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EXPONENT OF FLOW VELOCITY FOR THE TRANSPORT OF SEDIMENTS IN ALLUVIAL RIVERS

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ABSTRACT : Attempts have been made to derive the exponent "b" analytically for different total and bed load predictors with some assumptions. Seven total load and three bed load prediction formulae were considered in this analysis. The relation between the exponent "b" and the Shields parameter (θ) also analysed. The analysis shows that the exponent "b" varies between 3.1 to 8.5 for total load and 3.3 to 10.5 for bed load predictors. The sensitivity of the exponent "b" for variation with the grain sizes (d_{50}) is found to be significant for Ackers and White and van Rijn formula. The water level slope has a small influence on the exponent "b" as observed at low values of the Shields parameter (θ).

KEY WORDS : River morphology, Sediment flow, exponent, Alluvial rivers.

INTRODUCTION

Exponent of the flow velocity plays an important role in estimation of sediment transport. The importance of this exponent is well understood by the investigators in the field of river engineering but its significance in each of the predictors is often not well realised for practical use. Its sensitivity with different hydraulic parameters has to be understood properly.

For general application in morphological studies, either by mathematical analysis or by scale modeling it is attractive to use the schematized sediment transport formula in the following approximate form (de Vries, 1986) as a function of flow velocity :

$$q_s = a\,u^{b} \tag{1}$$

where q_s is sediment transport per unit width (m^2/s), 'a' is the coefficient depends on the exponent "b", u is the averaged flow velocity (m/s) and "b" is the dimensionless exponent of the flow velocity in sediment transport equation. In general, the exponent "b" varies between 3 and 7 and even

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more (high values for low transport rates), with "b" equal to 4 to 5 for high sediment rates (Breusers, 1988). The exponent "b" depends on the Shields parameter, which is a dimensionless shear-stress parameter. The objective of this paper is to derive the expression of exponent "b" for the selected total and bed load prediction formulae and thereby to obtained the range of exponent "b" for the alluvial river conditions and to compare this expressions with the measured data.

USE OF SCHEMATISED TRANSPORT EQUATION IN RIVER MORPHOLOGY : A REVIEW

The importance of exponent "b" in schematized transport equation had been demonstrated by Jansen et al (1979), Struiksma et. al. (1985), Struiksma and Klaassen (1988), Struiksma and Crosato (1989). They have shown that point bar formation in channel bends, bar dimensions, transition between meandering and braiding and the braiding index are closely liked to the exponent "b". Besides these, in morphological computations, the exponent "b" influences the prediction of the river bed response by any morphological model. In rivers with alluvial sediment, the slope of the channel bed and the flow depth depends on the value of the exponent "b", as demonstrated by de Vries (1986).

An approximate relationship between the exponent "b" and the Shields parameter (θ) was derived for the Meyer-Peter and Mueller transport prediction formula as corrected for form roughness (Struiksma and Klaassen, 1988). The relationship is presented in Figure 1, which shows that Shields parameter (θ') decreases with an increase value of the exponent "b" and vice versa. This evaluation indicates that the exponent "b" is not always constant but it is a function of (θ'). They introduced a corrected expression for Shields parameter (θ').

Struiksma et. al. (1985) have shown the importance of exponent "b" for bed deformation. They reported that which is because the exponent "b" affects the rate of damping in such a way that a decrease of the exponent "b" leads to a considerable increase of damping.

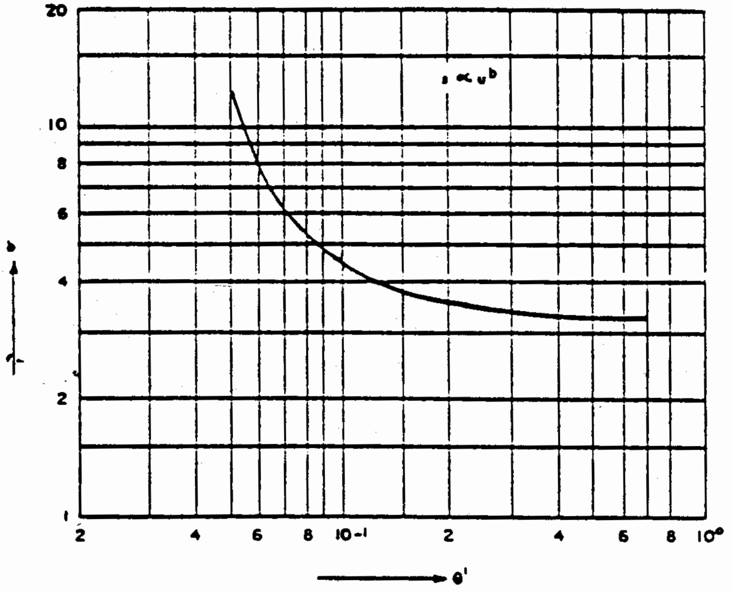

Fig 1. Approximate relationship between b and θ'

