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A NUMERICAL INVESTIGATION OF RATE OF LOADING FOR K_0-CONSOLIDATION OF LONDON CLAY

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ABSTRACT: Numerical analyses using finite differences have been carried out to model one-dimensional consolidation under continuous loading. Excess pore pressures generated at the mid-plane during K_0-consolidation of 102 mm dia. by 203 mm high London clay (coefficient of consolidation, $c_v = 0.25\ m^2/yr$) samples have been predicted for three different drainage conditions and for various rates of loading. Predictions indicate that the generation of excess pore pressures is very much dependent on the drainage conditions being provided for samples and also on the rate of loading the samples during consolidation. The better the drainage facilities, the lesser is the build up of excess pore pressures at the mid-plane of samples. Generation of excess pore pressures also increases with increasing rates of loading. Experimentally measured excess pore pressures developed during K_0-consolidation of four 102 mm dia. by 203 mm high London clay samples for a particular rate of loading (0.7 kPa/hr) and drainage conditions (drainage from one end and radial boundary) have been compared with the predicted values.

KEYWORDS: clays, K_0-consolidation, finite difference, excess pore pressure.

INTRODUCTION

Triaxial test has proved to be the most versatile test to investigate the stress-strain behaviour of soils. In triaxial tests samples are routinely reconsolidated under K_0-conditions before being sheared in compression or extension under undrained or drained conditions. In K_0-consolidation sample is allowed to consolidate by imposing anisotropic stress under conditions of zero lateral strain.

Anisotropic reconsolidation has been proposed by several investigators as an effective method of reducing sampling disturbance effects in clays (Davis and Poulos, 1967; Bjerrum, 1973; Ladd and Foott, 1974). K_0-consolidation to the in-situ stresses has been suggested by Davis and Poulos (1967) and Bjerrum (1974). Ladd and Foott (1974) proposed that samples should be consolidated under K_0-condition to a pressure equal to 1.5 to 2 times the in-situ vertical effective stress in order to eliminate the effects of sampling and sample preparation. This method of reconsolidation is well known method to geotechnical engineers and is called SHANSEP method (Ladd and Foott, 1974). While carrying out K_0-

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consolidation under continuous loading, one of the most important factors is the rate of loading being applied to the sample. The rate of loading during K_0-consolidation should be so chosen as to permit sufficient time to ensure dissipation of excess porewater pressure within the sample, especially at the mid-plane where the generation of excess pore pressure is the maximum. The rate of loading also controls the total testing time and it is particularly important while consolidating large clay samples (e.g., 102 mm diameter by 203 mm high), where considerable time is spent to consolidate a sample.

This paper presents the results of numerical analyses to determine the approximate rate of loading during K_0-consolidation stage for 102 mm dia. by 203 mm high London clay samples under three different drainage conditions. A comparison of numerical results with experimentally measured values has also been presented.

NUMERICAL SOLUTION FOR ONE-DIMENSIONAL CONSOLIDATION

The general differential equation governing one-dimensional consolidation is as follows:

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2} \tag{1}$$

where, z = depth; u = excess pore pressure and t = time.
Eq. (1) can be solved numerically using the method of finite differences (Scott, 1963; Das, 1983). The numerical errors associated with the method are negligible and the solution can be easily programmed for computer. The method is based on the depth-time grid as shown in Fig. 1. The oedometer sample has been modelled by dividing the depth into equal parts of nondimensionless thickness Δz and dividing time into equal intervals of nondimensionless time ΔT. Δz and ΔT are termed as depth factor and time factor respectively. The value of excess pore pressure at any depth after any time is denoted by $u_{i,j,}$ where i and j are subscripts denoting depth and time respectively. The excess porewater pressure at a point at time T + ΔT in terms of pore pressures at the point and adjacent points at time T is given by:

$$u_{i,j+1} = u_{i,j} + \frac{\Delta T}{(\Delta Z)^2}\left[u_{i-1,j} + u_{i+1,j} - 2u_{i,j}\right] \tag{2}$$

Eq. (2) is a explicit time integration relation which is conditionally stable. Since both ΔT and Δz are dimensionless, the factor $\frac{\Delta T}{(\Delta z)^2}$ (= M) is also dimensionless and termed as an operator of Eq. (2). It has been reported by Scott (1963) that for convergence the value of M must not exceed 1/2 and that the errors due to neglecting high order differences are reduced to a minimum value when the value of the operator M is 1/6.

