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TORSIONAL DESIGN ASPECTS OF ACI 318-89 AND ACI 318-95 BUILDING CODES WITH REFERENCE TO BNBC

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ABSTRACT: This paper makes a comparative review of the torsional design provisions in the 1989 and 1995 editions of the ACI Building Code. The recent code provisions are based on the analysis of a space truss model for the RC beam subjected to torsion. The previous code provisions, on the other hand, were empirical in nature, based mostly on the observed test results. It has been demonstrated that the space truss analogy provides a rational method for design of members under torsion and shear. A comparative design exercise by both the methods has revealed that the ACI 318-95 Code provisions yield a much economical design. The superiority of this approach warrants that Bangladesh National Building Code (BNBC) provisions for design of torsion, which is currently based on ACI, 1989, should be revised to take advantage of the recent development.

KEYWORDS: Torsion, shear, reinforced concrete.

INTRODUCTION

In the Bangladesh National Building Code (1993), the design provisions for torsional design is completely based on skewed bending theory and directly adopted from ACI Building Code (318-89). The torsion design provisions in the ACI Building Code 318-89 were proposed in a series of papers by ACI Committee 438, in 1968 and 1969 and later adopted in the 1971 ACI Building Code. This design provisions was empirical in nature and based on experimental results. Shortly thereafter, a radically different design procedure based on a thin-walled tube, space-truss analogy was proposed in Switzerland. ACI code continued to adopt older empirical design provisions upto its ACI 318-89 edition. The recent ACI 318-95 is based on the thin walled tube, space truss analogy. This design method is currently included in the Canadian Code (1984) and the CEB-FIB Model Code (1990), among others.

The space truss analogy method is considerably simpler to understand and apply and is more accurate than the previous method. This paper discusses the salient aspects of these two design provisions. The clear superiority of the later approach clearly advocates its inclusion in place of the existing one in future revisions of BNBC Code.

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BACKGROUND OF CODE PROVISIONS

The design method presented by American Concrete Institute in ACI 318-89 Code (1989) is empirical. This method gives a safe design but lacks a proper physical model for beam behavior, when subjected to shear, torsion combined with bending. From experimental test results, an empirically developed equation is followed in adopting the diagonal cracking load of a member without web steel as the "concrete contribution" to the shear strength of an otherwise identical member with web steel. There are restrictions on the range of applicability of specific equations which are empirically developed for specific classes of members.

The method of design for torsion and for combined torsion, shear and flexure in beams as adopted in ACI 318-95 Code (1995) is based on thin walled tube, space truss analogy. Its advantage over the previous method is that it is simple, easily visualized physical model for the behavior of a reinforced concrete member subjected to torsion alone or in combination and leads to more rational design with less dependence upon empirical adjustments.

COMPARISON OF BASIC THEORIES

Skewed bending theory which forms the basis of ACI 318-89, lacks a proper physical model. When members are adequately reinforced, the concrete cracks at a torque equal to or at a value only somewhat larger than that in an unreinforced member. A great number of spiral cracks develop at a close spacing. Upon cracking, the torsional resistance of the concrete drops to about half of that of the uncracked member, the remainder being resisted by reinforcement. Redistribution of internal forces occurs at the cracking torque T_{cr} , accompanied by large deformation at constant torque. Thereafter reinforcement picks up the portion of the torque no longer being carried by the concrete after cracking (Fig.1). Any further increase of applied torque is assumed to be carried by reinforcement. The torsional strength are analyzed by considering the equilibrium of internal forces which are transmitted across the potential failure surface with failure to occur when the crack reaches the extreme face.

In space truss analogy, once cracking has occurred, the concrete in the center of the solid member contributes little to the torsional strength of the cross-section and can be ignored. Fig.2 compares the torsional capacity of solid and hollow reinforced concrete beams. As can be seen from Fig.2 that in the post cracking range the torsional capacity of solid and hollow sections is the same. The beam, in effect, is considered to be an equivalent tubular member. The new torsion design provisions in ACI 318-95 are based on a thin walled tube, space truss analogy in which the beam cross-section is idealized as a tube. After cracking, the tube is idealized as a space truss consisting of closed stirrups, longitudinal bars in the corners and concrete compression diagonals and taken at 45 degree approximately centered on stirrups (Fig.3). Based on this physical model the capacity of the beam is assessed and code provisions have been formulated in the ACI 318-95.

