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Effect of reinforcement on punching shear of multi-panel flat slab

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Abstract

Effect of flexural reinforcement on punching shear behavior of reinforced concrete multi-panel flat slabs is investigated in this study. Non-linear finite element material model strategy based on past experimental investigations for several types of concrete strength and reinforcement ratio is used. It has been found that the flexural reinforcements embedded in the slab play significant roles on punching shear capacity. It has been observed that the unit punching shear strength of multi-panel slabs increases with increase of flexural reinforcement ratio. A proposal for calculating punching shear capacity of flat slab is incorporated in this paper based on the findings of investigation. The proposal includes the effect of flexural reinforcement in addition to concrete strength in estimating the punching capacity. The punching shear capacity estimated using the proposed expression is compared with the results of non-linear finite element analysis and has been found to be in good agreement. The estimated punching capacity is also compared with some code equations.

Keywords: Multi-Panel, Flat Slab, Flexural Steel, Numerical Analysis, Punching Shear

1. Introduction

Design codes such as the American code (ACI 318-2011), Canadian Standard (CSA-A23.3-04 (R2010)) and Australian code (AS 3600-2009) do not reflect the influence of the flexural reinforcement ratio on the punching capacity of slab-column connections. For the design of punching shear, these code provisions rely mostly on empirical methods derived from the test results on simply supported conventional and thin slab specimens (Kuang and Morley 1992, Alam *et al*. 2009). In continuous slab, all panel edges cannot rotate freely, in contrast to its simply supported counterpart. Investigations from multi-panel slabs will be more reasonable than the results obtained by using isolated single span slab specimens. However, multi-panel tests are time consuming, expensive and it is difficult to determine experimentally the shears and moments applied to the individual slab-column connections. An alternative to such expensive and difficult experimental procedure is to perform the investigation by means of numerical finite element analysis.

Nonlinear finite analysis procedures are reliable and popular in recent years as engineers attempt to more realistically model the behavior of structures subjected to all types of loading. Computer simulation makes the accuracy for describing actual behavior of the structures, compare the behavior with laboratory experimenting methods, prospects in the process of scientific research, and relation with experiment and analysis methods. It is very important that before practical application finite element analysis methods should be verified and validated comparing the analysis results with reliable experiment data.

In this study, an advanced non-linear finite element investigation of multi-panel flat slab considering full scale with practical geometry has been carried out on the behavior of punching shear characteristics of concrete slab in presence of flexural reinforcement. At first stage, FE model has been developed to simulate relevant experiments carried out earlier (Alam *et al.* 2009). Good agreement has been observed between numerical FE simulation and experiment, which establish the validity of FE model. Later on the same FE procedure has been used to analyze multi-panel slab models and the results are presented in this paper in an effort to understand the actual punching shear behavior of slab systems.

2. Experimental works

Experimental investigation and finite element analysis of 15 model slabs subjected to punching load was carried out by authors (Alam *et al.* 2009). The finite element simulation for these slabs has been performed by modeling the concrete with solid elements, and placing discrete reinforcing bars elements in the model (Alam and Amanat 2012). Details of FE material model and analysis procedure are included in the next sections of this paper. It was found that failure load and load deflection behavior of all model slabs predicted by FE analysis are reasonably matched with experimental result. Typical load-deflections of slabs are shown in Figure 1 while detailed results can be found in author's other papers.

The same FE analysis modeling has been use to simulate other independent experimental investigations of (Kuang and Morley 1992). Load-deflection curves of analysed and tested model by Kuang and Morley (1992) is shown in Figure 2. In this case also, very good agreement has been obtained between the FE analysis and the experimental data. Such good agreement between the FE modeling and several experimental data establishes the validity of the FE modeling technique in simulating the punching shear behaviour of flat slabs and thus such modeling can be further applied to numerically study the behaviour of multi panel flat plate systems as an alternative to experiments. Effect of edge restraint and flexural reinforcement were obtained from above study, which is used in multi panel flat slab effectively.

3. Finite element modelling

The non-linear finite element program DIANA (2003) is used in this study. This program is capable of representing both linear and non-linear behavior of concrete. For the linear stage, the concrete is assumed to be an isotropic material up to cracking. For the non-linear part, the concrete may undergo plasticity and/or creep. The total strain approach with fixed smeared cracking (i.e. the crack direction is fixed after crack initiation) is used in this study. For this approach, compression and tension stress–strain curve are used. The model based on total strain is developed along the lines of the Modified Compression Field Theory, originally proposed by Vecchio and Collins (1986). The three-dimensional extension of this theory is proposed by Selby and Vecchio (1993), which was followed during the implementation in FE modelling.