Struiksma and Klaassen (1988) studied the equilibrium bed topography in curved channels and developed the criteria for the spatial amplification of steady bed level perturbations. The exponent "b" was found to be an important factor for deriving the threshold values. They showed that wave length Lp and damping length L_D of perturbations are dependent on the ratio of the adaptation length λ_s of the sediment transport and the bed topography development and the adaptation length λ_w of the main flow. The ratio was also related to the exponent "b" as shown in Figure 2. The criteria derived for classification of plan forms are as follows :

$\lambda_s/\lambda_w < \lambda_s/\lambda_w^*$: meandering

$\lambda_s/\lambda_w^* < \lambda_s/\lambda_w < \lambda_s/\lambda_w^{**}$: transition between meandering and braiding

$\lambda_s/\lambda_w^{**} < \lambda_s/\lambda_w$: braided

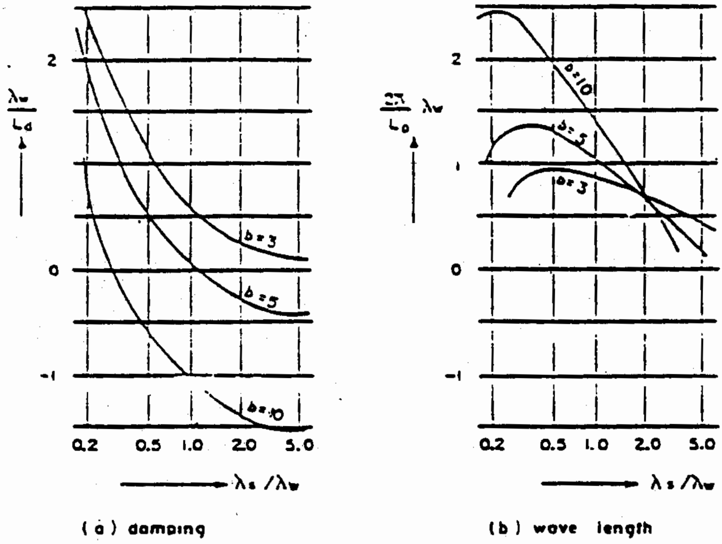

Fig 2. Damping and wave of perturbations in the linear analysis versus λ_s/λ_w

where $\lambda_s/\lambda_w{}^*$ and $\lambda_s/\lambda_w{}^{**}$ are lower and upper limit of harmonic range. The value of $\lambda_s/\lambda_w{}^*$ and $\lambda_s/\lambda_w{}^{**}$ depend on the value of the exponent "b", according to the following relationship :

$$\frac{\lambda_s}{\lambda^*_w} = \frac{2}{b-3} \tag{2}$$

and

$$\frac{\lambda_s}{\lambda_w{}^{**}} = [\frac{1}{2}(b+1) - \frac{1}{2}((b+1)^2 - 2(b-3))^{0.5}]^{-1} \tag{3}$$

It is important to mention here that Figure 1 was used for the transformation of θ' to exponent "b". The proposed criterion for planform classification merely consists of computing the values of λ_s/λ_w and "b", and comparing these with the appropriate values of $\lambda_s/\lambda_w{}^*$ and $\lambda_s/\lambda_w{}^{**}$. Thus it is necessary to determine the accurate value of exponent of "b" for a fair prediction of channel pattern.

For a constricted or a widened river section, the influence of the exponent "b" on the main parameters of flow and sediment transport in a river can be demonstrated easily. If the existing and the final future equilibrium situation are changed (before and after disturbance in river width) then leads the following expression :

$$\frac{h_1}{h_o} = \left(\frac{B_o}{B_1}\right)^{\frac{b-1}{b}} \tag{4}$$

and

$$\frac{S_1}{S_o} = \left(\frac{B_1}{B_o}\right)^{1-\frac{3}{b}} \tag{5}$$

Here it is seen that if "b" equal to 3, slope does not change but depth will change. However for higher values of "b" both slope and depth become influenced.