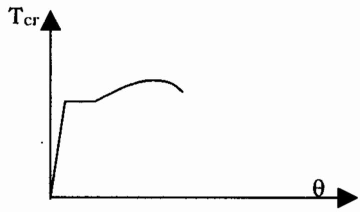


Fig 1. Torque twist curve in reinforced concrete

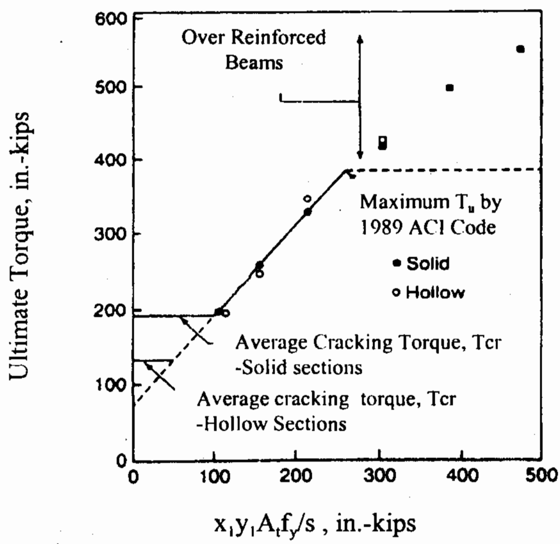


Fig 2. Comparison of torsional strengths of solid and hollow beams

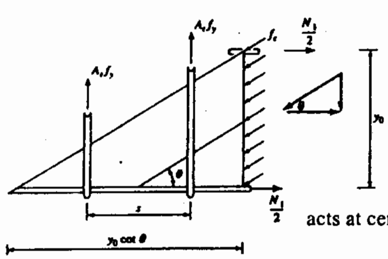


Fig 3. Physical model of space truss analogy in resisting forces

COMPARISON OF CODES

A comparison of the provisions of the ACI 318-89 and ACI 318-95 Codes is made below:

1) In both ACI 318-89 and ACI 318-95 Codes, design of cross section subjected to torsion is based on $T_u \leq \phi T_n$ where T_u is factored torsional moment at the section considered and T_n is nominal torsional moment strength of the section.

2) In ACI 318-89, torsion effects can be neglected with concurrent shear and flexure where factored torsional moment T_u is less than $\phi(0.5 \sqrt{f_c'} \Sigma x^2 y)$.

In ACI 318-95, for nonprestressed members, it is permitted to neglect torsion effects when the factored torsional moment T_u is less than $\phi \sqrt{f_c'} \left(A_{cp}^2 / p_{cp} \right)$.

Discussion: In ACI 318-89, the limiting torsional moment is based on a maximum torsional stress of $1.5\sqrt{f_c'}$. This stress corresponds to about 25 percent of the pure torsional strength of a member without torsion reinforcement. This is done assuming such magnitude of stress will not cause significant reduction in ultimate strength in either flexure or shear.

In ACI 318-95, it is assumed that torques that do not exceed approximately one quarter of the cracking torque (T_{cr}), will not cause significant reduction in ultimate strength in either flexure or shear. Cracking occurs when τ reaches $4\sqrt{f_c'}$, giving the cracking torque as

$$\phi 4 \sqrt{f_c'}\, A_{cp}^2 / p_{cp}$$

In both the codes, negligible torsion is 25 percent of the cracking torques. However, in ACI 318-95, tube analogy is less likely to be a proper model before cracking but it has been used to maintain the consistency of the model.

3) According to the ACI 318-89, a rectangular box section is taken as a solid section provided the wall thickness h is at least x/4. A box section with wall thickness less than x/4 but greater than x/10, shall be taken as solid section except that $\Sigma x^2 y$ shall be multiplied by 4h/x. When h is less than x/10 the stiffness of the wall shall be considered.

