

# Evaluate variations of deflections of fixed headed pile from non-linear load - deflection curve under lateral loading

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## Abstract

Lateral stability is the vital issue when the pile stands in several soft clay either sand layers. In reality, structure and soil both are show non-linear behaviours. Proper evaluation of non-linear behaviour has not so essay in theory. However, slight non-linear effects are included in pile deflection from load-deflection curve of pressure meter test in laboratory. Pressure meter modulus is influenced deflection of fixed headed piles. Predicting the variations of non-linear load deflection curve with the successive incremental lateral loading is the main scope of the present research. Variable parameters are diameter of pile, lateral load and length of pile. After a certain length of piles, deflection difference between any of two diameters piles is very low. Maximum deflection occurs at pile head for lowest diameter pile and this value is very low. Lower value of deflection indicates non-linear effects. For larger diameter pile, deflection variation shows like as exponential curve. In the present study, maximum deflection has been found to be approximately 1.6 $\mu$ m for 400mm diameter pile and 20000kN lateral load. Diameter increment is the major reason of deflection reduction at constant lateral load.

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## 1. Introduction

Pile behaves like as tall structures under lateral loading. Lateral load heats at pile head. In past, pile was analyzed and design considering only gravity loading. When pile length is exceeded 30m then lateral load analysis is essential for proper design of pile. Another point of view, when pile stands on several soft clay or sand layers then lateral non-linear analysis is essential for proper analysis and design of pile. Deflection of pile under lateral load was evaluated by several authors (Reese and Matlock, 1956; Davisson and Gill, 1963) within elastic limit. Lateral load resistance capacity of soft clay is lower than stiff clay and several methods (Matlock, 1970) have been developed to calculate deflection from lateral load-deflection curve. Sub-grade modulus is the function of non-linear lateral deflection. Several authors (Baguelin et al., 1978; Tomlinson, 2005) were founded sub-grade modulus based on

their research. Two stage pressuremeter tests (Baguelin et al., 1978) provide better response of soil under lateral loading. Experiments and case studies had shown good agreement (Tomlinson, 2005) to compute non-linear load-deflection curve. Non-linear effect on soil-pile interface is the updated technique which influenced deflection of pile under lateral loading.

Pile deflection is maximum at pile head and minimum at pile tip. Pile carries more than double curvatures under lateral loading. Pile deflection gradually decreases with the increment of length of pile because surrounding soils are influenced to the pile geometry and behaviors. Pile diameter is another factor to causes deflection under lateral loading. Non-linear analysis expresses the failure mechanisms of pile but linear analysis expresses pile behaviors within yield limits. So, pile deflection due to linear analysis has less than inelastic non-linear analysis. How pile diameters and lengths effect on pile deflection has the main objectives in the present study.

## 2. Analytical model of fixed headed pile

Fixed headed piles are used in building, bridge, flyover etc. structures. This pile is laterally stiffer than free headed pile. Analytical model has been performed by considering single fixed headed pile. Actual fixity of piles varies along its lengths. However, variable diameters reflect deflections of piles. In analytical model, diameter, length and variable dimension along length of pile are represented by  $B$ ,  $L$  and  $x$ . Overall deflected shape of pile shows like as tall structures deflected shape. Basic difference between tall structure and pile is medium. Tall structure stands in air medium and pile stands in soil medium. Fixed headed pile model is represented by Figure 1.

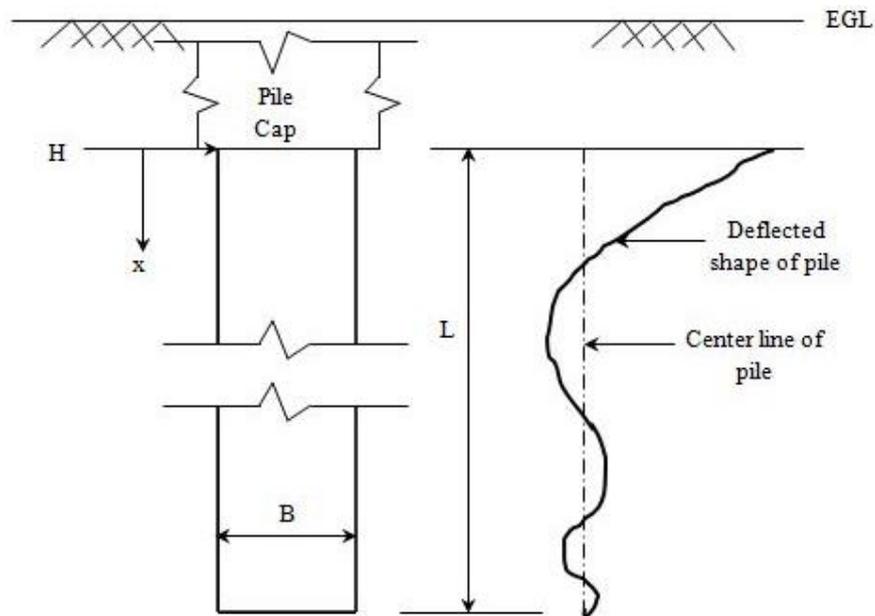


Fig. 1. Fixed headed pile analytical model.