The finite difference approximation has been used to model the theoretical consolidation behaviour of a clay subjected to transient continuous loading. A computer programme (written in Fortran 77) has

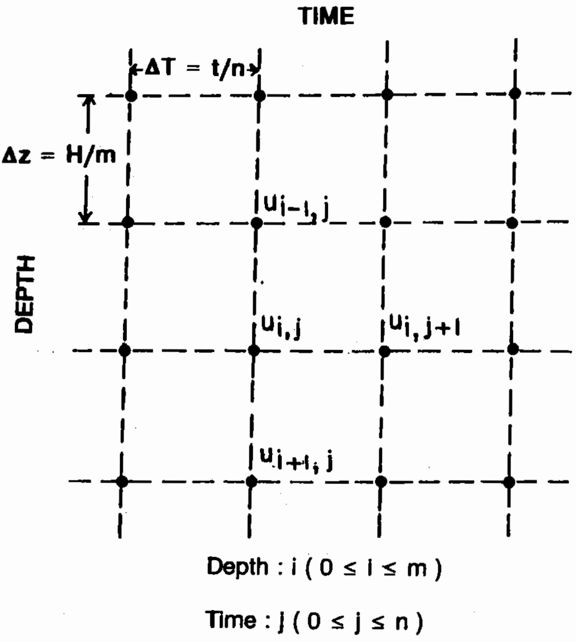

Fig1. Depth-Time Grid for Numerical Solution of One-Dimensional Consolidation Theory Using Finite Differences

been developed to compute nondimensionless excess porewater pressures at different depths of a oedometer sample as a function of time factor. In the numerical analyses, the oedometer sample has been divided into ten equal parts of nondimensionless thickness, i.e., $\Delta z = 0.1$. The value of the operator M has been taken as 1/6 to eliminate the errors due to neglecting high order differences. The time factor step ΔT (= $M\Delta z^2$) was therefore equal to 0.001667. Using the data the average degree of consolidation was determined. The predicted variation of the average degree of consolidation with time factor T for continuous loading with drainage from both ends from sample is presented in Fig. 2. The predicted dissipation of excess pore pressures at mid-plane of sample as a function of time factor is shown in Fig. 3(a) and (b).

NUMERICAL INVESTIGATION ON LONDON CLAY

Based on predictions shown in Figs. 3(a) and (b), the excess pore pressures for highly impermeable London clay samples of 102 mm dia. by 203 mm high have been determined during K_0-consolidation with continuous loading under three different drainage conditions. The coefficient of consolidation, c_v of London clay used in the analyses has been determined by Siddique (1990) and Hopper (1988) from oedometer tests on 76.2 mm dia. by 19.1 mm high samples. Siddique (1990) reported an average value of 0.27 m^2/yr for c_v in the normally consolidated range (100 kPa to 800 kPa) while Hopper (1988) reported an average value of 0.24 m^2/yr at a stress range of 50 kPa to 2000 kPa. Based on the experimental results, an average c_v-value of 0.25 m^2/yr has been used in the numerical analyses.

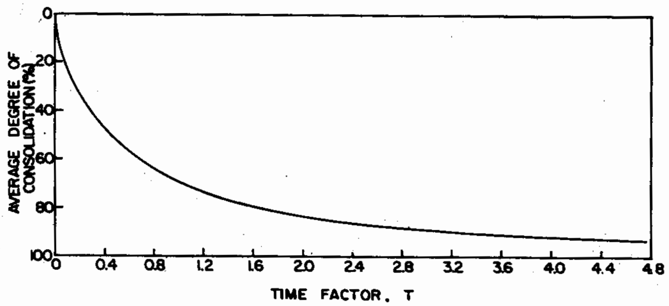

Fig 2. Average Degree of Consolidation-Time Factor Relationship for Continuous Loading