In ACI 318-95, actual section is replaced by an equivalent thin walled tube with a wall thickness t equal to $0.75A_{cp}/p_{cp}$ and an area enclosed by the wall centerline A_0 equal to $2A_{cp}/3$, prior to cracking. In hollow sections, A_0 is the area enclosed by the wall centerline.

Discussion: According to the provisions of previous code, hollow sections are replaced by equivalent solid sections based on test results whereas as per the ACI 318-95 code, solid sections are replaced by equivalent hollow tube based on theoretical comparison and test results.

4) In both ACI 318-89 and ACI 318-95 codes, design torsional

moment is equal to factored torsional moment in a member if it is required to maintain equilibrium.

For compatibility torsion, in indeterminate structures, according to ACI 318-89 and ACI 318-95 code design torsional moment is equal to $\phi 4\sqrt{f_c'}\sum x^2 y/3$ and $\phi 4\sqrt{f_c'}\left(A_{cp}^2/p_{cp}\right)$

Discussion: The equilibrium torsional moment cannot be reduced by redistribution of internal forces. The torsional moment due to compatibility torsion can be reduced by redistribution of internal forces after cracking.

For compatibility torsion, the torsional stiffness before cracking corresponds to that of uncracked concrete section. At torsional cracking, a large twist occurs under an essentially constant torque, resulting a large redistribution of forces in the structure. When T_u exceeds the cracking torque T_{cr}, a maximum factored torsional moment equal to T_{cr} assumed to occur at the critical sections near the faces of the support.

In previous method, cracking torque of a member is $6\sqrt{f_c'}\Sigma x^2 y/3$ but upper limit of compatibility torsion conservatively taken equal to $\phi(4\sqrt{f_c'}\Sigma x^2 y/3)$. In new method, cracking torque and upper limit of compatibility torsion are same and equal to $\phi 4\sqrt{f_c'}\left(A_{cp}^2/p_{cp}\right)$.

5) In previous method (Skewed bending theory), capacity of concrete in torsion alone is taken 40% of the cracking torque and equal to $0.8\sqrt{f_c'}\Sigma x^2 y$. Under combined torsion plus shear taken according to interaction diagram and contribution of concrete to torsion and shear equal to

$$T_c = \frac{0.8\sqrt{f_c'}\sum x^2 y}{\sqrt{1+\frac{0.4V_u}{C_t T_u}}} \qquad \text{..(1)}$$

$$V_c = \frac{2\sqrt{f_c'}bd}{\sqrt{1+2.5C_t\frac{T_u}{V_u}}} \qquad \text{...(2)}$$

In new method (tube analogy), V_c is assumed to be unaffected by the presence of torsion and T_c is always taken equal to zero.

Discussion: In previous method, no satisfactory theories of the complex interaction between shear and torsion was obtained. Reliance was placed on the extensive experimental investigations. Strength predictions and design done conservatively by the circular interaction equation which fits to test data well.

In new method, concrete is assumed to resist no tension. Shear capacity of the concrete remains unchanged. Torsional capacity of concrete is ignored before and after cracking.

In skewed bending theory, nominal torsional strength computed by $T_n = T_c + T_s$ where T_c is nominal torsional strength provided by concrete and T_s is nominal torsional strength provided by torsion reinforcement.

In space truss analogy, nominal torsional strength computed by $T_n = T_s$ where T_s is nominal torsional strength provided by torsion reinforcement.

This part of skewed bending theory was based on experimental evidences but that of thin walled tube analogy is more realistic. A significant part of the complexity of the previous ACI design procedure arises from the assumed circular interaction between V_c and T_c. New design method has greatly simplified the calculations assuming V_c remains unchanged and T_c equal to zero.

