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DESIGN FORCES AND MOMENTS IN CIRCULAR SILOS BASED ON FINITE ELEMENT ANALYSIS

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ABSTRACT: Analysis of full-scale circular silos having different types of supports and subjected to various loading conditions revealed that the conventional method is not adequate in predicting the values of some of the stress resultants required for the design of a silo. The conventional method is completely unable to compute the values of meridional and circumferential moments. Axisymmetric thick shell finite elements, on the other hand, can easily analyse circular silos to determine all the design forces and moments. A number of parameters (geometric or material properties) also affect the values of such forces and moments.

KEY WORDS: Full-scale silo, conventional, meridional, circumferential, axisymmetric, finite element.

INTRODUCTION

Concrete silos and bunkers may be single or multiple and of various plans (Fig. 1). The most common shape is circular, since under uniform lateral pressure around the circumference the circular wall is under tension with no bending moment. For this reason, circular silos are built with diameters far exceeding practical lateral dimensions of rectangular or square silos. A circular silo essentially consists of a number of axisymmetric structural elements, namely the roof, cylindrical vertical wall and the bottom . The top roof may be of concrete, doweled to the walls by providing full or partial continuity of walls or it may be supported in a manner permitting free expansion and contraction and slight movement due to lateral forces. The vertical wall may be of uniform or varying thickness. Flat bottom may create problem in the removal of material, while conical hopper is self cleaning and it may also be of uniform or varying thickness. Vertical wall and conical hopper of a silo may be monolithically constructed and supported on columns or continuous circular vertical wall . In some cases the vertical wall and conical hopper may be supported separately . The widely used three types of silos, depending on the type of support, are shown in Fig.2.

Whether isolated or in connected groups, circular silos are usually first analyzed and designed as single silos. Interaction among silos of the group is then considered, and modifications are made where necessary. The design of a reinforced concrete silo structure consists of analysis, selection of physical dimensions and calculations and placement of reinforcement. Among these three steps analysis is the most important. From analysis one gets the forces and moments required for design. This

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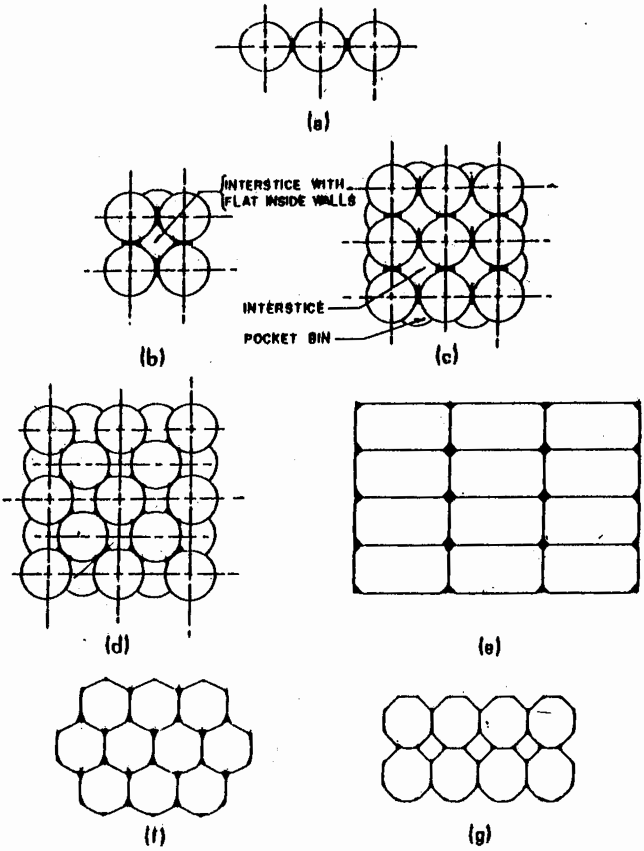
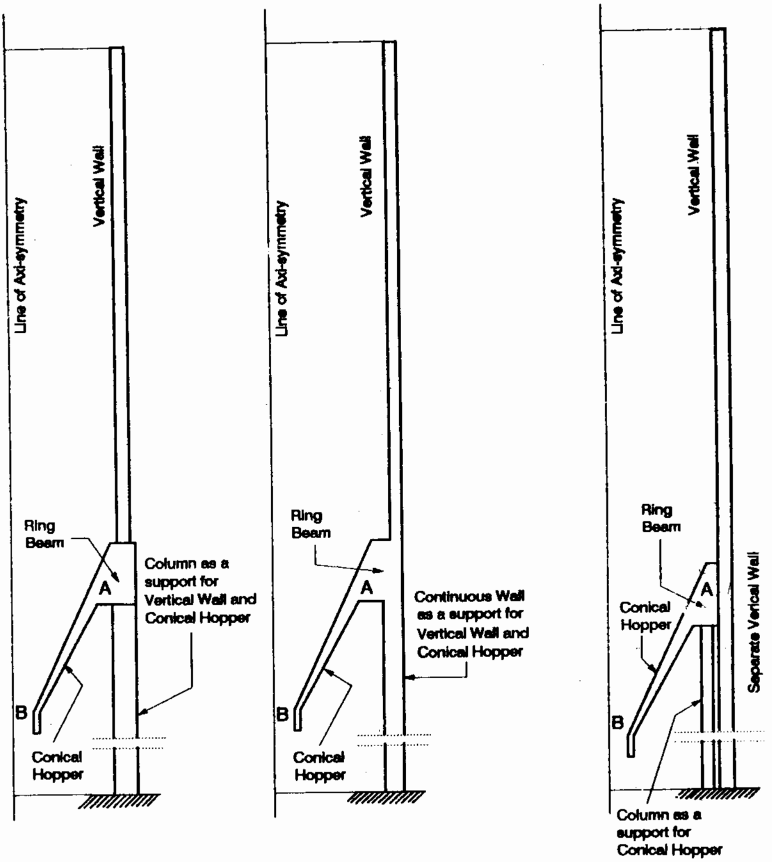


Fig 1. Typical Silos and Bunker Groups

paper concentrates on the determination of rational design forces and moments in silos.

Conventional methods of analysis of silos can deal well with axisymmetric loading due to gravity and stored materials . A silo, being an elevated structure, may be subjected to tremendous lateral loads due to wind and earthquake. The conventional methods cannot incorporate the effect of lateral loads in the design procedure effectively. Meridional and hoop forces developed in silo vary vertically. For an economic design, these variations must be taken into consideration. There are other stress resultants such as meridional moment or circumferential moment having considerable but localized effect. Prediction of various stress resultants at critical locations by approximate conventional methods may not always be acceptable .Besides, traditional approach of analysis can not



(a) Type - 1 (b) Type - 2 (c) Type - 3

Fig 2. Types of Silos Depending on Ring Beam Supports
(a) Ring beam supported by columns
(b) Ring beam supported by monolithic continuous wall
(c) Ring beam supported by separate columns

predict any type of moments at all. Despite all such approximations the conventional method of analysis has been used with considerable success in the past. Conservative design approach combined with high factor of safety can be attributed to such success.

With the advancement of the versatile and powerful techniques of finite elements it has now become easy to determine more accurately all the design forces at any section of a circular silo, in addition to the hitherto neglected moments.

PRESSURES IN SILO

Analytical methods normally give the static pressures (pressures when material is at rest) only. The structural designers need to know the final total pressure, or "Design Pressure". This design pressure can be estimated by modifying the computed static pressure to account for material movement, eccentric discharge, and other pressure-affecting conditions or by using analytical methods intended to give design pressures directly.

The analytical methods are based on equilibrium of the stored material in a static condition without considering the elastic interaction with the bin structure. In this investigation, the static pressure is computed using the Janssen method (Janssen 1895) [Appendix I] which is then converted to design pressure by multiplying the static pressure with overpressure factor (ACI 313-77). Two other methods used for computing static pressures in silo are the Reimbert method (Reimbert and Reimbert 1976) and the Airy method (Airy 1897).