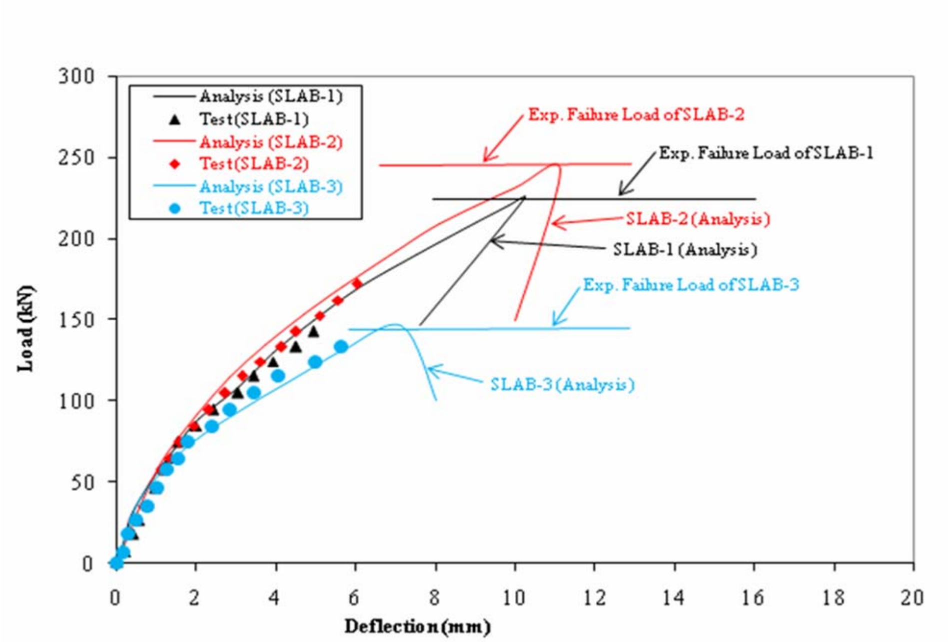

Fig. 1. Load-deflection curves of typical analyzed and tested model

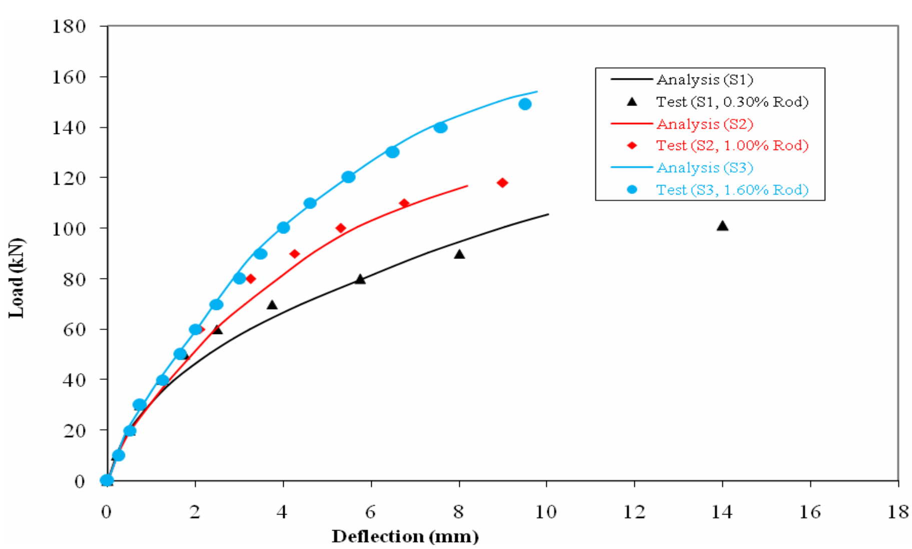

Fig. 2. Load-deflection curves of analysed and tested model by Kuang and Morley

3.1 Geometry of the model

Finite element model of the multi-panel full-scaled reinforced concrete flat slab, which has been studied in this paper, is shown in Figure 3. The model consists of four equal panels, each of 6000mm square with nine square columns of size 400mm x 400mm. The slab is extended 1500 mm outward from all columns to simulate continuous action beyond the column lines. All columns are extended by 1500mm from both top and bottom surface of slab. To achieve physical parameter as used in different building structures designed by different codes as well as to fulfill minimum slab thickness criteria, slab thickness of 200mm is used in this study.

All columns are vertically restrained at bottom ends and horizontally restrained both at top and bottom ends. Uniformly distributed load was applied on the top surface of slab to simulate actual behavior of practical slabs.

A total 30 model slabs with variation of compressive strength of concrete ($f_c^{'}$) and percentage of flexural reinforcement are analyzed in this study. Compressive strength of 24, 30, 40, 50 and 60 MPa for concrete are considered for analysis. Percentage of flexural reinforcements having 0.15%, 0.25%, 0.5%, 1%, 1.5% and 2% for each compressive strength of concrete are used.

3.2 FE mesh

The twenty-node isoparametric solid brick element (elements CHX60) was adopted for this study. Gaussian 2x2x2 integration scheme was used which yields optimal stress points. The typical full model and enlarged portion of same model after meshing are shown in Figure 4 and 5.