6) In ACI 318-89, torsional reinforcement is required to carry the excess torsional moment over that carried by the concrete. The torque to be resisted by the reinforcement $T_s = (T_u - \phi T_c)/\phi$ and the required cross sectional area A_t of one stirrup leg for torsion T_s is $A_t \alpha_t x_1 y_1 f_y / s$

In ACI 318-95, torsional reinforcement is required to carry the total torsional moment. Transverse reinforcement for torsion shall be designed using

$$T_n = \frac{2A_0 A_t f_{yv}}{s} \cot\theta \qquad (3)$$

Discussion: In previous method, torsion is partly carried by concrete and partly by the torsional steel. This equation is obtained by assuming crack angle equal to 45 deg. In new method, beam cross section is idealized as a tube. After cracking the tube is idealized as a space truss consisting of closed stirrups, longitudinal bars in the corners and concrete compression diagonals approximately centered on stirrups. Outward thrust of the compression diagonals must be equilibrated by tension in the transverse steel. Here, θ shall not be taken smaller than 30 deg. nor larger than 60 deg. usually, θ is taken 45 deg. for nonprestressed members.

Due to presence or absence of T_c and partial or full value of V_c in previous and new methods, respectively, design comparisons show that for combinations of low V_u and high T_u, with v_u less than about $0.8(\phi 2\sqrt{f_c'})$, the new method requires more stirrups than ACI 318-89. For v_u greater than this value, the new method requires marginally fewer stirrups than ACI 318-89.

7) In ACI 318-89, required area of additional longitudinal bars A_l distributed around the perimeter of the closed stirrups A_t shall be computed by $A_l = 2A_t(x_1 + y_1)/s$.

In ACI 318-95, the additional longitudinal reinforcement required for torsion shall not be less than $A_l = \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{yv}}{f_{yl}}\right) \cot^2\theta$.

Discussion: In previous method, volumes of longitudinal and torsional reinforcement are maintained equal. In this method, reasons behind the necessity of longitudinal reinforcements are

a) It anchors the stirrups, particularly at the corners, which enables them to develop their full yield strength.

b) It provides at least some resisting torque because of the dowel forces which develop where the bars cross torsional cracks.

c) It has been observed, after cracking, members subjected to torsion tend to lengthen as the spiral cracks widen and become more pronounced. Longitudinal reinforcement counteracts this tendency and controls the crack width.

According to new method, the diagonal compression in the concrete struts produces a horizontal component of thrust that must be equilibrated by the total tension force in the longitudinal steel. This horizontal components of thrust act at the middle of the walls and the resultant of these components, N acts along the centroidial axis of the cross section of the space truss. The line of action of the force in the longitudinal bars should coincide with that of N. As a result; longitudinal torsional steel must be distributed around the perimeter of the cross section.

New method provides a rational explanation for tension in the longitudinal reinforcement, while by the previous skewed bending theory, its role is not well defined.

8) In ACI 318-89, size of the cross section is limited by the criteria that torsional moment strength T_s shall not exceed $4T_c$.

According to ACI 318-95, the cross sectional dimensions should be such that,

For solid sections,

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f_c'}\right) \qquad (4)$$

For hollow sections,

$$\left(\frac{V_u}{b_w d}\right) + \left(\frac{T_u p_h}{1.7 A_{oh}}\right) \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f_c'}\right) \qquad (5)$$

Discussion: In previous method, torsion reinforcement designed, to ensure ductile behavior rather than brittle behavior, to reach the yield stress before the concrete crushes. Test data indicates that for pure torsion the maximum torsional stress should be limited to $12\sqrt{f_c'}$.

In new method, the size of cross section is limited for two reasons, first to reduce unsightly cracking and second to prevent crushing of the surface concrete due to inclined compressive stresses due to shear and torsion. The two terms on the left-hand side are the shear stresses due to shear and torsion. The sum of these stresses may not exceed the stress causing shear cracking plus $8\sqrt{f_c'}$. It is derived on the basis of crack control. It is not necessary to check against crushing of the web since this happens at a higher level of shear stresses.

Previous Code provides size limitation for shear and torsion separately. In new Code, size limitation is provided by a common equation by limiting the total shear stress.

9) In ACI 318-89, minimum longitudinal steel shall computed by

$$A_{l\,min} = \left[\frac{400\ xs}{f_y} \left(\frac{T_u}{T_u + \frac{V_u}{3 \times C_t}} \right) - 2A_t \right] \times \left(\frac{x_1 + y_1}{s} \right) \quad \ldots(6)$$

value of $A_{l,min}$ need not to exceed that obtained by substituting $(50 b_w s/f_y)$ for $2A_t$.