CONVENTIONAL METHOD OF ANALYSIS

Brief Review of Conventional Method

In the conventional method of analysis of a circular silo certain assumptions are necessary. Some of which are listed below:

i) The radial pressure from the stored materials is uniform around the circumference at a particular elevation.

ii) Silo is a thin-walled cylinder stressed in circumferential tension only due to lateral pressure and there is no bending moment or shear.

iii) If the ring beam is monolithic with the vertical wall or conical hopper, there is no effect of the restraint provided by the ring beam either on the vertical wall or on the conical hopper.

iv) The vertical wall can expand freely at the bottom of pressure zone.

v) The conical hopper can expand freely at its junctions with the bottom of ring beam.

(a) Analysis for self weight and material pressure

In conventional method of analysis both vertical wall and conical hopper are considered to be subjected to meridional and cicumferential membrane forces only. For vertical wall the circumferential membrane force (horizontal tensile force) per unit height of cylindrical wall at any height is given by

$$F = p_{des}\, D/2 \qquad \text{(WSD method)} \qquad (1)$$

$$F_u = k_l p_{des} D/2 \text{ (USD method)} \qquad (2)$$

where k_l = design value of lateral pressure at the corresponding point,
D = inside diameter of the silo.

Total factored (ultimate)meridional membrane force F_{vu} per unit width of the wall is given by (not considering wind)

F_{vu}=1.7 x(Vertical friction force + roof and other vertical live load) + 1.4 x (wall self weight above + roof and other dead load).

All terms in parenthesis are per unit length of wall.

Conical hopper of a concrete silo may be monolithically constructed with the vertical wall. Normally, it is supported by a concrete ring-beam around the upper perimeter of the hopper, and the bottom of the hopper preferably should not be restrained or supported. Fig.3 shows dimensions to be used in computing vertical pressures and pressures normal to the walls of a conical hopper. Two membrane forces, the meridional force F_m and the circumferential force F_t, acting in conical hopper wall according to USD method are given by (Safarian and Harris 1985)

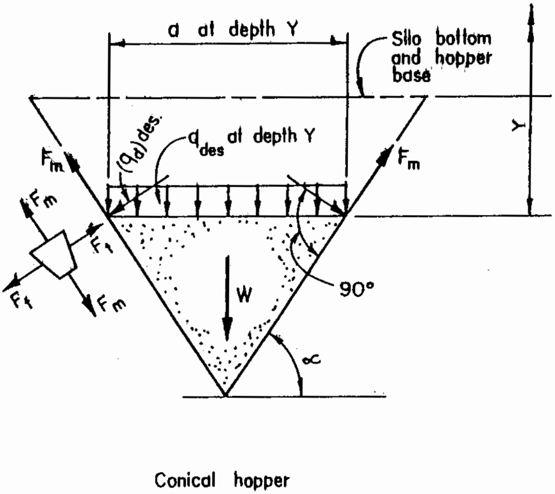


Fig 3. Forces in Conical Hopper By Conventional Method of Analysis

$$F_{mu} = 1.7\left[\frac{q_{des}D}{4\sin\alpha} + \frac{W_L}{\pi D\sin\alpha}\right] + 1.4\left[\frac{W_g}{\pi D\sin\alpha}\right], \qquad (3)$$

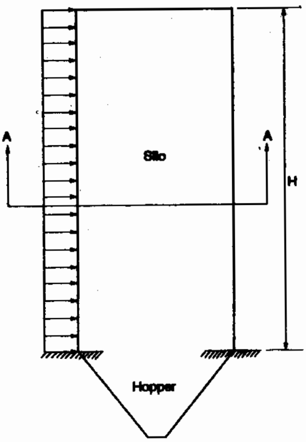
$$F_{tu} = 1.7\left[\frac{q_{\alpha des}D}{2\sin\alpha}\right] \qquad (4)$$

Both forces are maximum at the upper edge of the hopper, and approach zero at the lower edge.

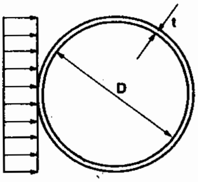
(b) Wind load

Conventional method of analysis can compute the meridional membrane force only due to wind load. In this case the vertical wall is considered as a cantilever beam fixed at the level of bottom ring beam and subjected to the lateral wind pressure which is uniformly distributed in the horizontal direction on the diametrial projection of the cylindrical wall. Vertical distribution of the wind pressure intensity may be taken uniform or varying depending on the height of the respective point above ground (Fig.4).

For the present analysis of a full-scale silo the wind pressure has been assumed 36.9 psf (corresponding to a wind velocity of 120 mph) on a projected surface normal to the directions of wind all over the depth and a reduction factor (Safarian and Harris 1985) of 0.6 has been used to take into account the effect of circular geometry.



(a) Vertical distribution



(b) Section A-A

Fig 4. Wind Pressure Distribution in Conventional Method of Analysis

(c) Thermal stresses

Due to temperature difference between inside and outside of a silo wall, both meridional and circumferential bending moments develop and their ultimate value is given by (Safarian and Harris 1970)

$$M_{xt,u} = M_{yt,u} = 1.4E_c h^2 \alpha_t \, \Delta T/(L\nu) \qquad (5)$$

(in - lb per ft when E_c is psi and h is in.)

where $M_{xt,u}$ = ultimate circumferential bending movement,
$M_{yt,u}$ = ultimate meridional bending moment,
E_c =modulus of elasticity of concrete,
α_t =coefficient of thermal expansion of concrete,
ΔT = temperature difference across the wall.
The factor of 1.4 in the above equation is the load factor K_g.

Using Poisson's ratio $\nu = 0.2$, the above equation can also be written as

$$M_{xt,u} = M_{yt,u} = 1.25E_c h^2 \alpha t \, \Delta T K_g.$$

Analysis of a Full scale Silo

A full-scale silo has been analysed using both conventional method and finite element method, and a comparison has been made thereafter. For this purpose a silo of standard dimensions with common properties has been used. *The geometric parameters of the silo are shown in Fig.* 5. Data used for the silo are given below.

Dimensions:

Height of vertical wall, H = 160 ft.

Diameter of the silo (internal), D = 30 ft.

Overall depth of silo (From bottom

of hopper to top of vertical wall) = 180 ft.

Size of Bottom Ring beam

(a) Depth of bottom ring beam d = 42 in.

(b) Width of bottom ring beam at top, b = 12 in.

(c) Width of bottom ring beam at bottom = 30.4 in.

Thickness of vertical wall at top, T_{top} = 6 in.

Thickness of vertical wall at bottom, T_{bottom} = 9 in.

Thickness of hopper at top, t_{top} = 9 in.

Thickness of hopper at bottom, t_{bottom} = 5 in.

Angle of conical hopper with horizontal, α = 55 degree

Properties of stored material (grain):

Unit weight of material,	$\gamma =$	50	lb/cft.
Angle of internal friction.,	$\rho =$	30	degree
Coefficient of wall friction,	$\mu' =$	0.45	
Temperature difference across the wall,	$\Delta T =$	200°	F

Properties of construction material:

Modulus of elasticity of concrete,	E_c	=	$3\text{x}10^6$ psi.
Unit weight of concrete,	γ_c	=	150 lb/cft.
Ultimate strength of concrete,	$f'c$	=	3000 psi.
Poisson's Ratio of concrete,	v	=	0.2
Coefficient of thermal expansion of concrete,	α_t	=	$5.5\text{x}10^{-6}/°F$
Modulus of Elasticity of steel,	E_s	=	$29\text{x}10^6$ psi.
Ultimate strength of steel,	f_y	=	60000 psi.
Air pressure due to wind		=	36.9 psf.