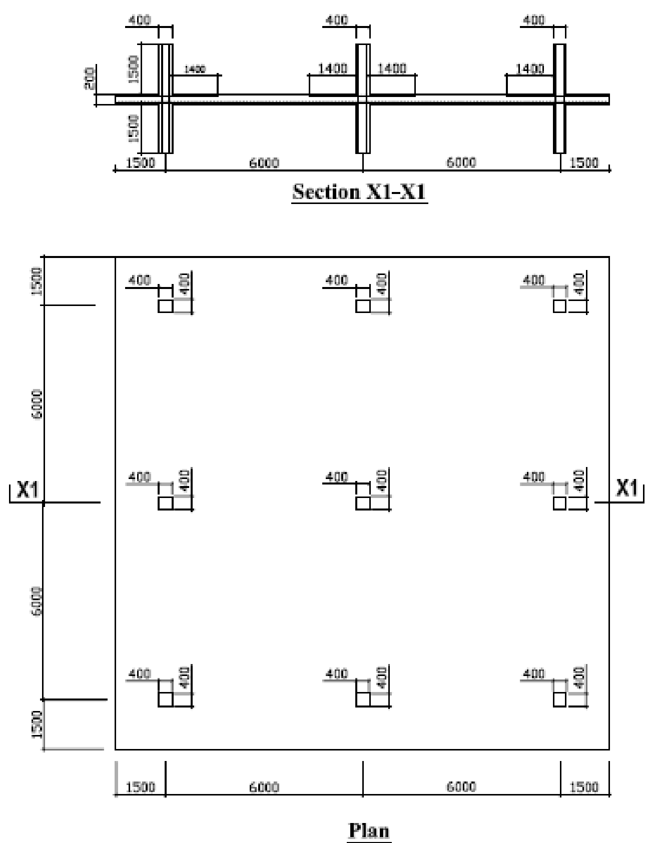

Fig. 3. Geometry of model slab

3.3 Material Model

3.3.1 Concrete

The constitutive behavior of concrete material is characterized by tensile cracking and compressive crushing, yielding of the reinforcement. The input for the total strain crack models comprises two parts: (1) the basic properties like the Young's modulus, Poisson's ratio, tensile and compressive strength, etc and (2) the definition of the behavior in tension, shear, and compression. Cylinder compressive strength of concrete at 28 days age ($f_c^{'}$) is considered as ideal properties of concrete. Relationship of compressive strength of concrete with Young’s modulus ($E_c = 4730\sqrt{f_c^{'}}$) and tensile strength ($f_t = 0.333\sqrt{f_c^{'}}$), Poisson’s ratio for concrete = 0.15 are used in this study.

The compressive behavior is in general a nonlinear function between the stress and the strain in a certain direction. Concrete subjected to compressive stresses shows a pressure-dependent behavior, i.e., the strength and ductility increase with increasing isotropic stress. Due to the lateral confinement, the compressive stress-strain relationship is modified to incorporate the effects of the increased isotropic stress. Thus, the base function in compression and tension can be modeled with a number of different pre-defined and user-defined curves. The pre-defined curve according to Thorenfeldt et al. (1987) and nonlinear constant tension-softening curve is used in the present study. The modeling of the shear behavior is only necessary in the fixed crack concept where the shear stiffness is usually reduced after cracking. A constant shear retention factor = 0.01 is considered for the reduction of shear stiffness of concrete due to cracking.

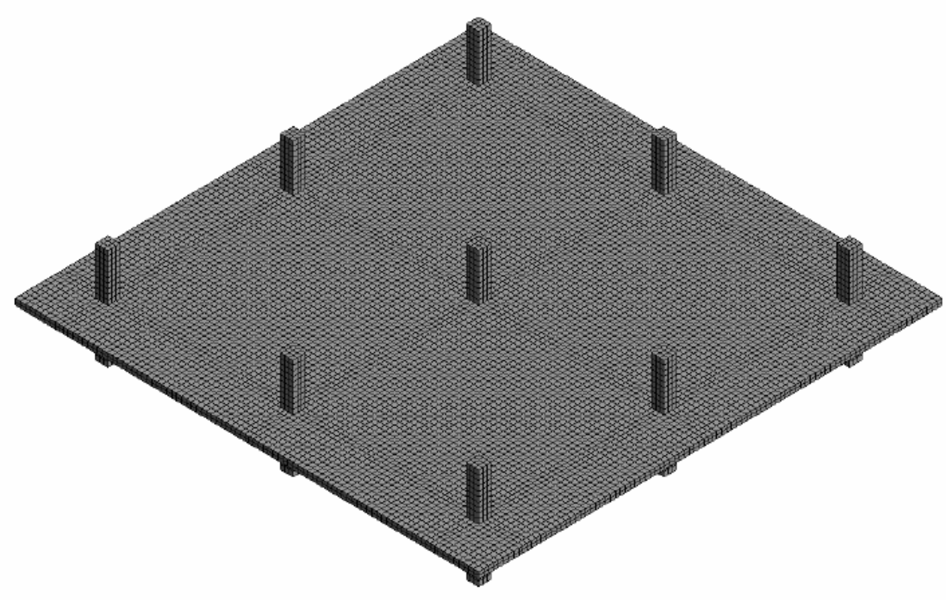

Fig. 4. Model slab after meshing

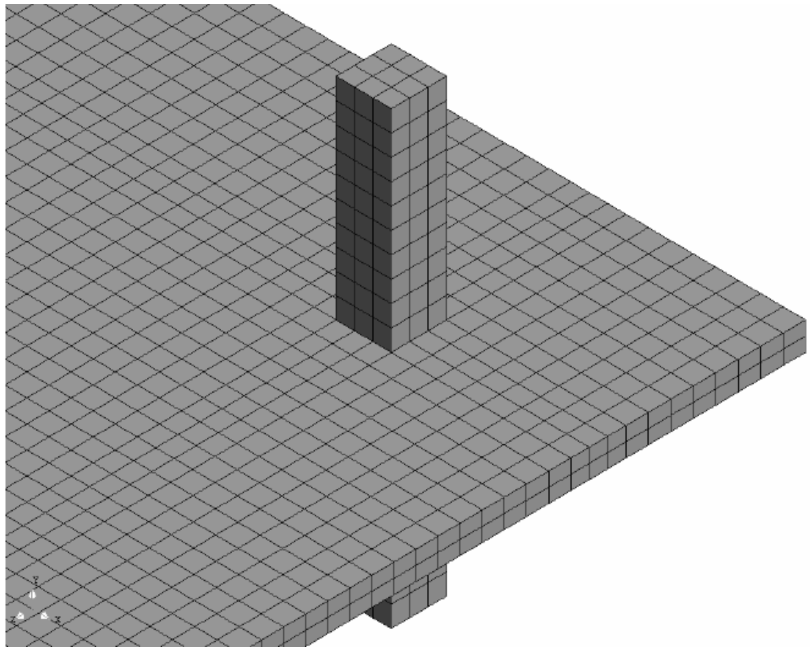

Fig. 5. Enlarged corner of meshed model

3.3.2 Reinforcement

The reinforcement in a concrete slab is modeled with the bar reinforcement embedded in the solid element. In finite element mesh, bar reinforcements have the shape of a line, which represents actual size and location of reinforcement in the concrete slab and beam. Thus in the present study, reinforcements are used in a discrete manner exactly as they are normally provided in the real test specimens. Typical reinforcement of the model at central column is shown in the Figure 6.