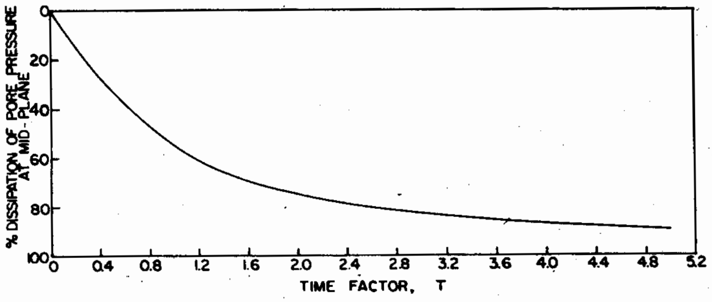

Fig 3. (a) Dissipation of Excess Pore Pressure-Time Factor Relationship

Excess pore pressures at mid-plane of London clay samples have been predicted for the following drainage conditions:

(1) drainage from one end and radial boundary of sample;
(2) drainage from both ends and radial boundary of sample;
(3) drainage from only radial boundary of sample.

The predicted variation of dissipation of excess porewater pressure at the mid-plane of London clay samples with consolidation time for three different drainage conditions are presented in Fig. 4. It can be seen from Fig. 4 that excess pore pressure dissipates quite rapidly during the early stage of consolidation and then it dissipates very slowly as consolidation continues. It is also evident from Fig. 4 that the dissipation of excess pore pressure depends on the drainage conditions provided for samples. The better the drainage facilities, the faster is the rate of dissipation of excess

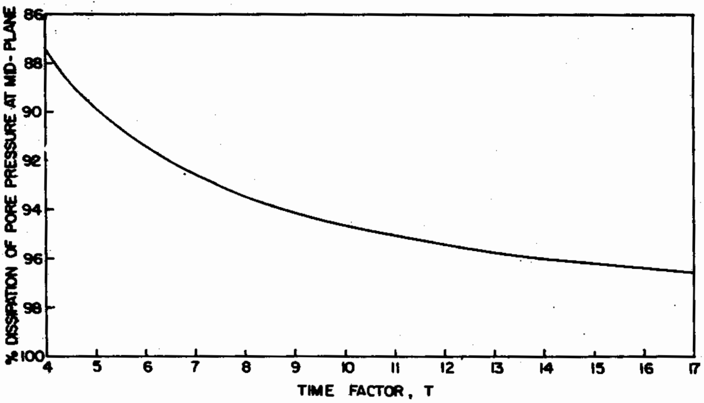

Fig 3. (b) Dissipation of Excess Pore Pressure-Time Factor Relationship

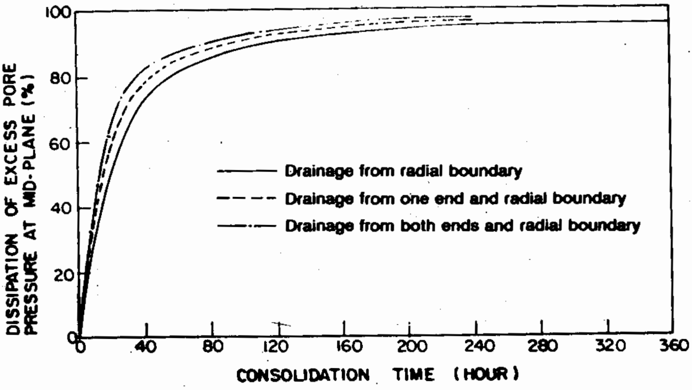

Fig 4. Variation of Excess Pore Pressure at Mid-Plane with Consolidation Time for Different Drainage Conditions