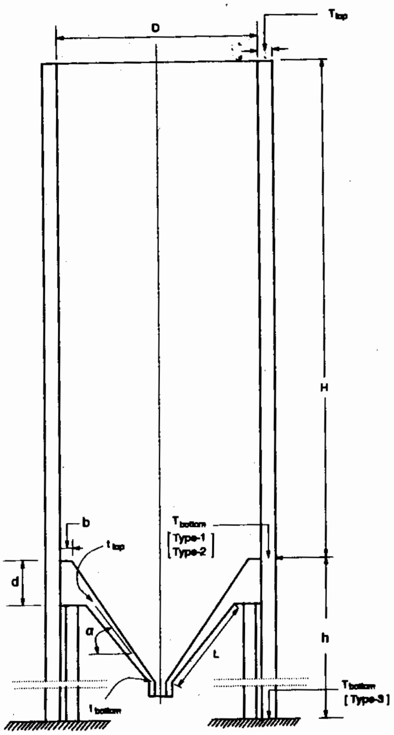


Fig 5. Diagram Showing Various Dimensions of a Silo

The results of analysis by the conventional method have been presented along with those obtained from the finite element analysis for the sake of comparative study.

FINITE ELEMENT METHOD OF ANALYSIS

The Finite Element Program

Ahmad (Ahmad 1968, Ahmad 1969) developed a very general computer program with parabolic and cubic type of Thick Shell Finite Elements capable of analysing axisymmetric shells loaded axisymmetrically as well as non-symmetrically. It can also deal adequately with thin shells. This program has been adapted and used for the analysis of silo for the following load cases:

i) Gravity (Self weight), considered axisymmetric;
ii) Stored material pressure, considered axisymmetric;
iii) Wind load, considerd non symmetric;
iv) Temperature difference, considered axisymmetric.

For finite element analysis the wind pressure distribution around the circumference has been taken the same as used by Ahmad (Albasiny and Martin 1967) and is shown in Fig.6. About seven Fourier harmonics represent the above distribution quite accurately and the Fourier coefficients used are given in Table 1. Observe that at θ = 180° the wind direction is perpendicular to the surface and the diametral line is parallel to the wind direction.

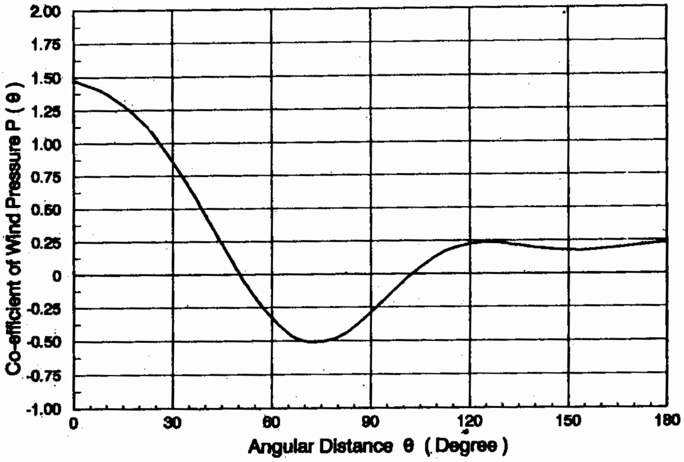


Fig 6. Wind Pressure Distribution Around the Circumference of a Silo

Finite Element Idealisation of Silo

The silo is represented by a chain of axisymmetric shell elements placed end to end. Since the program can not deal with branching, the actual structure needs idealisation near the joints. In this case the following assumptions are made.

Table 1. Fourier Coefficients for the Pressure Distribution of Fig.6

Harmonics	Coefficients
0	0.24706
1	0.31387
2	0.58763
3	0.42213
4	0.02466
5	-0.11481
6	-0.00451

Type-1 and Type-2

i) The mid-surface of vertical wall meets with the mid-surface of the bottom ring beam.

ii) The bottom supporting ring beam is divided into two parts. One part is associated with the vertical wall and another part is associated with the conical hopper.

iii) The bottom of the vertical wall is assumed to be supported by columns for Type-1 and by continuous wall for Type-2. The whole structure is supported at the junction of vertical wall and conical hopper in such a way that there is no vertical displacement, but there may be radial movement.

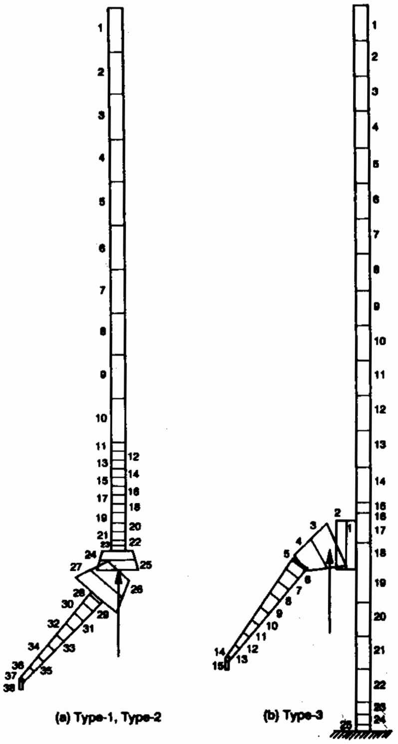
Idealisation of Type-1 and Type-2 is shown in Fig. 7 and length of elements are given in Table 2.

Table 2. Length of Elements in Finite Element analysis.

	Vertical Wall		Conical Hopper	
Silo Type	Element No.	Length of Element	Element No.	Length of Element
	1 to 10	8.0% of H	26, 27	d / 3
TYPE-1	11 to 21	(18.0/11)% of H	28, 37	L / 64
&	22 to 23	1.0% of H	29, 36	7L / 64
TYPE-2	24 to 25	d / 6	30 to 35	L / 8
			38	12″
	1 to 14	{H-0.025(H+h)}/14	1 to 2	b/2
	15	1.5% of (H+h)	3 to 4	b1/2
	16	1.0% of (H+h)	5, 14	L / 64
TYPE-3	17 to 18	d / 2	6, 13	7L / 64
	19 to 22	{h-d-0.04(H+h)}/4	7 to 12	L / 8
	23 to 24	1.5% of (H+h)	15	12″
	25	1.0% of (H+h)		

Type -3

i) The vertical wall and the conical hopper with ring beam are considered to be completely separate components of silo. The



(a) Type-1, Type-2

(b) Type-3

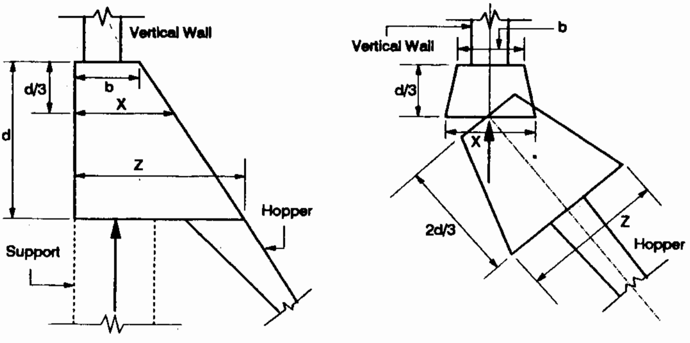
Fig 7. Idealisation and Element Numbering of Silos

whole analysis is performed separately and the element numbering of the conical part is independent of the vertical wall. During analysis the vertical wall is considered as problem-1 with the stored material pressure in the pressure zone only, and the wind load all over the depth. The conical part is considered as problem -2 and subjected to symmetric loads with no wind load.

ii) The vertical wall is considered to be supported on foundation and fixed at the bottom.The conical hopper is also move inward supported vertically by ring beam on separate columns. The centre line of column passes through the node which is closest to the centroid of the ring beam section.

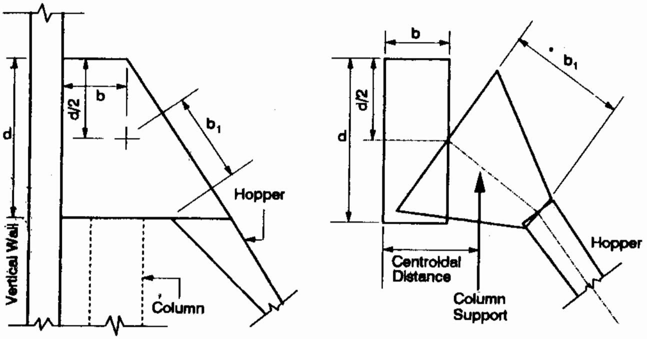
Idealisation of Type-3 is also shown in Fig. 7 and length of elements are given in Table 2.