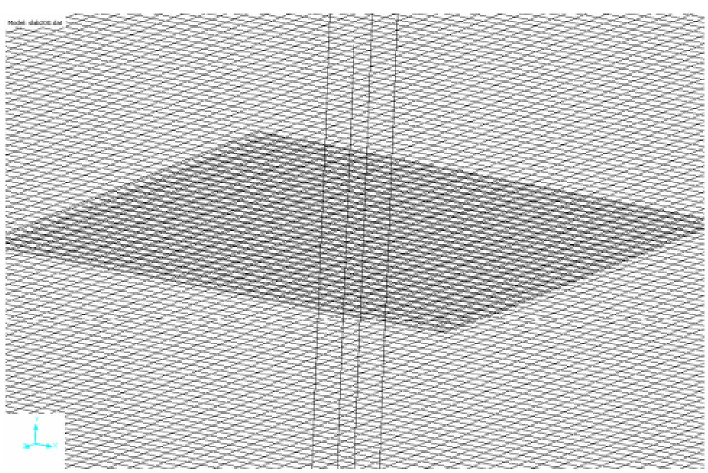

Fig. 6. Typical reinforcement of the model at central column

The constitutive behavior of the reinforcement is modelled by an elastoplastic material model with hardening. Tension softening of the concrete and perfect bond between the bar reinforcement and the surrounding concrete material was assumed. The Von Mises yield stress of 421 MPa Young's modulus of 200000 MPa and Poisson's ratio = 0.30 for steel reinforcement is used in this study. Similar types of non-linear parameters were also used in FE analysis of slab by Bailey *et al.* (2008).

3.4 Analysis procedure

Modified Newton–Raphson solution strategy was adopted in this analysis, incorporating the iteration based on conjugate gradient method with arc-length control. The line search algorithm for automatically scaling the incremental displacements in the iteration process was also included to improve the convergence rate and the efficiency of analyses. Second order plasticity equation solver solved physical non-linearity with total strain cracking. Reinforcement was evaluated in the interface elements. Accuracy checked by the norms of residual vector.

4. Discussions on FE analysis

The present study is focused primarily on the effect of concrete strength and flexural reinforcement on the punching shear capacity of slabs. In the FE model, the punching behavior of the slab as well as the detailed stress condition and failure modes is studied around the central column. For this reason nonlinear material behavior for all slab elements around the central column upto 1/4th of the adjacent span was applied. To make the FE modeling and analysis numerically efficient and less time consuming, linear material behavior was applied to other elements of the model.

Load deflection curve of various nodes adjacent to central column is shown in Figure 7. In this figure nodes B640, B480, B320, B160, B80 and B00 are located at bottom surface of slab at a distance 640mm, 480mm, 320mm, 160mm, 80mm and 0mm respectively from edge of middle column. Similarly nodes T640, T480, T320, T160, T80 and T00 are located at top surface of slab. Deflections of node located same section of slab such as B640 and T640, B320 and T320, B160 and T160, B80 and T80, B00 and T00 are almost matched. Similar deflect of top and bottom fibre at any load is indicating no differential horizontal movement in same section of slab. No differential horizontal movement of top and bottom chord at same section of slab during failure load indicates that failure due to bending moment is not occurred for model slab in this study.

It is clear from Figure 7 that punching type brittle failure occurs at and around 80mm from the edge of column. Deflected shape of a typical model slab before failure load is shown in Figure 8. Punching type deflected shape before failure adjacent to central column is observed as shown in Figure 8. Later on, serious shear cracks at the bottom surface of slab around the central column before failure are visible as shown in Figure 9. Thus, punching failure of slab at middle column is confirmed and ultimate failure load is obtained from load-deflection curve of slab adjacent to middle column. Ultimate punching failure load of all model slabs is shown in Figure 10.

Results of FE analyses obtained from this study shows that ultimate punching shear capacity and behavior of slab samples are dependent on flexural reinforcement ratio as well as compressive strength of concrete of slabs which are discussed in detail in the following paragraphs. In this paper, slab deflections are also studied to evaluate the actual punching shear behavior of slabs.

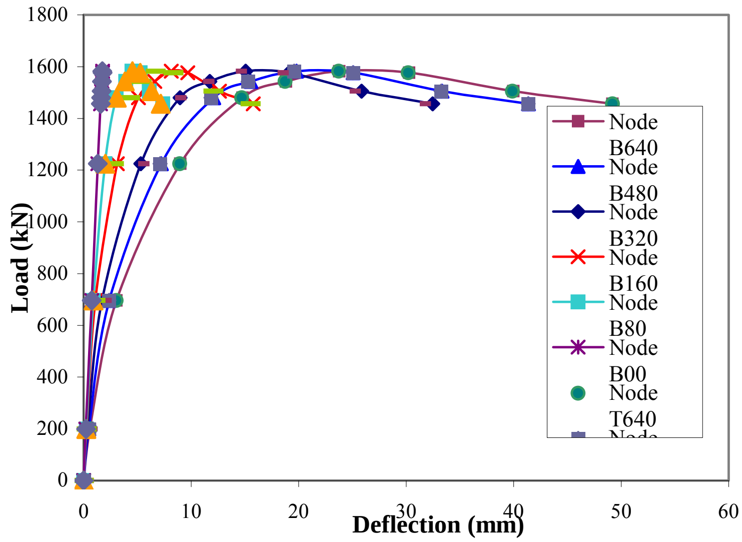

Fig. 7. Load-deflection curves of various nodes of slab for $f_c^{'}$=30 MPa and 0.50% reinforcement.