porewater pressure. The theoretical relationships presented in Fig. 4 have subsequently been used to predict pore pressures at the mid-plane of London clay samples as a function of vertical effective consolidation stresses. For each drainage condition, excess pore pressures were calculated for five different rates of increase of vertical effective stress. The rates used in numerical calculations are 0.5 kPa/hr, 1 kPa/hr, 1.5 kPa/hr, 2 kPa/hr and 3 kPa/hr. The predicted excess porewater pressure versus vertical effective consolidation stress relationships are presented in Figs. 5 to 7 for three different drainage conditions. The curves shown in Figs. 5 to 7 clearly indicate that for a given drainage condition, the excess pore pressures generated at the mid-plane of sample increase with increasing rate of loading. These curves also demonstrate that at the initial stages of loading there is a sharp increase in excess porewater pressure which eventually decays and becomes approximately constant at higher consolidation stresses. Fig. 5 shows that if drainage from one end and radial boundary are provided, the excess pore pressures generated at vertical effective consolidation stress of 200 kPa are of the order of 5 kPa to 28 kPa for loading rates between 0.5 kPa/hr and 3 kPa/hr. For loading rates of above 1 kPa/hr, the excess pore pressures generated at the mid-plane of samples are always more than 5% of the current vertical effective consolidation stress. For loading rates of 0.5 kPa/hr and 1 kPa/hr, however, the excess pore pressure at the mid-plane at higher consolidation stresses are less than 5% of vertical effective consolidation stresses. During K_0-consolidation, an excess pore pressure generation of less than 5% of current vertical effective stress is tolerated. It can be seen from Fig. 6 that, if drainage is provided from both ends and radial boundary of sample, then the loading rate may be increased up to 2 kPa/hr to keep the build up of pore pressures within acceptable limits. However, if sample is allowed to drain from radial boundary only, it can be seen from Fig. 7 that the rate of loading should be sufficiency slow and preferably less than 1 kPa/hr. The relationships shown in Figs. 5 to 7 can also provide a valuable basis for the preliminary selection of appropriate rate of loading during K_0-consolidation of large clay samples under various drainage conditions.

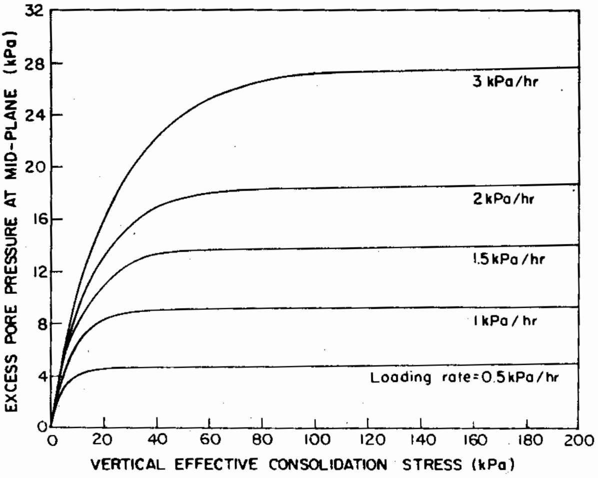

Fig 5. Excess Pore Pressure-Vertical Effective Stress Relationship for Drainage from One End and Radial Boundary

K_0-CONSOLIDATION OF LONDON CLAY IN THE LABORATORY

A laboratory testing programme of K_0-consolidation of normally consolidated reconstituted soft London clay was carried out (Siddique, 1990). The London clay (liquid limit, LL = 69, plasticity index, PI = 45) sample was prepared in the laboratory by K_0-consolidation from slurry having water content equal to 1.5 times the liquid limit of the clay.

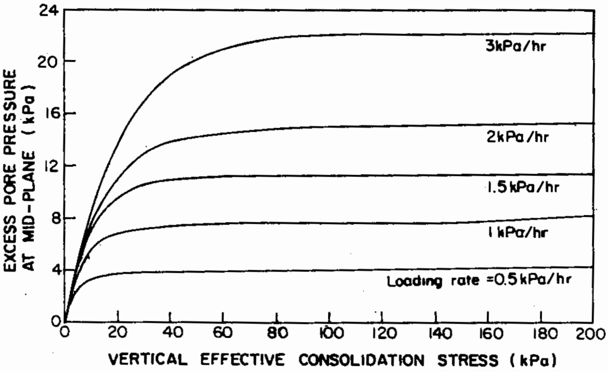

Fig 6. Excess Pore Pressure-Vertical Effective Stress Relationship for Drainage from Both Ends and Radial Boundary