Near the junctions the element shapes become trapezoidal due to lack of continuity of slopes of the middle surface of the two elements on the two sides of a junction. Such elements suffer in their performance. Huda's technique (Huda 1984) illustrated in Fig. 8 as applied to joint 'A' and joint 'B' has been successfully used to eliminate the shortcomings.



(a)-i. Joint A, Type-1, Type-2

(b)-i. Joint A, Type-1, Type-2



(a)-ii. Joint A, Type-3

(b)-ii. Joint A, Type-3

(a)-iii. Joint B

(b)-iii. Joint B

Fig 8. Idealisation of Joints (a) Actual Joints (b) Idealised Joints

This makes the nodal normals of each element perpendicular to its middle surface at the node. This technique consists of removing small quantity of material from one side of the middle surface and adding it to other side so that the odd shaped element now assumes a normal shape. Shifting a small quantity material from tension side to compression side or vice versa does not change the total quantity of strain energy so long as the behaviour of the material is linearly elastic. Since the finite element formulation is based on the minimisation of strain energy the above idealisation does not affect the stiffness term, thought it ensures gentle behaviour of the element.

COMPARATIVE STUDY

Mode of Comparison

In order to reveal the merits of finite element method of analysis in relation to conventional method of analysis of silo, a comparative study has been made. Results obtained from both conventional method and finite element method are presented in the same figure. At first, the stress resultants which can be obtained from both conventional and finite element methods are presented. A number of forces and bending moments, which can not be computed by conventional method, but can easily be obtained from finite element analysis are presented separately later. In case of meridional moments and circumferential moments due to wind load, the maximum values are shown considering 13 equidistant points (at intervals of 15°) along half of the circumference at each node. The symbols (H, h, L,l) used in this presentation are shown in the Fig. 9.

Forces Obtained from Both the Methods of Analysis

(a) Meridional force

Vertical wall: In the vertical wall meridional forces along vertical section due to self weight, obtained from conventional method and finite element method, are indistinguishable, and it is negative all over the depth (Fig. 10a). So is the case with meridional forces due to stored material Pressure (Fig. 11a).

Various stress resultants such as meridional force, hoop force, meridional moment and circumferential moment due to wind load vary circumferentially as a result of non symmetric distribution of wind pressure in circumferential direction (Fig.6). The circumferential variations of various stress resultants on a horizontal plane at different levels for different types of silos are shown in Fig. 12 and Fig. 13. The locations of maximum wind effect for different stress resultants are visible from these figures. It is observed that the maximum positive meridional forces occur at $\theta = 0^\circ$ for all types of silos and the maximum negative meridional forces, away from the bottom supports, occur at $\theta = 180^\circ$. Near the bottom support maximum negative meridional forces occur between $\theta = 105^\circ$ to $\theta = 120^\circ$. This indicate that the locations of maximum effect may change depending on the level of the horizontal plane. The variation of meridional forces in the vertical direction is shown in Fig. 14. Fig.14a shows positive meridional forces at $\theta = 180^\circ$ for all types. Fig.14b shows negative meridional forces at $\theta = 105^\circ$ for Type-1 and Type-2 and at $\theta = 120^\circ$ for Type-3. Finite Element analysis and conventional analysis are in close agreement in respect of positive meridional force for the upper part of the vertical wall (Fig.14a). But for

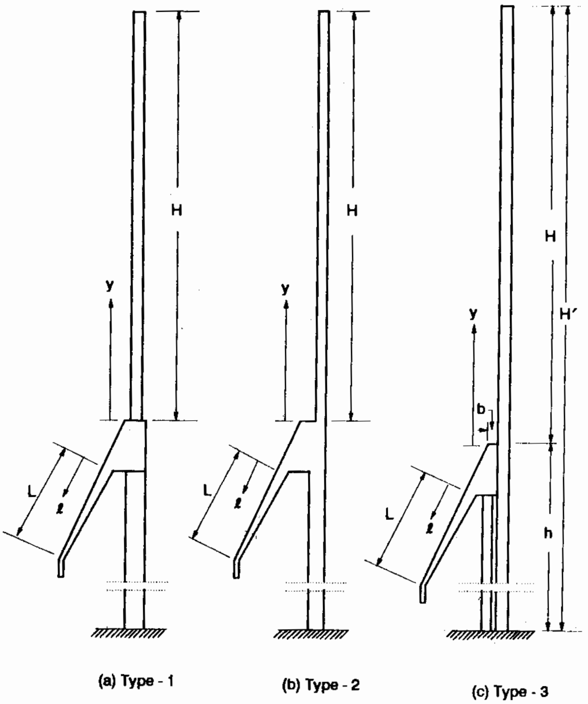
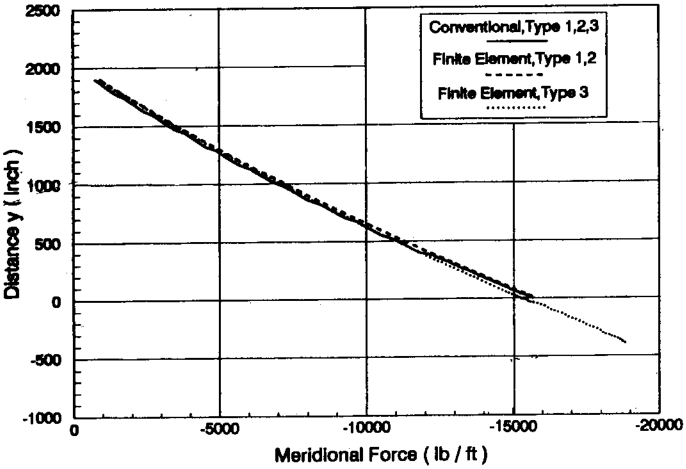


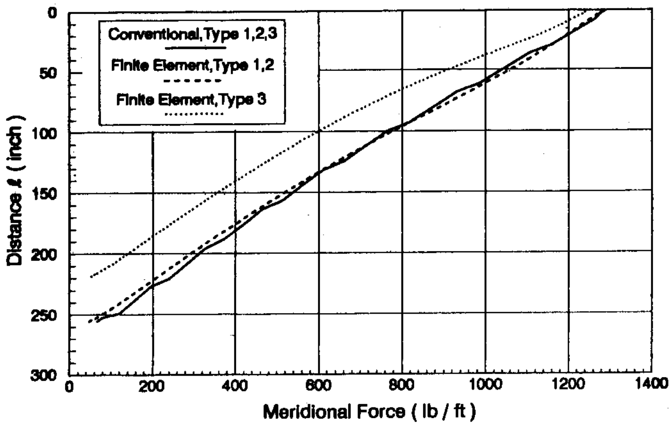
Fig 9. Types of Silos Depending on Ring Beam Supports with Various Symbols
(a) Ring beam supported by columns
(b) Ring beam supported by monolithic continuous wall
(c) Ring beam supported by separate columns

lower portion of the vertical wall the conventional method underestimates the value of positive meridional force. The negative meridional force due to wind predicted by conventional method is always much greater than that obtained from finite element method (Fig. 14b).

Conical hopper: In the conical hopper the meridional forces due to self weight obtained from finite element analysis of Type-1 and Type-2 are almost identical to those obtained from conventional method (Fig. 10b). However, this is considerably smaller in Type-3. On the other hand, the conventional method underestimates the value of meridional force due to material pressure, in comparison to finite element analysis (Fig. 11b). Again, finite element analysis yields smaller meridional force in Type-3 than what develops in Type-1 and Type-2.



(a) Vertical Wall, y = Distance from bottom of pressure zone (upward positive)



(b) Conical Hopper, ℓ = Distance from bottom of ring beam

Fig 10 Meridional Force Due to Self Weight Along Vertical Section

(b) Hoop force

Vertical Wall: Due to grain load, hoop forces predicted by conventional method and finite element analysis are almost identical for the upper part of the vertical wall (Fig.15). Near the ring beam some discrepancy is observed. In conventional method there is no negative hoop force for the vertical wall. But in finite element analysis there exists considerable negative hoop force near the ring beam in Type-1 and Type-2, and near the foundation in Type-3.