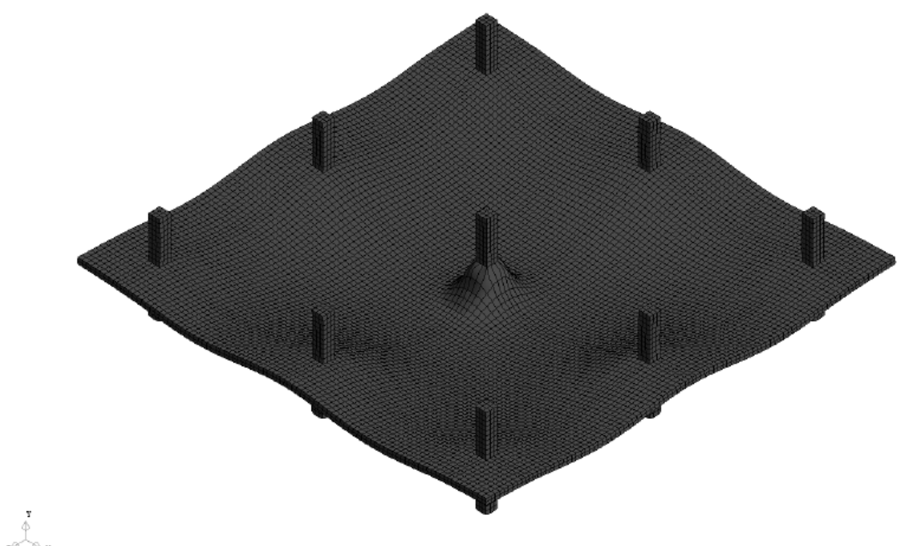

Fig. 8. Deformed shape of a typical slab before failure load

4.1 *Load-deflection behaviour*

Slab deflection is measured at bottom surface of slab 320mm apart from the edge of central column. Shortening of column for each load is deducted from slab deflection to calculate actual slab deflection. Reaction of central column for each load step is considered as punching load. Typical load-deflection curve of the model slab for concrete strength of 30 MPa is shown in Figure 11. Load deflection behavior of other strength of concrete is similar. It is found that, deflection of slab having 2% flexural reinforcement is smaller than 0.15% flexural reinforcement at same applied load. Similarly slabs of compressive strength 60 MPa deflect smaller than those of 24 MPa for same applied loading.

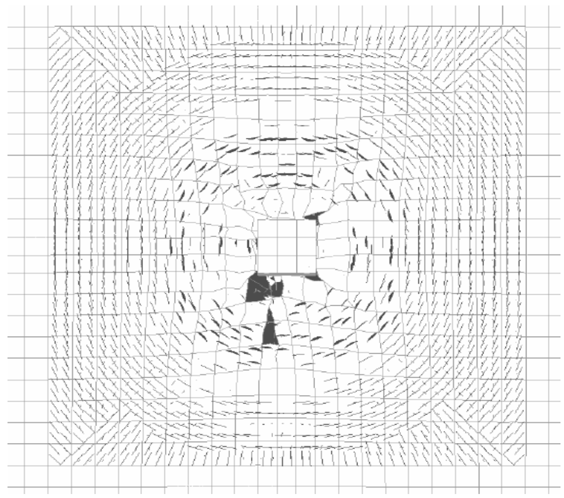

Fig. 9. Typical crack pattern at the bottom surface of slab before failure

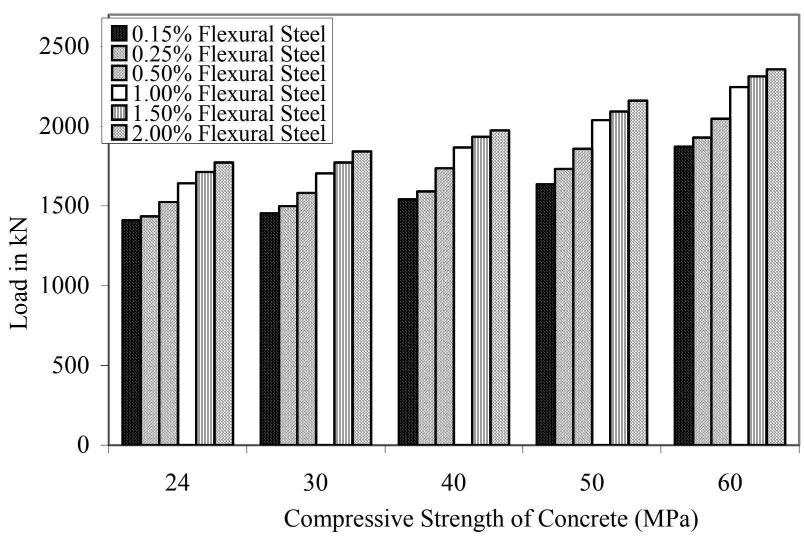

Fig. 10. Ultimate punching failure load of all model slabs

Value of deflection is decreased in general with the increase of reinforcement ratio and compressive strength of concrete. The heavily reinforced slabs, on the whole, showed slightly higher stiffness and underwent lesser deflections. Higher reinforcement and compressive strength of concrete increase tensile strength capacity at extreme fibre of slab, which causes lesser deflection. Similar trend of load deflection behavior of numerical analysis indicates to have similar nature of other parameters for structural designing of slab.