## 3. Material properties and analytical formulae

Material properties have been found from laboratory pressuremeter test. Elastic modulus, sub-grade reactions have been found from tri-axial test and pressuremeter modulus has been found from pressuremeter test. Rheological factor (Tomlinson, 2008) has been obtained from past study. Reference diameter of pile is used to be 600mm. Material properties from laboratory test results are represented by Table 1.

Table 1  
Material properties of soil

Specifications	Test Result values	Unit
Sub-grade reaction, k	15000	kN/m <sup>3</sup>
Modulus of elasticity of pile, E	4.53E+11	kN/m <sup>2</sup>
Rheological factor of clay, $\alpha$	0.75	-
Pressuremeter modulus, $E_m$	4.10E+08	kN/m <sup>2</sup>

Analytical formulae have been developed by closed formed solutions (after Tomlinson, 2008). Non-linear behavior has been considered during developing analytical formulae. Stiffness factor, coefficient of sub-grade reaction and deflection are expressed by Eq. (1), Eq. (2-a), Eq. (2-b) and Eq. (3).

$$R = \sqrt[4]{\frac{EI}{kB}} \quad (1)$$

$$\frac{1}{k_m} = \frac{2}{9E_m} B_0 \left( \frac{B}{B_0} \times 2.65 \right)^\alpha + \frac{\alpha B}{6E_m} \quad (B \geq 600mm) \quad (2-a)$$

$$\frac{1}{k_m} = \frac{B}{E_m} \left( \frac{4(2.65)^\alpha + 3\alpha}{18} \right) \quad (B < 600mm) \quad (2-b)$$

$$y_{(z)} = \frac{H}{Rk_m B} F_2 \quad (3)$$

Where,

$E$  = modulus of elasticity of pile (kN/m<sup>2</sup>)

$I$  = moment of inertia of Centroidal axis of pile (m<sup>4</sup>)

$k$  = sub-grade reaction (kN/m<sup>3</sup>)

$B$  = diameter of pile (m)

$B_0$  = reference diameter of pile (m)

$E_m$  = pressuremeter modulus (kN/m<sup>2</sup>)

$\alpha$  = rheological factor

$F_2$  = coefficient

$H$  = Lateral force (kN)

#### 4. Interpretation of analytical results

For analysis, three variable parameters are considered. Lateral loads are varying from 0kN to 20000kN. Diameters of piles are 400mm, 450mm, 500mm, 600mm, 750mm and 900mm. Pile lengths varies from 0m to 15m during analysis. Deflection rates are very low with the increment of lateral forces within ranges. Maximum deflection has been found to be 1.6 $\mu$ m at pile head which is very low. Non-linear based analytical formulae have been decreases lateral deflection of pile. Maximum deflection occurs when diameter of pile is 400mm. Deflections are gradually decreases with the increment of pile diameter and incremental length of pile. Diameter variation of pile effects on deflection from length of pile 0m to 6m. After crossing these lengths, some diameter shows equal deflection of various lateral loading. Deflection difference between any of two diameters of pile has been found to be 0.2 $\mu$ m for several cases. This difference shows when the length of pile varies from 0m to 6m. Similar variations of deflections have been found for two various diameters such as 450mm and 500mm at length 7m. At lengths 7m and 9m, deflections are similar for 400mm, 450mm and 500mm diameters of piles.