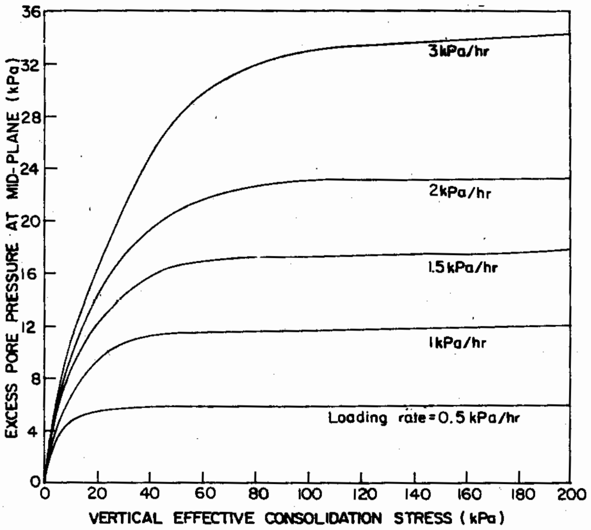

Fig 7. Excess Pore Pressure-Vertical Effective Stress Relationship for Drainage from Radial Boundary Only

K_0-consolidation of slurry was carried out in a large hydraulic consolidation cell of 1000 mm dia. by 490 mm deep. A pressure of 100 kPa was used during K_0-consolidation. Block samples were cut by hand from the consolidated sample using wire saws. Samples were trimmed from the block samples to nominal dimensions of 102 mm dia. by 203 mm high. Water content and bulk density of the samples were respectively 45 ± 1% and 1.76 ± 0.01 Mg/m^3.

K_0-consolidation was carried out using a 102 mm triaxial cell in a stepless compression machine. During K_0-consolidation the samples were allowed to drain through the bottom end and radial boundary. Axial forces were measured using an internal load cell with a resolution of 1 N. Cell and back pressures were measured adjacent to the triaxial cell using pressure transducers with a resolution of 0.25 kPa. A miniature pressure transducer with a resolution of 0.25 kPa was used to measure mid-plane pore pressures (Hight, 1982). Local deformations were measured using Hall effect devices (Clayton, Khatrush, Bica and Siddique, 1989). Two axial Hall effect devices having a gauge length of 70 mm and one radial caliper were used. The resolutions of these axial strain and radial strain devices were about 1 mm and 0.5 mm respectively. Axial strains were also measured externally using a displacement transducer with a resolution of approximately 6 mm. All instrumentation were monitored using a microcomputer. An automated stress path system (Khatrush, 1987; Siddique, 1990) was used to control stresses imposed on samples during K_0-consolidation. The system was controlled by a microcomputer. Two automated pressure controllers controlled the air pressure applied to the cell and back pressure air/fluid Bellofram rolling diaphragm. A third automated pressure controller regulated the air supply to a double acting Bellofram rolling diaphragm air actuator that was connected to the internal load cell. Air pressure increments of 0.1 kPa was attainable which corresponds to about 0.07 kPa for the axial pressure on the specimen.

In K_0-consolidation tests, samples were initially brought back to their in-situ stresses from their initial set up isotropic stresses by applying an undrained stress path at constant radial stress. The samples were then consolidated under K_0-conditions ($K_0 = 0.64$) up to 1.8 to 1.9 times the maximum past vertical effective consolidation pressure (i.e., 100 kPa). During K_0-consolidation vertical effective stress was increased at a rate of 0.7 kPa/hr. A back pressure of 250 kPa was used during K_0-consolidation.