METHODOLOGY

The value of the exponent "b" can be assessed straightforward analytically from any sediment transport equation where the exponent of the flow velocity appears directly. For example the England-Hansen prediction formula directly yields the value equal to 5. The same is the case for the simplified prediction formula of Colby where "b" equal to 3.1. After some simplification, it is possible to obtain the exponent "b" for the Hossain formula in which "b" is equal to 3.2. However there are other sediment transport prediction formulae where the exponent of the flow velocity does not appear directly.

The exponent of the flow velocity can be assessed by using the following relation :

$$b = \frac{dq_s}{du} \cdot \frac{u}{q_s} \tag{6}$$

Which follows by differentiation of equation (1). Here dq_s/du is the derivative of sediment transport per unit width with respect to 'u', 'u' is flow velocity and 'q_s' is the sediment transport per unit width. This relation can be applied to sediment transport prediction formula like Ackers and White (1973), Bagnold(1966), Meyer-Peter and Mueller (1948), Parker and Klingeman (1982), van Rijn (1984) and Yang (1973). The above analytical expression can only be used when it is possible to find the differential dq_s/du from the sediment transport prediction formula. However there are some complicated sediment transport equations where there is a problem to find the differential dq_s/du. This problem can be solved numerically with different form of derivatives of equation (6).

In order to make the graphical relation between the exponent "b" and the Shields parameter(θ), it is necessary to compute the Chezy coefficient (C) and the water surface slope (S) for a given value of water depth and the depth averaged flow velocity. Here the two variables have been computed by Engelund-Hansen roughness predictor.

DERIVATION OF THE EXPONENT "b"

Seven total load and three bed load prediction formula have been selected for the derivation of exponent "b. These are Ackers and White (1973), Bagnold (1966), Colby (1957), Engelund-Hansen (1967), Hossain (1992), van Rijn (1984) and Yang (1973) for total load and Meyer-Peter and Mueller (1948), Parker and Klingeman (1982) and van Rijn (1984) for bed load formulae. These formulae were selected based on the criteria such that it is suitable for the application in alluvial rivers and most of the sediment transport formula are described as a function of the bed shear-stress and influenced by the accelerating and decelerating flow conditions. The methodology described above have been utilized to derive the exponent "b" analytically for the total and bed load prediction formulae. Dey (1995) analysed these prediction formulae for the expression of exponent "b" and this results are presented in Tables 1and 2.

Table 1 Expression of exponent "b" for total load formulae

Type of formula, Total load	Expression of exponent "b"
Ackers and White (1973)	$1 + \dfrac{m}{1 - \dfrac{Ycr}{Y}}$
Bagnold (1966)	$3 + \dfrac{1}{1 + \dfrac{\theta_b}{\theta_a(1-\theta_b)\tan\phi} \cdot \dfrac{w_s}{c\sqrt{\Delta d_{50}\,\theta}}}$
Colby (1957)	3.1
Engelund-Hansen (1967)	5
Hossain (1985),	3.25
van Rijn (1984), original	$3 + \dfrac{3}{1 - \dfrac{\theta_{cr}}{\theta}\left(\dfrac{c'}{c}\right)^2}$
van Rijn (1984), simplified	$3 + \dfrac{2.4}{c\,\dfrac{\sqrt{\Delta d_{50}\theta}}{u_{cr}} - 1}$
Yang (1973)	$6.397 - 1.227\log\left(\dfrac{w_s d_{50}}{v}\right) - 0.942\log\left(\dfrac{\sqrt{g\Delta d_{50}\theta}}{w_s}\right)$

Table 2 Expression of exponent "b" for bed load formulae

Type of formula, Bed load	Expression of exponent "b"
Meyer-Peter and Mueller (1948)	$\dfrac{3}{1 - \dfrac{0.047}{\mu\theta}}$
Parker and Klingeman (1982)	$3\left(\dfrac{\phi_{50} + 1.644}{\phi_{50} - 0.822}\right)$
van Rijn (original form) (1984)	$1 + \dfrac{3}{1 - \dfrac{\phi_{cr}}{\theta}\left(\dfrac{c'}{c}\right)^2}$
van Rijn (simplified form) (1984)	$0.6 + \dfrac{2.4}{c\,\dfrac{\sqrt{\Delta\, d_{50}\theta}}{u_{cr}}}$

DATA USED IN THE ANALYSIS

Depth and Froude number ranges expected in a alluvial river has been considered in order to make the graphical relation between the exponent "b" and the Shields parameter (θ). These data are presented in Table 3.