Conical hopper: Hoop forces in conical hopper due to self weight, obtained from conventional method and finite element analysis are in close agreement for Type-1 and Type-2 beyond certain distance from the ring beam (Fig. 16a). Similar agreement is found with the hoop forces due to stored material pressure (Fig. 16b). Near the ring beam finite element analysis predicts hoop force much smaller than that form conventional method. In conventional method the maximum hoop force occurs at the junction of conical hopper and ring beam but in finite element analysis the maximum value of hoop force is found at a distance away from the ring beam. For Type-1 and Type-2, the distance of maximum hoop force is about 25% of the length of the conical hopper and for Type-3, it is about 15% of the length of conical hopper. The maximum value of hoop force in conventional method is about 20% greater than that of finite element value in Type-1 and Type-2. But the maximum value of hoop force in Type-3 from finite element analysis is about 10% greater than that obtained from conventional method.

Moments obtained from Finite Element Analysis

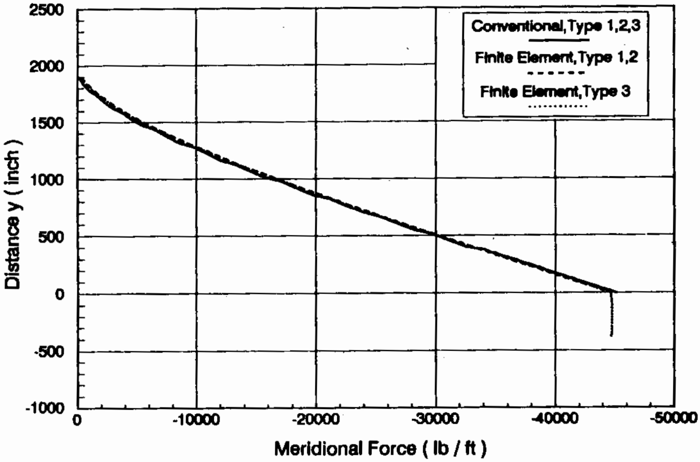
The variation of meridional moment and circumferential moment predicted by finite element analysis for different load cases are discussed below. None of these values are obtainable from conventional method.

(a) Meridional moment

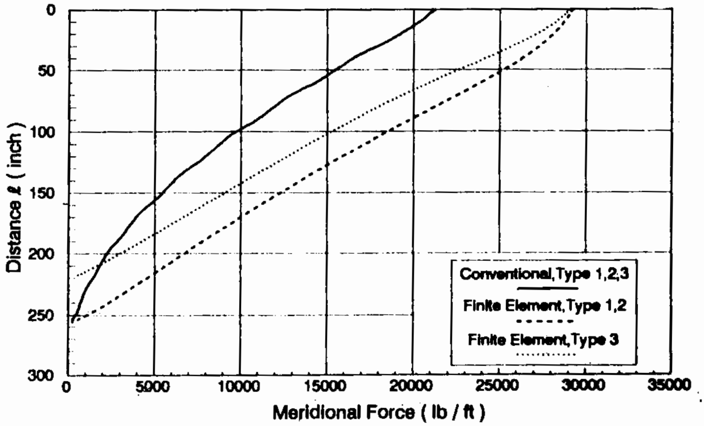
Vertical Wall: The value of meridional moment due to grain load is considerable near the ring beam (Fig.17). Both negative (tension inside) and positive (tension outside) meridional moments in Type-1 and Type-2 are much greater than those in Type-3. In Type-1 and Type-2 the maximum negative value occurs at the junction of vertical wall with the ring beam and maximum positive value occurs at a small distance above this. In Type-3 the maximum negative bending moment occurs at the bottom of vertical wall.

Analysis of vertical wall in Type-1 and Type-2 are similar for self weight and grain load, but for wind load there is slight difference. In Type-2, due to wall support of the ring beam, the conical hopper is not subjected to the wind pressure. But in Type-1, through the opening of column, wind may create pressure on the conical hopper. Results of wind load analysis are shown in Fig. 18, separately for the three different cases. However, it is seen that the results of Type-1 and Type-2 are indistinguishable.

Conical hopper: Considerable amount of meridional moments occur in conical hopper (Fig.19). For Type-1 and Type-2, at the junction of conical hopper and ring beam, negative meridional moment develops due to grain pressure. In Type-3, at the same point positive meridional moment is predicted. For all the types the meridional moment reduces sharply as the distance increases from the junction of hopper and ring beam.



(a) Vertical wall, y = Distance from bottom of pressure zone
(upward positive)



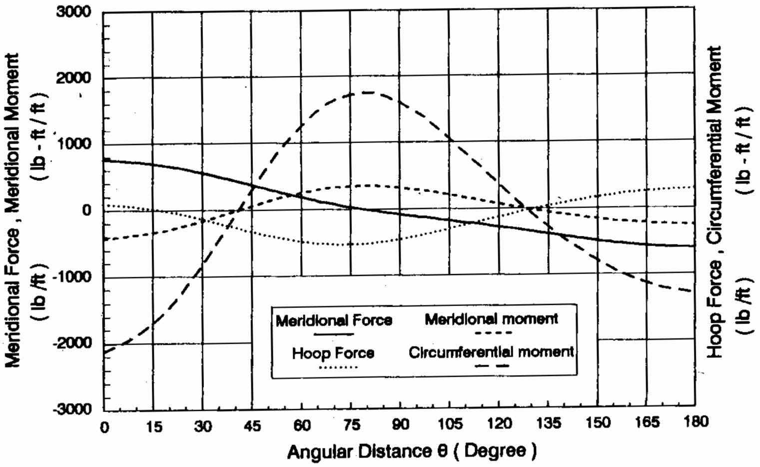
(b) Conical Hopper, ℓ = Distance from bottom of ring beam

Fig 11. Meridional Force Due to Stored Material Pressure Along Vertical Section

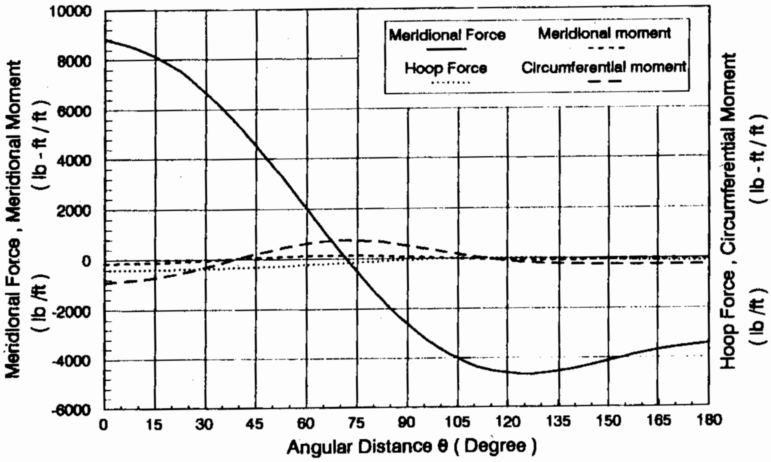
(b) Circumferential moment.

Vertical wall: Circumferential bending moments, both positive and negative, develop due to wind load on the vertical wall (Fig.20a and Fig 20b). The maximum values of circumferential bending moment develop in the top portion of the vertical wall and it reduces to zero at the bottom of vertical wall.

Conical hopper: Circumferential bending moments developed in conical hopper due to wind load are negligible for all the types.