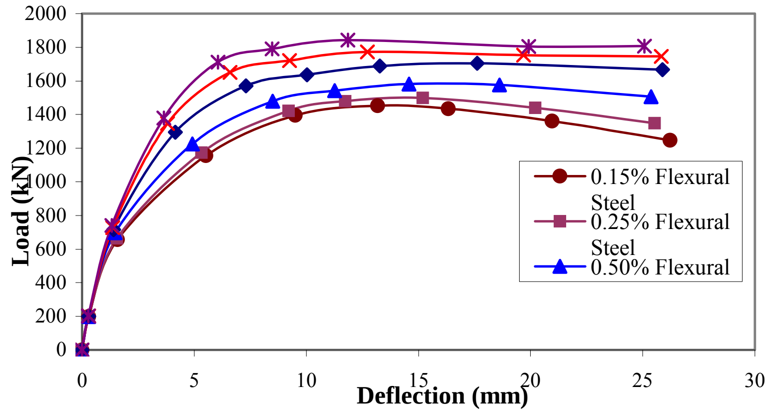

Fig. 11. Load-deflection curves of slab for $f_c^{'}$=30 MPa

4.2 *Effect of concrete strength*

The normalized punching shear strength in accordance with ACI and Canadian code formula ($V/\sqrt{f_c^{'}}b_0d$) [where, V = punching failure load, d=effective depth of slab, b_0= 4 * (side of column + d)] of various slab, have been analyzed in this research work. Normalized punching shear strengths are plotted for different compressive strength of concrete of specimen having same percentage of flexural reinforcement as shown in Figure 12. Normalized punching shear strengths for all slabs shown in Figure 12 are higher than those of ACI ($V = 0.33\sqrt{f_c^{'}}b_0d$) and Canadian ($V = 0.40\sqrt{f_c^{'}}b_0d$) codes. The normalized punching shear capacity of the all slab panels decreases with increase of compressive strength of concrete upto around 48 MPa. Very small or no increase of normalized punching load carrying is observed from concrete strength of 48 MPa to 60 MPa. Thus, contribution of concrete strength for punching shear capacity decreases with the increase of concrete strength and after 48 MPa, it is very small.

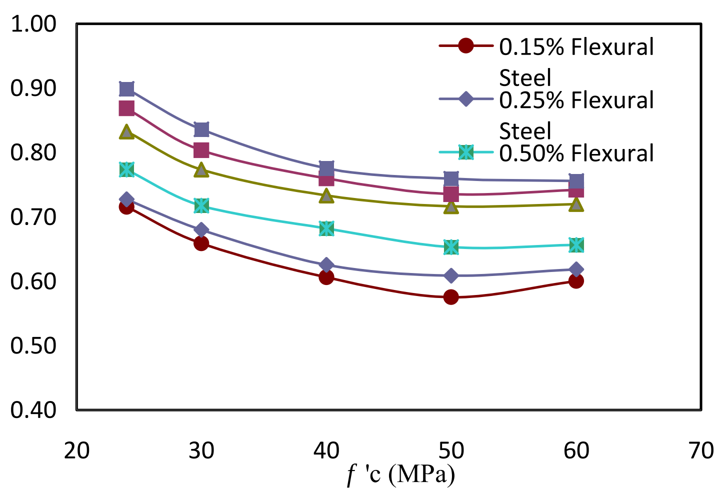

Fig. 12. Normalized punching shear at various compressive strength of concrete

4.3 *Effect of Flexural Reinforcement*

The normalized punching shear strengths of various slabs are plotted against percentage of flexural reinforcement and shown in Figure 13. It has found that having same concrete strength, normalized punching shear strength is increased with addition of flexural reinforcement ratio from 0.15% to 2% percent. Although rate of increase of punching load carrying capacity is higher upto 1% reinforcement ratio than the above of this ratio. Thus, punching load-carrying capacity of the all slab panels increased with the increase of steel reinforcement. This is also clear from Figure 12 as well.

Due to increase of applied load, cracking of concrete propagates at the tension zone of concrete, which decrease the effective depth of slab for resisting the shear. If it is assumed that little or no shear can be transferred through the portion of the depth of slab that is cracked, it is easy to conclude that the width and hence the depth of the crack have a significant influence on the shear capacity of the connection. With present of flexural reinforcement, this propagation crack will be reduced, thus the load carrying capacity increased.

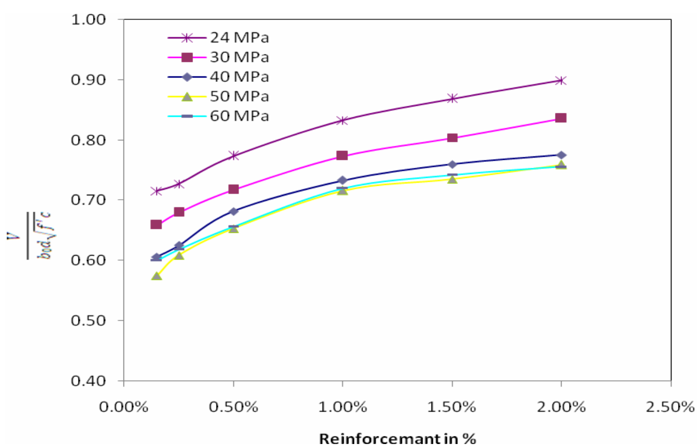

Fig. 13. Normalized punching shear at various reinforcement ratio

5. Proposal for punching shear capacity

From the analysis of all 30 model slabs, it is established that punching shear capacity is dependent on both the compressive strength of concrete and flexural reinforcement used in that slab. Thus, following empirical formula to calculate punching shear capacity has been proposed. Safety factor is not included in the proposed formula.