COMPARISON OF PREDICTED EXCESS PORE PRESSURES WITH EXPERIMENTALLY MEASURED VALUES

Fig. 8 presents the variation of excess porewater pressure (difference between midplane and base pore pressures) at mid-height with increase in vertical effective consolidation stress for four samples. The predicted variation of excess pore pressures at mid-plane is also shown as solid line in Fig. 8. The prediction is based on a loading rate (0.7 kPa/hr) and drainage conditions (drainage from one end and radial boundary) similar to those used in testing the samples in the laboratory. It can be seen from Fig. 8 that the excess pore pressures at the mid-height of the four samples at the end of K_0-consolidation are between 8 and 11 kPa, which is about 4 to 6% of the vertical effective consolidation stress at the end of consolidation, which being about 200 kPa. It can be observed from Fig. 8 that the nature of the predicted curve does not agree with the experimental curves (except for sample 4). However, although the experimentally measured excess pore pressure values at the end of K_0-consolidation overestimate the predicted value, the values are comparable. In fact, the numerical analyses have been conducted before carrying out the experiments to select an appropriate rate of loading during K_0-consolidation so that the generation of excess pore pressure at the end of consolidation does not exceed 5% of the vertical effective stress.

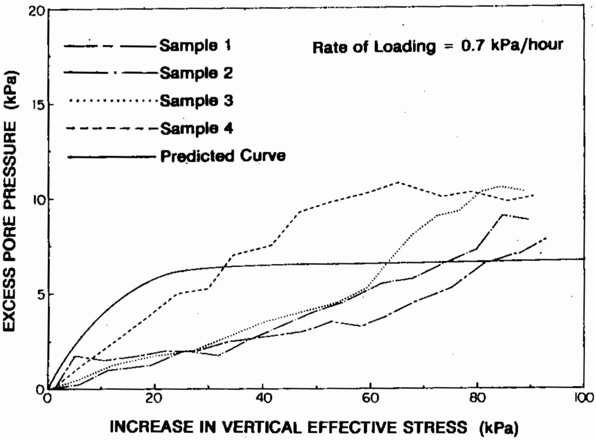

Fig 8. Measured Excess Pore Pressure-Vertical Effective Stress Relationship for Normally Consolidated London Clay Samples

CONCLUSIONS

Numerical analyses using finite differences have been carried out to investigate one-dimensional consolidation under continuous loading. Excess pore pressures generated at the mid-plane during K_0-consolidation of 102 mm dia. by 203 mm high London clay (c_v = 0.25 m^2/yr) samples have been predicted for three different drainage conditions and for various rates of loading. Predictions clearly indicate that the generation of excess pore pressures is very much dependent on the drainage conditions provided for samples and also on the rate of loading the samples during consolidation. The better the drainage facilities, the lesser is the generation of excess pore pressures at the mid-plane of samples. Generation of excess pore pressures increases with increasing rates of loading. If drainage is provided from one end and radial boundary of the samples, the rate of loading could be 1 kPa/hr to keep the build up of excess pore pressures within tolerable limits. In contrast, the rate of loading could be doubled if the samples are allowed to drain from both ends and radial boundary. The rate of loading, however, should be slow enough and preferably less than 1 kPa/hr when drainage is being permitted from only the radial boundary of samples. The predicted relationships showing the variation of excess pore pressures with vertical effective stress for different drainage conditions could also provide a valuable basis for the preliminary selection of appropriate rate of loading during K_0-consolidation of large clay samples.

A laboratory testing programme of K_0-consolidation of normally consolidated reconstituted soft London clay (LL = 69, PI = 45) was carried out. All the tests were performed using an automated stress path system incorporating local axial and radial strain and mid-plane pore pressure measuring devices. Experimentally measured excess pore pressures developed during K_0-consolidation of four 102 mm dia. by 203 mm high samples for a particular rate of loading (0.7 kPa/hr) and drainage conditions (drainage from one end and radial boundary) have been compared with the predicted values. Results indicate that although the experimental values of excess pore pressure at the end of K_0-consolidation are comparable with the predicted value, the nature of the predicted curve does not agree well with the experimental curves.

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NOTATION

c_v	coefficient of consolidation
LL	liquid limit
PI	plasticity index
t	time
T	time factor
ΔT	time factor increment
u	excess pore pressure
z	depth
Δz	depth factor increment