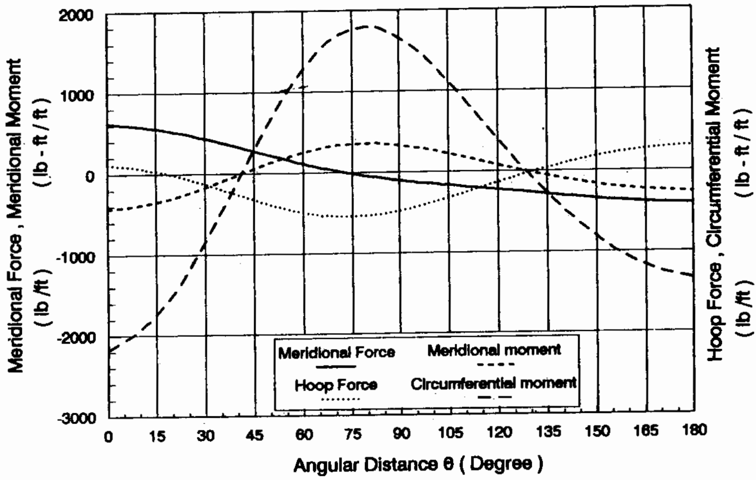


(a) At y = 1455 inch.

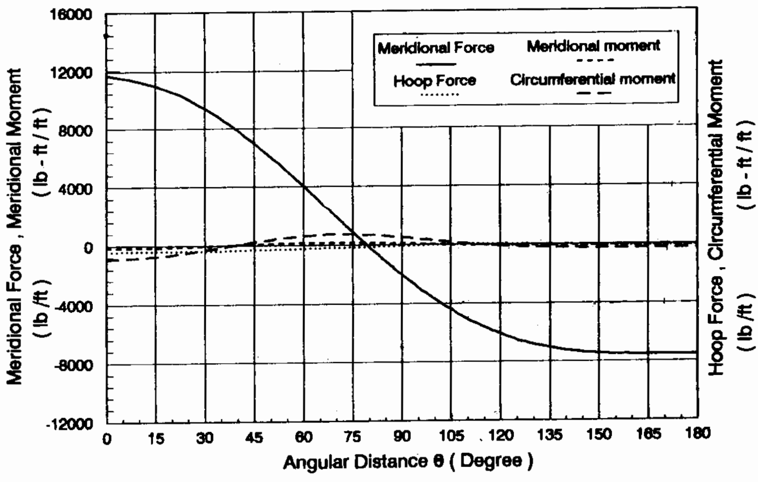


(b) At y = 383 inch.

Fig 12. Forces and Moments Along the Circumference Due to Wind Pressure (Type-1, Type-2)



(a) At y = 1517-inch.

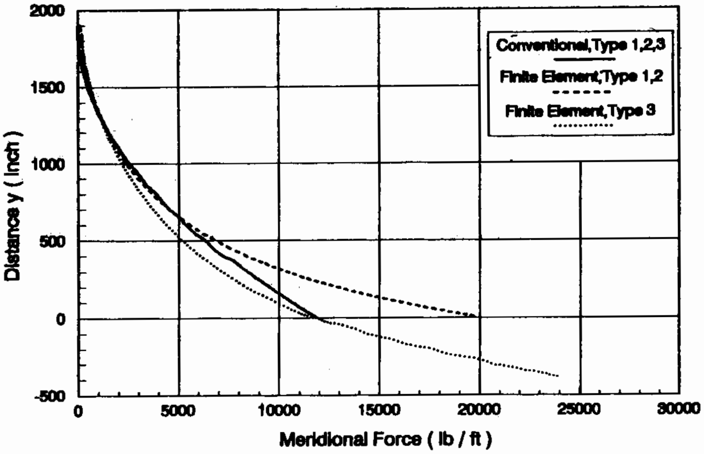


(b) At y = 8 inch.

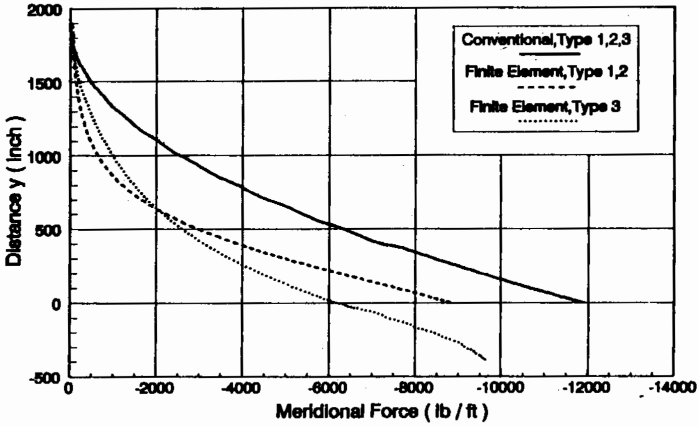
Fig 13. Forces and Moments Along the Circumference Due to Wind Pressure (Type-3)

Moments Developed due to Temperature Difference Across the Wall

Due to temperature difference across the wall of a silo, both meridional and circumferential bending moments develop. Conventional method of analysis suggests a single equation (Eq.6) for the computation



(a) Positive (Tensile) Meridional Force



(b) Negative (Compressive) Meridional Force

Fig 14. Meridional Force in Vertical Wall Due to Wind Pressure

of meridional and circumferential bending moments. But finite element analysis predicts different values of meridional and circumferential moments due to temperature difference at the same point. Fig.21 to Fig.22 show the values of meridional and circumferential moments developed in the full-scale silo.

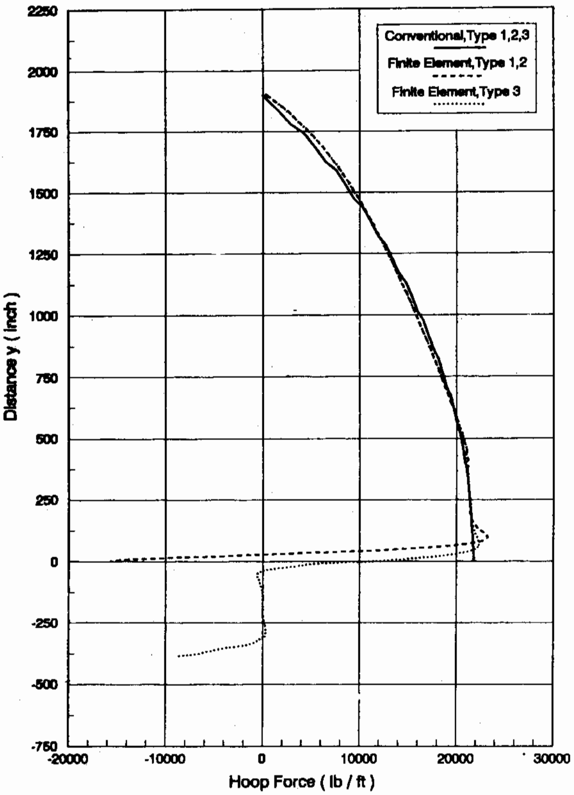
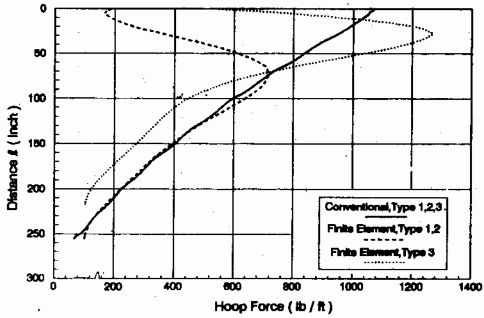


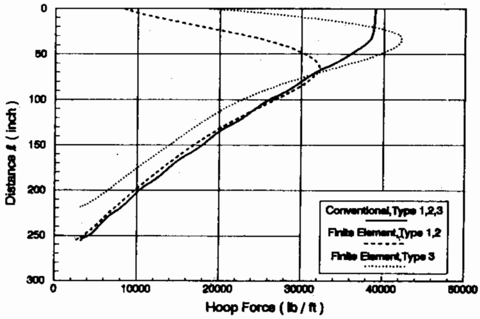
Fig 15. Hoop Force in Vertical Wall Due to Stored Material Pressure

<u>Vertical wall:</u> In Type-1 and Type-2 both the moments obtained from finite elements analysis are greater than those predicted by conventional analysis for the whole length of the vertical wall (Fig.21a). The meridional moment in finite element analysis is slightly greater than the circumferential moment, and both the moments sharply increase near the bottom of vertical wall (top of ring) beam). In Type-3 it is assumed that below the pressure zone(below top of ring beam) inside and outside temperatures are the same. For the upper half of the vertical wall the finite element method gives slightly greater moments than the conventional method, but for the lower half the reverse occurs (Fig.22a). Near the top of ring beam the moments predicted by finite element analysis increase sharply and then reduce to zero with a little negative value just below the top of ring beam.