Table 3 Data used for the graphical relation of exponent "b"

Type of sediment transport formula	Depth range, h (m)	Froude number range (-)	Sensitivity analysis	
			slope range (S) (cm/km)	grain size range (d_{50}) (mm)
Total load	0.50-20.00	0.20-0.40	5-10	0.12-0.35
Bed load	2.00-20.00	0.088	-	-

RESULTS AND DISCUSSION

The ranges of exponent "b" for the selected prediction formulae were computed utilising the data shown in Table 3. In the computation of exponent "b", the Chezy coefficient(C) and the water surface slope (S) were calculated using Engelund-Hansen roughness predictor. The median particle size (d_{50}) was taken 0.22 mm which is the value for the Jamuna river.

For each total load prediction formula, the values of "b" were plotted against the Shields parameter (θ) as shown in Figure 3. It is seen that most of the formula show an increase of "b" with a decrease of the Shields parameter (θ). But the exception is seen for the Bagnold formula where "b" is decreasing with a decrease of Shields parameter (θ). For the Colby, Engelund-Hansen and Hossain formulae, "b" is found to be constant, because a term related to initiation of motion were not included with these formulae. It is also evident from Figure 3 that for a higher values of θ, variation of exponent "b" is almost constant. However for smaller values the variation is significant. The ranges of exponent "b" for each total load prediction formulae are shown in Table 4.

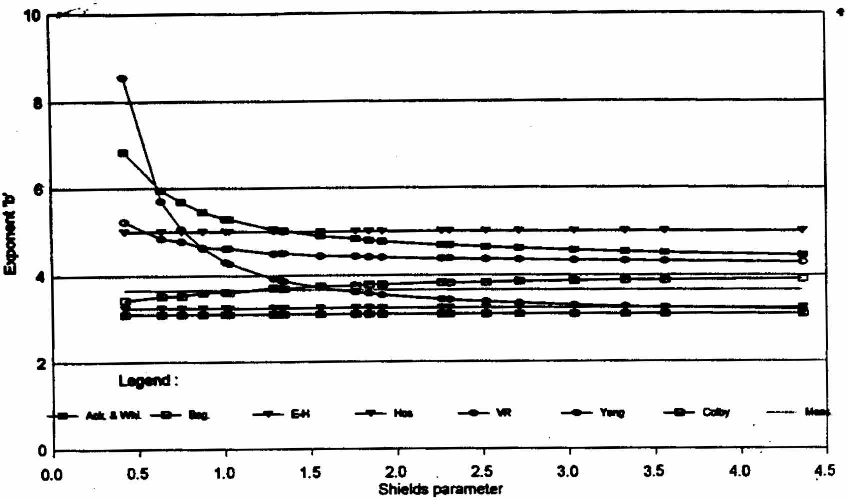

Fig 3. Variation of exponent 'b' with Shields parameter for total load formula

The exponent "b" has also been computed for the Jamuna River, utilising the data at Bahadurabad from 1984-87 and also shown in Table 4.

Table 4 Ranges of exponent "b" obtained from total load formulae (after Dey, 1995)

Sediment transport equation	Type of formula	Ranges of exponent "b"
Ackers and White (1973)	Total load	4.4-6.8
Bagnold (1966)	Total load	3.4-3.9
Colby (1957)	Total load	3.1
Engelund-Hansen (1967)	Total load	5.0
Hossain (1992)	Total load	3.25
Measured	Suspended load	3.66
Van Rijn (1984), original form	Total load	3.2-8.5
Yang (1973)	Total load	4.3-5.2

For the selected bed load prediction formulae, the values of exponent "b" were plotted against the Shields parameter (θ), as shown in Figure 4

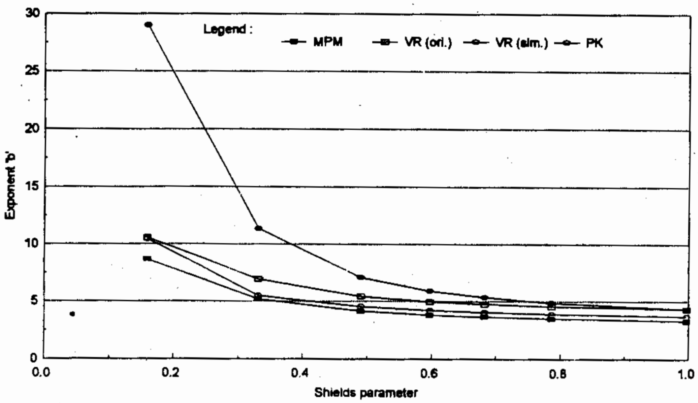

Fig 4. Variation of exponent 'b' with shields porameter for bed load formula.