In ACI 318-95, where torsional reinforcement required, minimum total area of longitudinal torsional reinforcement shall be computed by

$$A_{l,min} = \frac{5\sqrt{f_c'}\,A_{cp}}{f_{yl}} - \frac{A_t}{s} p_h \left(\frac{f_{yv}}{f_{yl}} \right) \quad \ldots(7)$$

where (A_t/s) shall not be taken less than $25b_w/f_{yv}$.

Discussion: A minimum amount of longitudinal reinforcement is always maintained in both Codes. Reinforced concrete beam specimens with less than 1 percent torsional reinforcement by volume failed in pure torsion at torsional cracking. In ACI 318-89, a relationship was presented which required about 1 percent torsional reinforcement in beams loaded in pure torsion and less in beams with combined shear and torsion. In ACI 318-95,. this was simplified by assuming a single value of this reduction factor and results in a volumetric ratio of about 0.5 percent.

New method is the simplification of ACI 318-89 and prior codes considering previous experiences and experimental results.

10) In ACI 318-89, spacing of closed stirrups shall not exceed the smaller of $(x_1+y_1)/4$ or 12 in. In ACI 318-95, spacing of transverse torsion reinforcement shall not exceed the smaller of $p_h/8$ or 12 in.

In both codes, spacing of longitudinal bars, not less than #3, distributed around the perimeter of the closed stirrups shall not exceed 12 in. At least one bar shall be placed in each corner.

Discussion: The spacing of the stirrups is limited to ensure the development of the ultimate torsional strength of the beam, to prevent excessive loss of torsional stiffness after cracking and to control crack widths. Same limitation, one eighth of perimeter of closed stirrup or 12 in. taken as the maximum spacing. In ACI 318-89, corner bars are required to provide anchorage for the stirrups. But in ACI 318-95, the longitudinal reinforcement is needed to resist the sum of the longitudinal tensile forces due to torsion in the walls of the thin walled tube. Since, the force acts along the centroidial axis of the section; the centroid of the additional longitudinal reinforcement should coincide with the centroid of the section. For this, additional longitudinal reinforcement is to be distributed around the perimeter of the closed stirrups.

Reason of distribution of longitudinal steel around the perimeter of the closed stirrups is well defined and explicit in new method.

11) New code allows to reduce the area of longitudinal torsion reinforcement in the flexural compression zone by an amount equal to $M_u/(0.9df_{yl})$ where M_u is the factored moment acting at the section in combination with T_u when longitudinal torsion reinforcement exceeds minimum steel in spacing and amount.

Discussion: The longitudinal tension due to torsion is offset in part by the compression in the flexural compression zone, allowing a reduction in the longitudinal torsion steel required in the compression zone.

New code allows logical reduction in longitudinal torsion in the compression zone. Such reduction is not allowed in previous code.

12) Previous code recommends for members subject to axial tension, torsion reinforcement shall be designed to carry the total torsional moment, unless a more detailed calculation is made in which T_c and V_c given by interaction equation that shall be multiplied by $(1+N_u/500A_g)$ where N_u is negative for tension.

Discussion: According to previous code, conservatively designer may provide torsional reinforcement to carry the total torque, disregarding the contribution of the concrete. On the basis of effect of axial force on torsional strength code permits calculation of T_c and V_c by interaction equation, resulting values to be reduced by the factor $(1+N_u/500A_g)$ where N_u is the axial force, negative for tension.
New code disregards the contribution of concrete in all cases. According to new method, effect of axial tension will not bring any change in the design procedure but in calculation of V_c its effect is considered by multiplying a factor.

Previous code allows the designer to apply his judgement to consider axial tension in part or in full. But in new method, its role and design procedure is well established.