<u>Conical hopper:</u> In Type-1 and Type-2 both the methods predict maximum moments at the junction of conical hopper with ring beam, but finite element analysis gives moments greater than those predicted by



(a) Due to self weight



(b) Due to stored material pressure

Fig 16. Hoop Force in Conical Hopper

conventional analysis and values of meridional moment are slightly greater than those of circumferential moments (Fig. 21b). Moments predicted in Type-3 by both the methods are shown in Fig. 22b. Finite element analysis predicts maximum values at some distance away from the top of hopper.

CONCLUSIONS

Full-scale silos with different types of supports at ring beam level were subjected to various loading conditions and analysed for the design forces and moments by both the conventional method and the axisymmetric thick shell finite elements. In the conventional method the vertical wall and the conical hopper are considered as separate structures which are subjected to membrane action only. But actually these are not so. The restraint provided either by the ring beam or by the ground support have significant effect on the overall behavior of silo. The conventional method cannot deal effectively with some of the possible loadings such as wind and earthquake because of its analytic limitations. Finite element method can easily deal with all such cases.

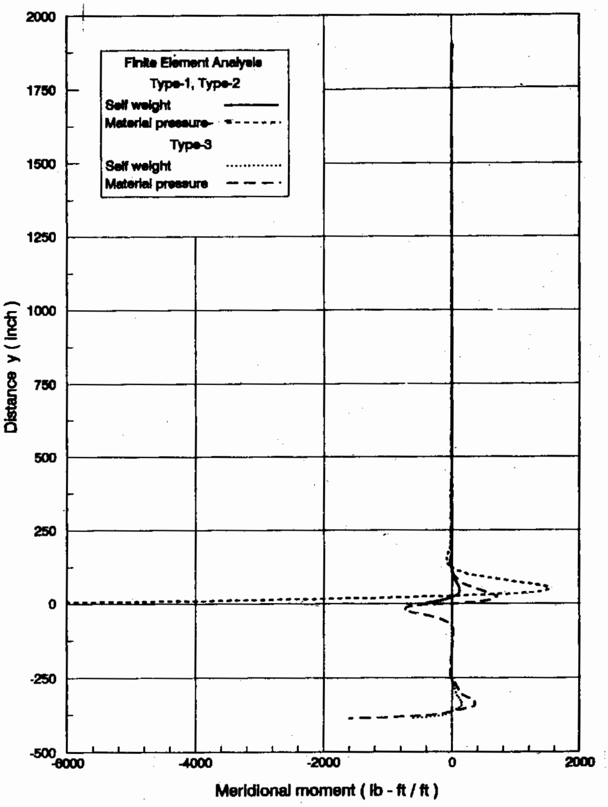
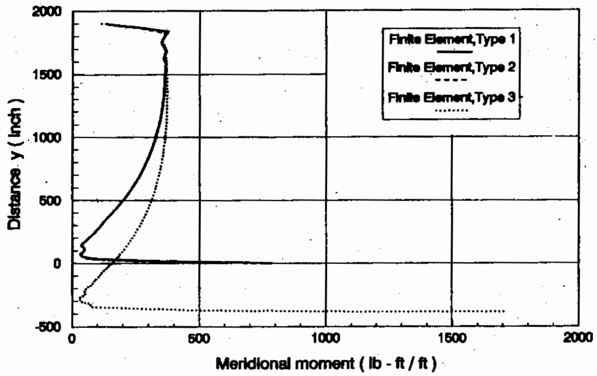
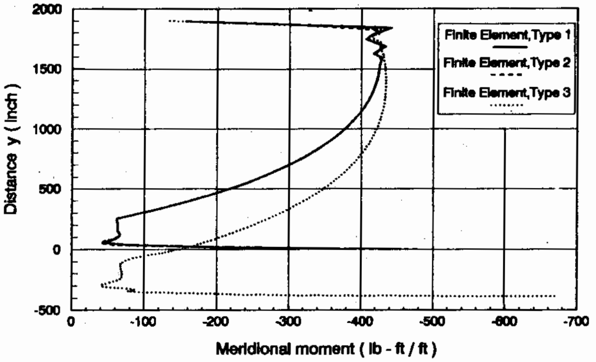


Fig 17. Meridional Moment in Vertical Wall Due to Self Weight and Stored Material Pressure

The investigation has revealed that certain forces and moments, hitherto neglected, can easily be incorporated in the design of silo. Conventional method cannot predict any negative hoop force in the vertical wall. But finite element analysis has shown that due to the stored material pressure considerable amount of negative hoop force develops at the bottom of vertical wall. Also negative and positive meridional moments occur at the bottom of vertical wall. This is due to the partial fixity provided by the ring beam and these are easily obtainable from finite element analysis. Conventional method predicts maximum meridional force and hoop force due to material pressure at the top of the hopper. Finite element analysis has shown that this is not true for the hoop force which is maximum at some distance away from the junction of ring beam and hopper Positive and negative meridional moments developed in the conical hopper are also appreciable. Finite element



(a) Positive Meridional Moment



(b) Negative Meridional Moment

Fig 18. Meridional Moment in Vertical Wall Due to Wind Pressure

analysis has shown that wind produces considerable amount of both tensile and compressive meridional forces in vertical wall. Also significant amount of circumferential moment develops in the vertical wall due to wind load. Conventional method of analysis is completely unable to predict such moments. Finite element analysis has further shown that meridional moment due to temperature effect may be much higher than the value predicted by conventional method.

The present investigation has revealed the drawbacks of the conventional approach, in comparison to the merits and potentials of finite element analysis. This has led to an extensive parametric study by finite element analysis culminating to the development of a simple and direct way of finding forces and moments required to design the various structural elements of a silo (Alauddin, 1994). The reader should consult a separate paper (Alauddin and Ahmad) for further details.

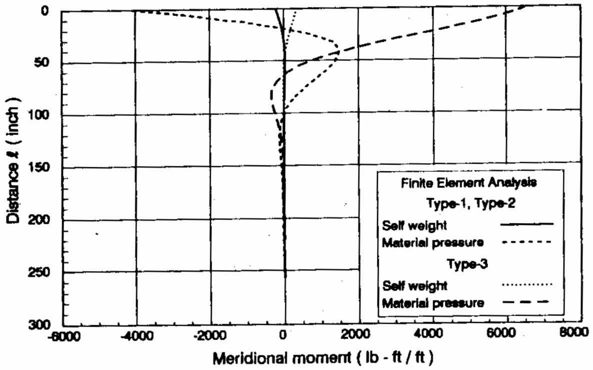
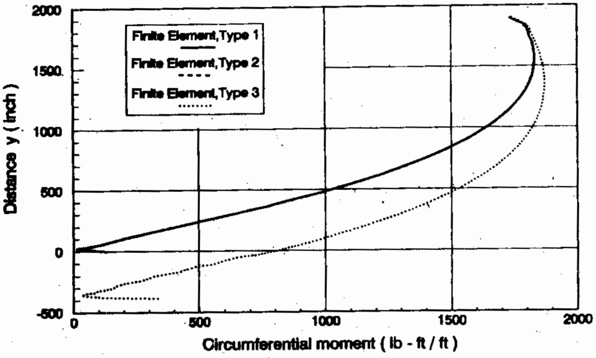
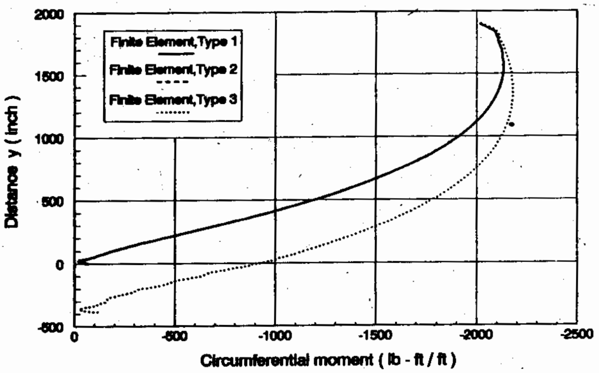


