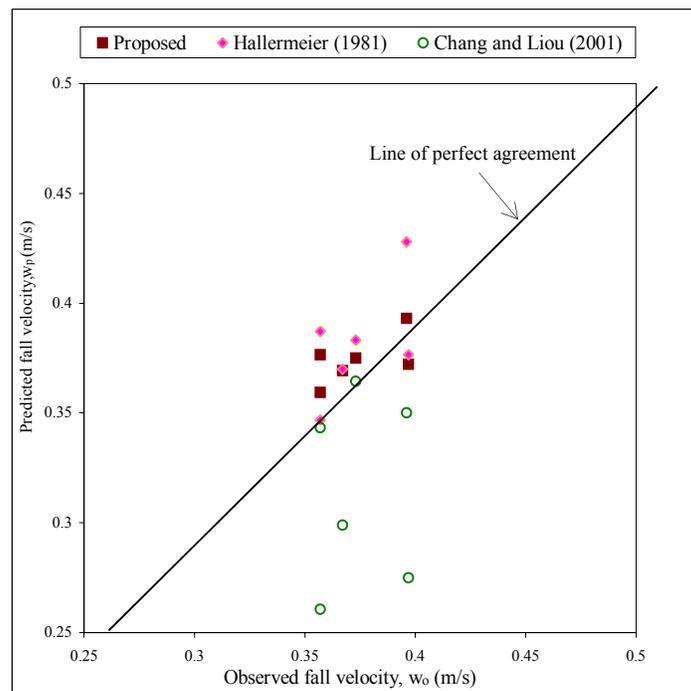


Fig. 8. Comparison of observed fall velocities with those calculated from formula proposed by Hallermeier (1981) and Chang and Liou (2001) for geobag

Table 5  
Performance of various settling velocity prediction formulas

Settling velocity formulas	Error (%)	
	Block	Geobag
Present study	3.91	2.38
Cheng (1997)	10.95	9.87
Van Rijn (1989)	12.97	7.06
Hallermeier (1981)	21.78	4.67
Chang and Liou (2001)	38.55	15.67
Ruby (1933)	30.27	14.01
Gögüş et al. (2001)	23.25	high

The errors in predicting settling velocity by the previously mentioned formulas is mainly due to the fact that the size of the elements used in this experiment is quite larger than those used in other studies. Most of the equations are proposed for natural sand particles (e. g. Ruby, 1933; Hallermeier, 1981; Cheng 1997; Chang and Liou, 2001). Cheng (1997) used the largest size of particle of 4.5 mm and Van Rijn (1989) formula is for particles larger than 1 mm. Gogus et al. (2001) conducted experiments using larger particles but the proposed characteristic dimensions of a particle is not proficient to predict the settling velocity of the elements used in this experiment.

## 7. Conclusion

Analysis of various hydro-dynamic phenomena of an element under flowing water will be more expedient incorporating the settling velocity. Simple empirical relationships have been developed to estimate of the settling velocity of individual regular shaped protective elements. The formulas are applicable to very high particle Reynolds number. Various investigators reported settling velocity data that are mostly for small sediment particles with a relatively low particle Reynolds number. The settling velocity data for the size of elements measured in this study is scarce in literature. Comparisons with formula available in literature show that the proposed formula predicts the settling velocity with reasonable accuracy. It is expected that the simple relationships obtained here can be useful for estimating the settling distance of individual protective elements for underwater construction. Scope of detailed analyses in this area of interest is in progress.

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