COMPARISON OF APPLICATIONS

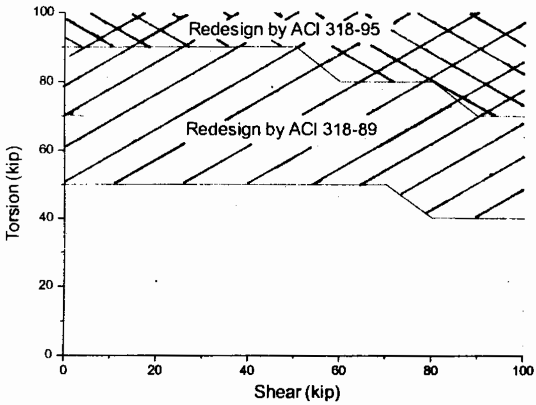
Torsion should be considered in design of structural members when it exceeds certain value. ACI 318-95 Code is more conservative about this negligible torsion than ACI 318-89 in that the new code neglects the contribution of concrete in resisting torsion.

Due to better understanding the beam behavior in post cracking range it has been possible to allow higher value of compatible torsion for indeterminate structures in new method.

Skewed bending theory assumes that spiral cracks occurs at an angle 45 deg. The method calculates stirrup size and spacing and amount of longitudinal steel assuming θ = 45 deg. The new space truss analogy allow the use of θ from 30 deg. to 60 deg. to allow designers to optimize the relative amounts of stirrups and longitudinal reinforcement if they wish. The same value of θ must be used when designing a given member for torsion.

New method is more flexible about the upper limit of torsion and Shear. Fig.2 shows the maximum T_u allowed by ACI 318-89 Code. The new ACI 318-95 permits use of over reinforced beams.

Fig.4 indicates the range of redesign of a 10 in. wide and 30 in. deep beam by both ACI 318-89 and ACI 318-95. Superiority of the later is clearly evident.



f_y=50,000 psi, f_c=4,000 psi

Fig 4. Range of redesign of a 10 in. by 30 in. beam by ACI 318-89 and ACI 318-95.

COMPARISON OF STRENGTH PREDICTIONS BY NEW AND PREVIOUS METHODS TO TEST RESULTS

The results of tests of reinforced concrete beams were compared to the failure torsions predicted by the two design procedures. The results are summarized in the following table (MacGregor and Ghoneim, 1995)

Histograms of the test to predicted torsional strength ratios of beams are given in Fig.5 for 100 beams loaded in pure torsion. The lowest strength ratios for these 100 beams was 0.453 for ACI 318-89 and 0.895 for the new design method. The average strength ratios for these 100 beams was 1.028 for the ACI 318-89 and 1.276 for the new design method.

The ratios of test to predicted torsional strength of beams subjected to combined shear, bending and torsion are shown in Fig.6. The extreme strength ratios for the beams examined were 0.940 and 1.827

for the 1989 ACI design procedure and 1.034 and 1.698 for the new design procedure.

Table 1. Comparison of tests to new design procedures

Loading	No. of tests	Test / calculated strength			
		New Method ACI 1995		Previous Method ACI 1989	
		Mean	Coefficient of variation	Mean	Coefficient of variation
Pure torsion	100	1.276	0.161	1.028	0.165
Combined bending and torsion	42	1.383	0.168	1.332	0.227
Combined bending, shear and torsion	38	1.359	0.106	1.382	0.156

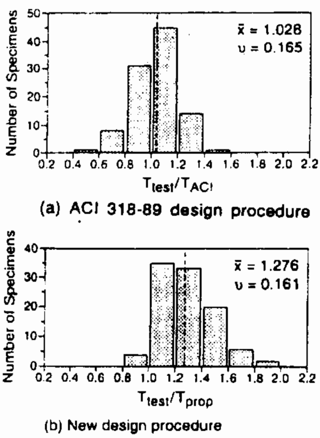


Fig 5. Comparison of measured and computed failure torsions for 100 reinforced beams in pure torsion

In Fig.7, the limit on shear stresses given by the right hand side of eq. (4), with v_c set equal to $2\sqrt{f_c'}$ is compared to shear stresses at failure of 20 reinforced concrete beams that failed due to web crushing. The fact that the computed values of the shear stresses at failure for all the beams, except one, are the larger than the suggested limiting value of

$10\sqrt{f_c'}$ indicated that these specimens failed at higher level of shear stress than that predicted by the criteria given in eq. (4). Although the limit $10\sqrt{f_c'}$ is extremely conservative for some cases, it is a safe and easily applied lower bound value for reinforced concrete members subjected to pure torsion and experiencing web crushing.