Fig 19. Meridional Moment in Conical Hopper Due to Self Weight and Stored Material Pressure

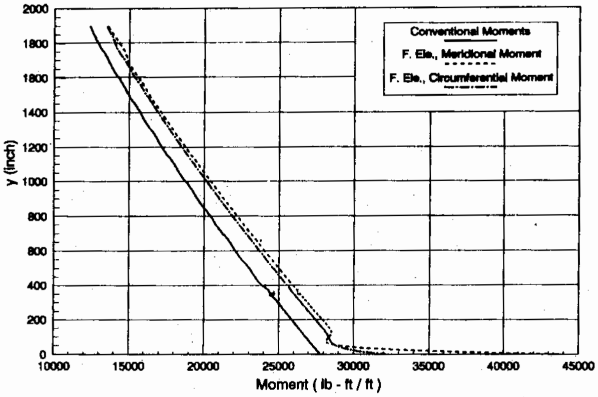


(a) Positive Circumferential Moment

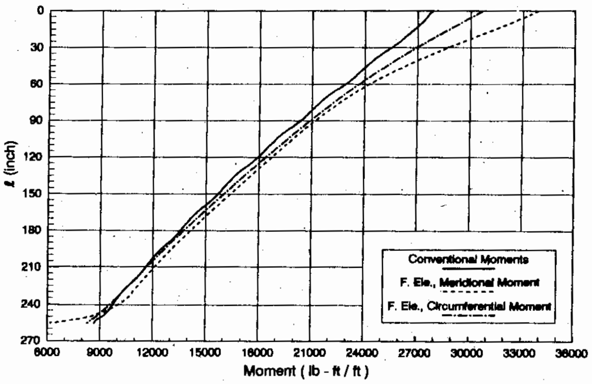


(b) Negative Circumferential Moment

Fig 20. Circumferential Moment in Vertical Wall Due to Wind Pressure

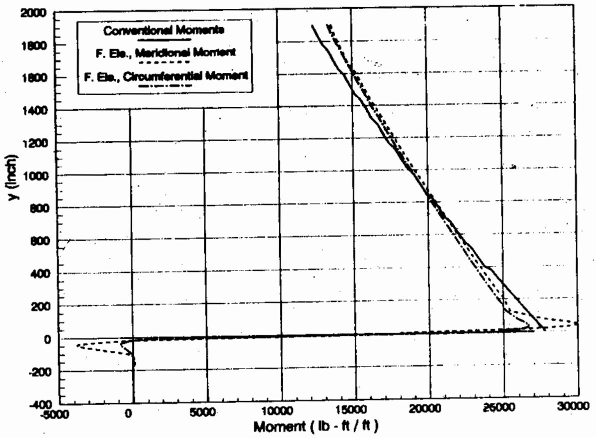


(a) Vertical Wall

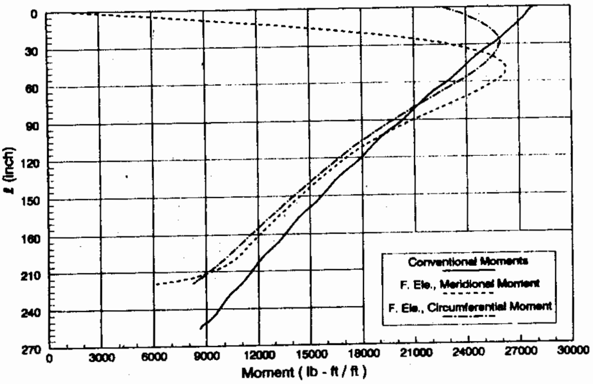


(b) Conical Hopper

Fig 21. Meridional and Circumferential Moments Due to Temperature Difference (Type-1, Type-2)



(a) Vertical Wall



(b) Conical Hopper

Fig 22. Meridional and Circumferential Moments Due to Temperature Difference (Type-3)

Appendix I. The Janssen Method

The major breakthrough in computation of stored material pressure came in 1895 when H.A. Janssen developed equations for computing lateral and vertical pressures of granular material in deep bins (Janssen 1895). The Janssen formula for vertical pressure at depth Y (Fig.3) below top of fill is given by

$$q = \frac{\gamma R}{\mu' k}\left[1 - e^{-\mu' kY/R}\right] \qquad \text{(A-1)}$$

where $k = \dfrac{(1 - \sin \rho)}{(1 + \sin \rho)}$

or simply $K = \tan^2 (45^\circ - \rho/2)$

which is the Rankine coefficient for active earth pressure - the ratio of horizontal pressure to vertical pressure. Hence, to compute the horizontal pressure P, Eq. A-1 is multiplied by k. Thus, the Janssen equation for horizontal pressure is

$$p = \frac{\gamma R}{\mu'}\left[1 - e^{-\mu' kY/R}\right] \qquad \text{(A-2)}$$

The wall friction force is $\mu' p$ per unit area of wall at depth Y. Vertical friction forces cause vertical force in the wall : compression if the wall is supported from below, tension if suspended from above. The vertical force in the wall (per unit of wall perimeter) at depth Y is given by

$$V_y = \mu' \int p\,dy = \gamma R\,[\,Y - \frac{R}{\mu' k}\;(1 - e^{-\mu' k\,Y/R})] \qquad \text{(A-3)}$$

If the cross section is circular, then the hydraulic radius is

$$R = \text{area/perimeter} = \frac{\pi D^2/4}{\pi D} = D/4$$

in which D is the inside diameter.

The designer of silo wall needs to know the total vertical force applied to the wall by friction from the stored material. This force, from materials above any depth Y, is equal to the weight of those materials minus the upward force from vertical pressure q. The friction force, per unit length of wall, from above is

$$V = R(\gamma Y - q). \qquad \text{(A-4)}$$

Appendix II. References

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Appendix III. Notations

C_d	Overpressure coefficient
D	Diameter of silo, Dead load
E	Modulus of elasticity
F	Force, Meridional force, Circumferential force
H	Height of storage zone
H'	Total height of vertical wall (Type -3)

K_L, K_g	load factors for live and dead load
L	Length of conical hopper
M	Moment, Meridional moment, Circumferential moment
R	Hydraulic radius
Y	Depth of stored material above point in question
b	Width of bottom ring beam at top
d	Depth of bottom ring beam
$f'c$	Unit compressive strength of concrete
g	Subscript meaning "gravity"
h	Wall thickness, subscript indicating "hopper", depth of vertical wall below pressure zone
k	Ratio of horizontal pressure to vertical pressure
l	Distance measured downward from top of conical hopper
m	Subscript meaning "meridional"
p	Lateral pressure due to stored material
q	Vertical pressure due to stored material, intensity of wind pressure
s	Subscript meaning "static"
t	Subscript meaning "total", "top"
u	Subscript meaning "ultimate"
v	Subscript indicating "vertical"
x	Subscript for "x-direction"
y	Subscript for "y-direction", upward distance from bottom of vertical wall
ΔT	Temperature difference, outside and inside wall faces
α	Angle of hopper slope, subscript for forces or pressures on sloping surface
α_t	Linear coefficient of thermal expansion

γ	Unit weight of stored material
θ	Angle around perimeter, subscript meaning "tangential" direction
μ'	Coefficient of wall friction
ν	Poisson's ratio
ρ	Angle of internal friction for stored material
ϕ	Strength-reduction factor, subscript for "meridional" direction

Appendix IV. FPS - SI Conversion factors

To convert	To	Multiply
inches (in.)	millimeters(mm)	25.4
feet (ft.)	meters(m)	0.305
pound (lb)	newtons(N)	4.45
pounds per feet (lb/ft.)	newton per meter (N/m)	14.59
pounds per square inch (psi.)	kilopascals(kPa.)	6.89
pounds per cubic feet (lb/cft.)	newton per cubic meter (N/m^3)	156.84