Nominal Punching Shear Capacity, $V = (1-\psi)(1+\sqrt[3]{\rho})\sqrt{f_c^{'}}\,b_0 d$

Here,

$\psi = \dfrac{\sqrt[3]{f_c^{'}}}{7.6+77\rho}$, for $f_c^{'}$ = 21 MPa to 48 MPa

$\psi = 0.47(1-7.3\rho)$, for $f_c^{'}$ = above 48 MPa

$f_c^{'}$ = Cylinder compressive strength of concrete at 28 days

d = Effective depth of slab

b_o = Perimeter at a distance d/2 from the side of column

ρ = Flexural reinforcement ratio

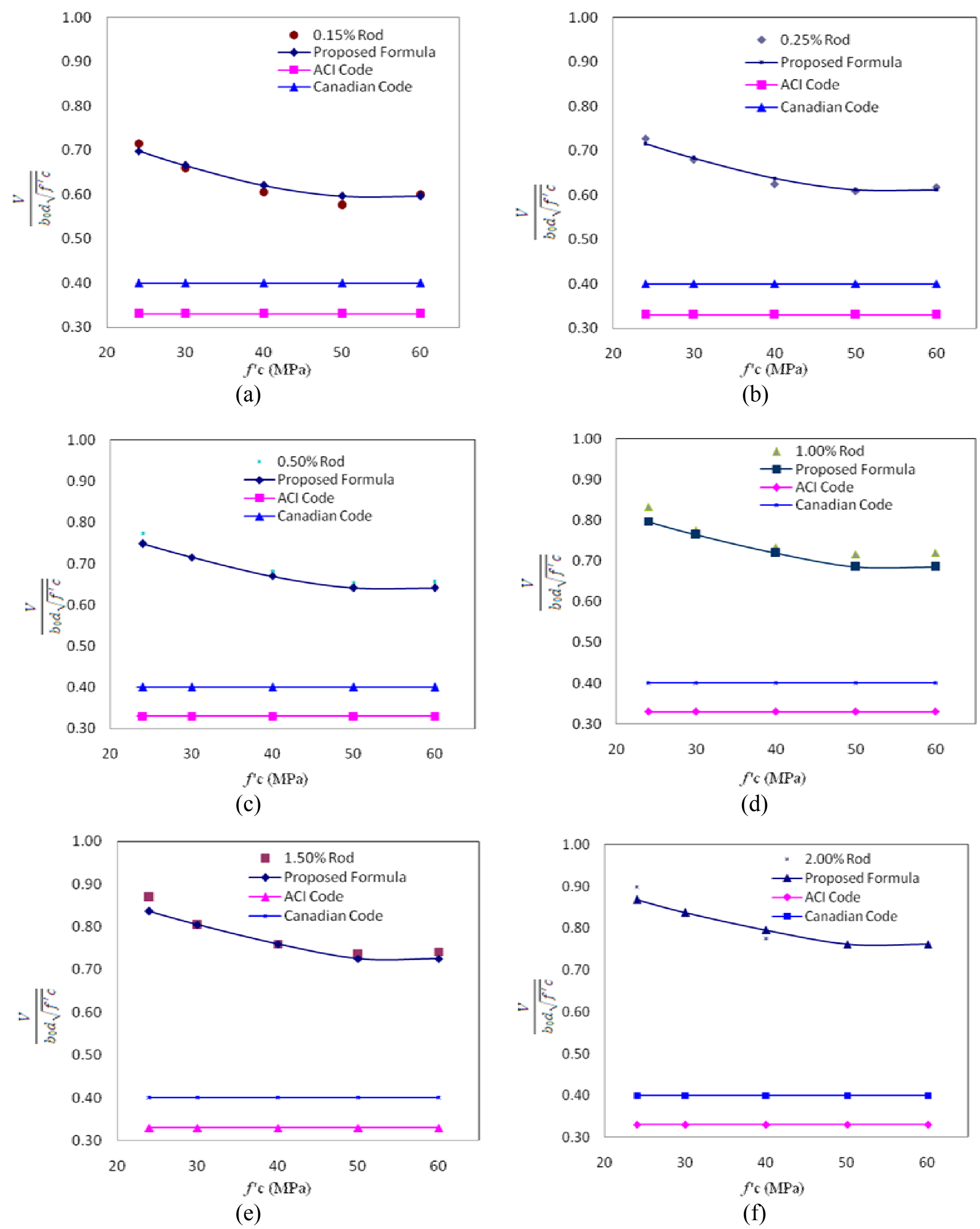

Fig. 14. Application of proposed formula for variable strength of concrete of (a) 0.15% , (b) 0.25% , (c) 0.50%, (d) 1%, (e) 1.5% and (f) 2% flexural steel.