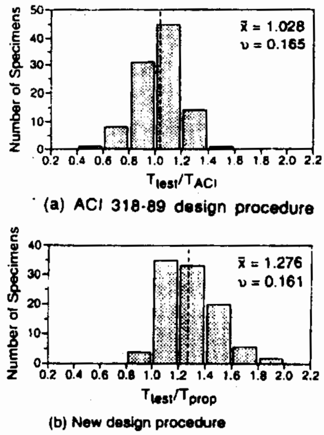


Fig 6. Comparison of measured and computed failure torsions of 38 reinforced concrete beams –Combined torsion, shear, and moment.

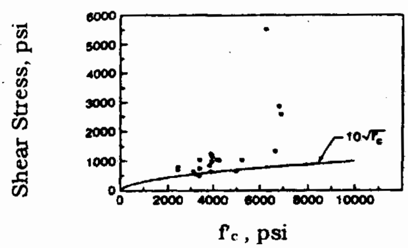


Fig 7. Comparison of shear stress limit in eq.(4) to shear stresses at failure of reinforced concrete beams failing due to web crushing -pure torsion

Comparison of the predicted strengths with test data for reinforced concrete beams suggest that, although the new design procedure is simpler to understand and apply, it predicts the test strengths at least as well as the ACI 318-89 procedure.

CONCLUSIONS

A comparative study of the torsion design provisions of the 1989 and 1995 codes reveals that new design method presented in ACl 318-95 Code is simpler to understand and is based on a proper physical model. In most cases, new method leads to a rational and economical design over previous method. Comparison of predicted strengths by new methods demonstrates its superiority than the old method.

As the new method is more accurate and economical, more flexible and easy to understand, and is based on a proper physical model for representing the behavior of members subjected to torsion and shear, the new design method for torsion in RC members may also be included in the BNBC.

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NOTATION

A_o = gross area enclosed by shear flow path, in.^{2}

A_{oh} = area enclosed by centreline of the outermost closed transverse torsional reinforcement, in.^{2}

A_{cp} = area enclosed by outside perimeter of the concrete cross section, in.^{2}

A_g = gross area of section, in.^{2}

A_l = total area of longitudinal reinforcement to resist torsion, in.^{2}

A_s = area of nonprestressed tension reinforcement, in.^{2}

A_t = area of one leg of a closed stirrup resisting torsion with in distance s, in.^{2}

A_v = area of shear reinforcement with in distance s, in^2

b = width of compression face of a member, in.

b_w = web width, in.

c_t = factor relating to shear and stress properties = $\dfrac{b_w d}{\sum x_2 y}$

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement, in.

f'_c = compressive streength of concrete, psi

f_y = yield strength of nonprestressed reinforcement, psi

f_{yv} = yield strength of closed transverse torsional reinforcement, psi

f_{yl} = yield strength of longitudinal torsional reinforcement, psi

h = overall thickness of member, in.

N_u = factored axial load normal to cross section

N_1 = axial Tension forces in sides of space truss

p_{cp} = outside perimeter of concrete cross section, in.

p_h = perimeter of centreline of outermost closed transverse torsional reinforcement, in.^{2}

s = spacing of shear and torsion reinforcement, in.

t = thickness of wall of a hollow section, in.

T_c = nominal torsional strength provided by concrete

T_n = nominal torsional moment strength.

T_u = factored torsional moment in section

v = coefficient of variation

V_c = nominal shear strength provided by concrete

V_n = nominal shear strength

V_s = nominal shear strength provided by shear reinforcement

V_u = factored shear force at section

v_n = nominal shear stress, psi

x = shorter dimension of rectangular part of cross section

x_1 = shorter centre to centre dimension of closed rectangular stirrup

$\bar{x}$ = average of the test to predicted torsional strength ratios.

y = longer dimension of rectangular part of cross section

y_1 = longer centre to centre dimension of closed rectangular stirrup

α_t = coefficient equal to $(2+y_1/x_1)/3$ but not more than 1.5

θ = angle of compression diagonals in truss analogy for torsion