This figure indicates that "b" computed by the simplified form of van Rijn formula gives extremely high range of exponent "b" compared to the van Rijn original form. The reason could be that the simplified form of the formula was based on computational approach and regression analysis (van Rijn, 1992). The range of exponent "b" for Parker and Klingeman formula resembles the original formula of van Rijn, whereas Meyer-Peter and Mueller formula gives smaller values than the original van Rijn formula. The ranges of exponent "b" for each bed load prediction formula are shown in Table 5.

Table 5 Ranges of exponent "b" obtained from bed load prediction formulae (after Dey, 1995)

Sediment transport equation	Type of formula	Ranges of exponent "b"
Meyer-Peter and Mueller (1948)	Bed load	3.3-8.6
Parker and Klingeman (1982)	Bed load	3.7-10.4
van Rijn (1984)-original form	Bed load	4.3-10.5
van Rijn (1984)-simplified form	Bed load	4.3-29

The sensitivity of exponent "b" for the selected prediction formulae were copared utilising the data shown in Table 3. This sensitivity analysis were based on the variation of grain size (d_{50}) and water surface slope (S).

The range of exponent "b" with variation of grain size for each total load prediction formulae were plotted against the Shields parameter (θ), as shown in Figure 5 to 8. From these figures, it is seen that the value of exponent "b" obtained from Ackers and White and van Rijn formulae are more sensitive than other formulae used.

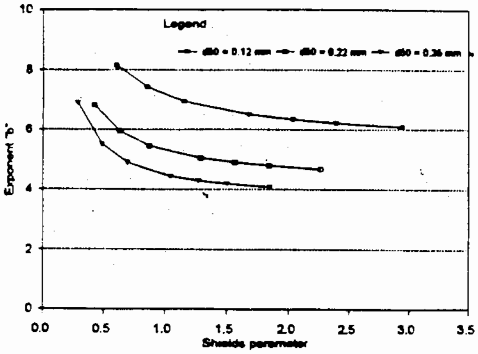

Fig 5. Sensitivity of exponent 'b' by variation of grainsize, Van Rijn formula.

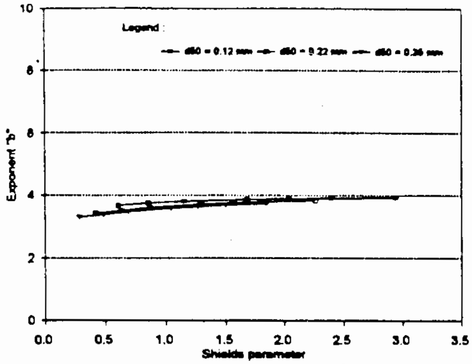

Fig 6. Sensitivity oj exponent 'b' by variation of grainsize, Yang formula.

White formula

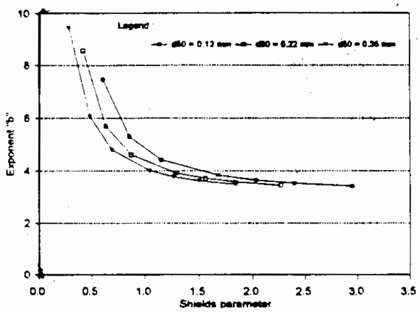

Fig 7. Sensitivity of exponent 'b' by variation of grainsize, Ackers White forrmula.

Also values of "b" obtained with different water surface slopes (S), considering the average condition of the Jamuna River, were plotted against the Shields parameter(θ), as shows in Figure 9. In this figure, it is observed that the influence of slope on the exponent "b" is sensitive for van Rijn formula at lower values of Shields parameter (θ), but less sensitive for the other formulae.

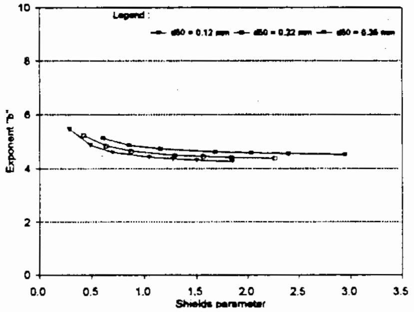

Fig 8. Sensitivity oj exponent 'b' by variation of grainsize, Ackers White forrmula.