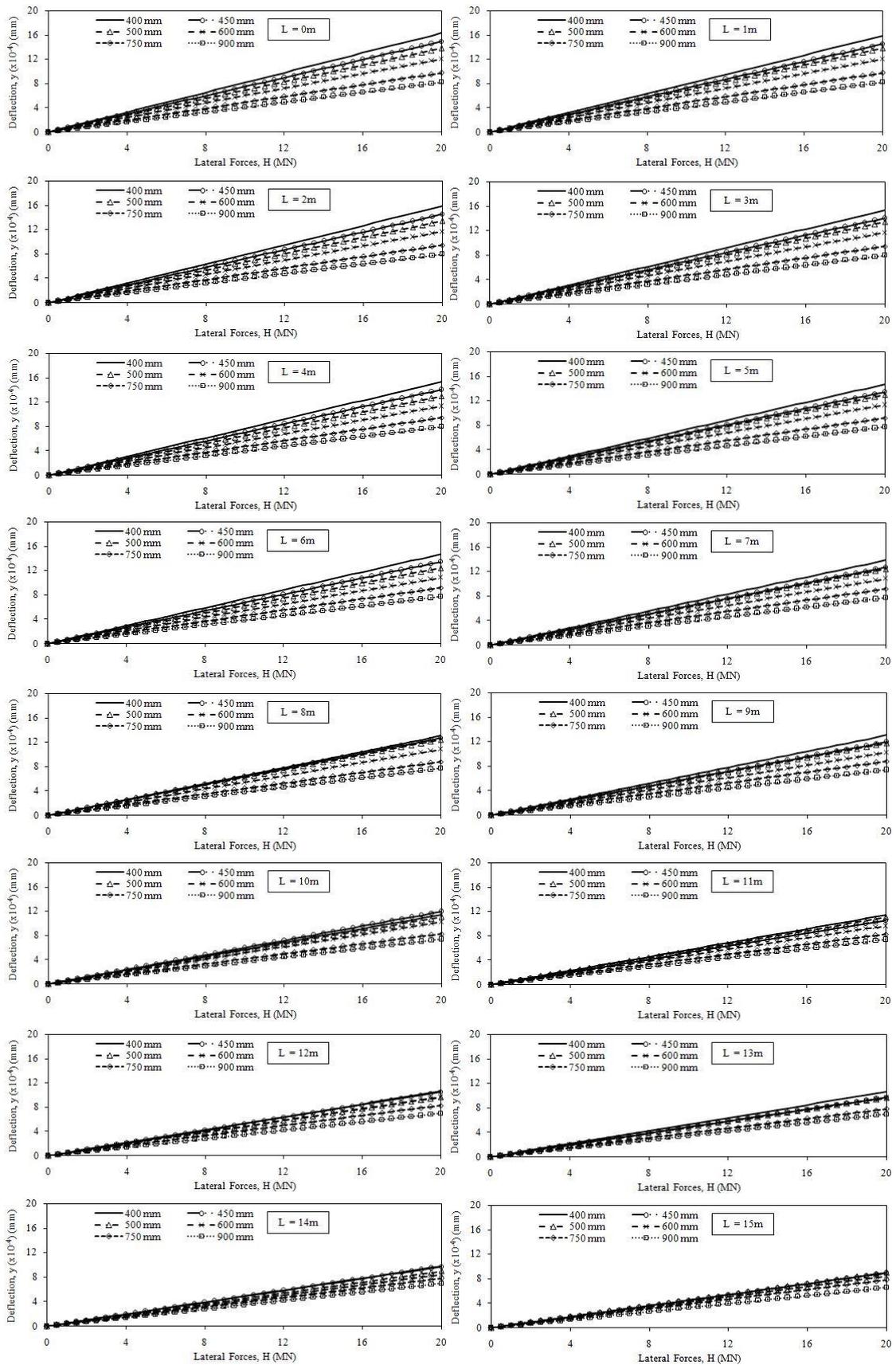


Fig. 2. Variations of deflections of various lengths and diameters of piles.

When pile lengths vary from 10m to 15m, difference of deflection between any of two diameters piles are very small. Uniform variations have been seen in non-linear load-deflection curves. Variations of deflections with the increment of lateral loads are represented by Figure 2.

## 5. Conclusion

Lateral deflection effects capacity of pile. Inelastic response of pile under lateral loading expresses capacity of piles until failure. Higher distance along pile length from pile head shows very small difference of deflection of any two different diameters of piles. Maximum deflection value is 1.6 $\mu$ m for 400mm diameter which is very small. Deflections are decreasing gradually from pile head and after crossing a certain distance, it is increasing in the opposite direction. Several points along pile body, deflections are very small. Diameter effects to the occurrence of deflections under lateral loading are very small for length of pile varies from 10m to 15m. Difference of deflection is gradually decreasing with the increment of pile diameter which indicates stability of pile. It can be concluded that maximum variations of deflections vary from 0mm to 10000mm along the length of pile.

## References

- Baguelin F., Jezequel J.F. and Shields D.G., (1978). *The Pressure meter and Foundation Engineering*, Trans Tech Publications, Clausthal.
- Davisson M.T. and Gill H.L., (1963). "Laterally-loaded piles in a layered soil system", *Journal of the Soil Mechanics Division, American Society of Civil Engineers*, Vol. 89, No. SM3, pp. 63–94.
- Hansen J.B., (1961). "The ultimate resistance of rigid piles against transversal forces", *Danish Geotechnical Institute, Bulletin No. 12, Copenhagen*, pp. 5–9.
- Matlock H., (1970). "Correlations for design of laterally loaded piles in soft clay", *Proceedings of the Offshore Technology Conference, Houston, Texas, Paper OTC 1204*.
- Prakash S., (1962). "Behaviour of pile groups subjected to lateral loads", *Thesis University of Illinois, Urbana, Illinois*.
- Randolph M.F., (1981). "The response of flexible piles to lateral loading", *Geotechnique*, Vol. 31, No. 2, pp. 247–59.
- Reese L.C. and Matlock H., (1956). "Non-dimensional solutions for laterally-loaded piles with soil modulus assumed proportional to depth", *Proceedings of the 8th Texas Conference on Soil Mechanics and Foundation Engineering, Austin, Texas*, pp. 1–41.
- Reese L.C., Cox W.R. and Koop F.B., (1974). "Analysis of laterally loaded piles in sand", *Proceedings of the Offshore Technology Conference, Houston, Texas, Paper No. OTC 2080*.
- Reese L.C. and Welch R.C., (1975). "Lateral loading of deep foundations in stiff clay", *Journal of the Geotechnical Engineering Division, American Society of Civil Engineers*, Vol. 101, No. GT7, pp. 633–49.
- Tomlinson M. and Woodward J., (2005). *Pile Design and Construction Practise*, Fifth edition, Taylor and Francis, New York and London.