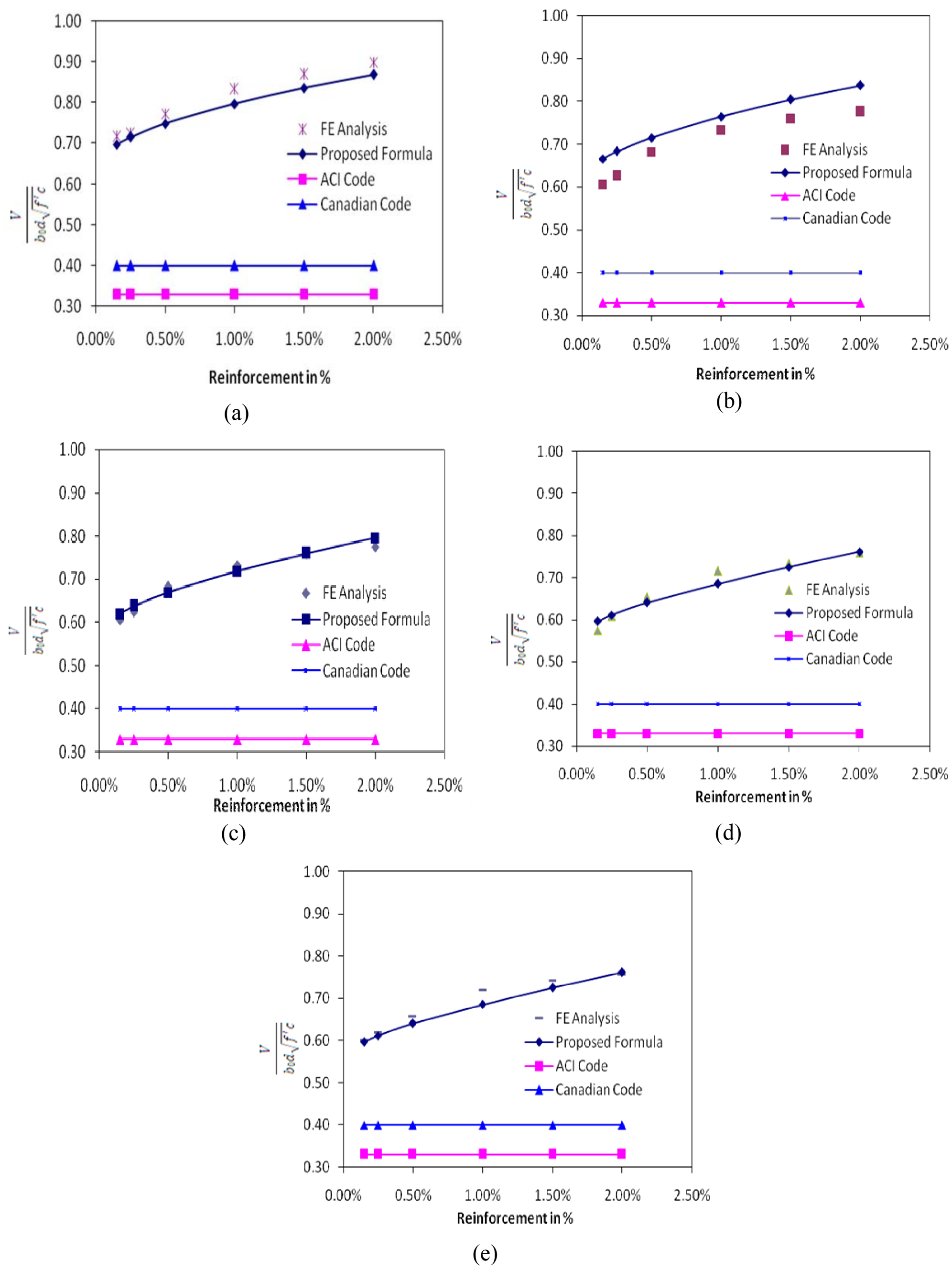

Fig 15. Application of proposed formula for variable flexural reinforcement of (a) 24 MPa, (b) 30 MPa, (c) 40 MPa, (d) 50 MPa and (e) 60 MPa concrete strength.

The normalized punching shear capacity using proposed formula is compared with non-linear analysis and shown in Figures 14. Normalized punching shear as calculated by ACI and Canadian codes are also included in those figures. It is shown that normalized punching shear strength is almost matched with non-linear finite element analysis and those are higher than ACI and Canadian code formula.

Proposed formula for calculating normalized punching shear capacity is applied with variable reinforcement is shown in Figures 15. In this case proposed formula is also almost matched with analysis. In some cases, analytical punching shear capacity is slightly higher than the proposed formula. Thus, the proposed formula is on safe site in those cases as well.

From the discussion, it can be concluded that the proposal for estimating punching shear capacity made in this paper predicts the capacity more reasonably taking into account the effect of flexural steel which some well practiced codes do not account for. It should be kept in mind that the present investigation corresponds to the study of FE models having a fixed slab thickness and column size. However, it can be expected that more detailed study with other slab thickness and column sizes would result in similar findings.

6. Conclusions

An advanced non-linear finite element study of multi-panel flat slab for punching shear capacity with different concrete strengths and reinforcement ratios is presented in this paper. An empirical formula for calculating punching shear capacity of flat slab is proposed. It has been found that compressive strength of concrete and flexural reinforcements embedded in the flat slab play a significant role on punching shear capacity of slab.

Though the absolute magnitude of punching load carrying capacity of slabs increases with the increase of compressive strength of concrete, the normalized load-carrying capacity of the all slab panel decreases with increase of compressive strength of concrete upto 48 MPa. Very small or no increase of normalized punching load carrying is observed above the concrete strength of 48 MPa.

It has found that punching shear capacity is increased with addition of flexural reinforcement ratio from 0.15% to 2.00% percent. Although rate of increase of punching load carrying capacity is higher upto 1% reinforcement ratio than the above of this ratio.

The punching shear capacity using proposed formula has been found to match well with the results of non-linear finite element analysis. Corresponding capacities predicted by the codes are much smaller and over-conservative due to not including the effect of flexural steel. The proposed empirical equation can be used for estimating the punching capacity of slabs more reasonably and after applying appropriate safety factor, the proposal may be incorporated in codes so that designer may use the proposed formula for calculating punching shear capacity of slab for economy building structure.

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