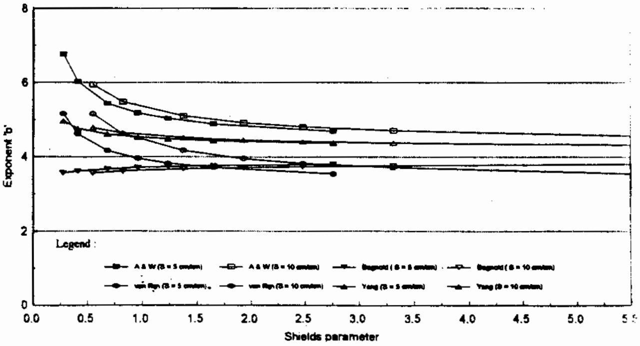

Fig 9. Sensitivity of exponent 'b' for variation of longitudinal slopes, total load formula

CONCLUSIONS

The importance of the use of exponent "b" in river morphology is focused in this study. Results show that the exponent of the flow velocity "b" varies between 3.1 to 8.5 for the total load formulae and 3.3 to 10.5 for the bed load formulae under normal conditions in alluvial rivers. The exponent "b" obtained from the measured data of the Jamuna river at Bahadurabad are found to lie within the range of exponent "b" derived analytically. Most of the prediction formulae show an increase of the exponent "b" with a decrease of the Shields parameter (θ). At higher depth the variation of exponent "b" is negligible. A small influence of water level slope on the exponent "b" was found at low values of Shields parameter (θ). However, the influence of grain sizes on the exponent "b" is found to be significant for the cases of Ackers and White and van Rijn formula.

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REFERENCES

Ackers, P. and W.R. White, (1973), Sediment Transport : New approach and analysis. JHD, Proc. ASCE, Vol. 99, No. HY-11, Nov. 1973.

Bagnold, R.A., (1966), An approach to the sediment transport problem from General physics. Geological survey professional paper 422-I (C.S.A), 1966.

Breusers, H.N.C., (1988), Lectures notes on sediment transport I, Delft, the Netherlands.

Colby, B.R.,(1957), Relationship of unmeasured sediment discharge to mean velocity. Trans. AGU, Vol. 38, No. 5, Oct. 1957.

Dey, K.C., (1995), Schematized sediment transport predictor for study of alluvial rivers : An application to the Jamuna. M.Sc. Engineering thesis, Department of Water Resources Engineering., BUET.

Engelund, F. and E. Hasen, (1967), A monograph on sediment transport in alluvial streams, Teknisk Forlag, Copenhagen, Denmark.

Hossain, M.M., (1992), Total sediment load in the Lower Ganges and Jamuna, Jour. Institution of Engineers, Bangladesh, Vol. 20 (1-2), pp 1-8.

Jansen, P. Ph., (1979), Principles of River Engineering, the non tidal alluvial river, Pitman, London.

Parker, G. and P.C. Klingeman, (1982), Bed load and size distribution in paved gravel bed streams. Journal of the Hydraulics Div., ASCE, Vol. 108, No. HY4.

Rijn, L.C. van, (1984), Sediment transport, part I : Bed load transport, Journal of Hydr. Engg. ASCE, Vol. 110, No. 10, Part II Suspended load transport, Journal Hydr. Engg. ASCE, Vol. 110. No. 11.

Rijn, L.C. van, (1992), Lectures notes on Principles of Sediment Transport in Rivers,. Estuaries, Coastal Seas and Oceans.

Struiksma, N., K.W. Ollesen, H.J. de Vriend and C. Flokstra, (1985), Bed deformation in curved alluvial channels, Journal of Hydraulic Research, Vol. 23, No. 1, pp. 57-79.

Struiksma, N. and G.J. Klaassen,(1988), On the threshold between meandering and braiding, Proc. Intern. Conf. on River 16(2) : 17-26.

Struiksma, N. and A. Crosato, (1989), Analysis of 2-D bed topography model for river, In : Ikeda, S. and G. Parker (1989), River meandering, Am. Geoph. Union, Water Resources Monograph 12, pp. 153-180.

Vries M, de, (1986) Morphological Computations, Lecture notes, Dept. of Hydraulic Engineering, Civil Engg. Faculty, Delft University of Technology, Delft, The Netherlands.

Yang, C.T., (1973), Incipient motion and sediment transport. JHD, Proc. ASCE, Vol. 99, No. HY-10, Oct, 1973.