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Simplified calculation method for serviceability deflection of edge supported slabs; Part 1: Modelling Slab Response

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ABSTRACT: Although deflection is an important parameter in the design of structure, enough emphasis has not been given to the calculation of slab deflection. Several methods are presently available for the calculation of deflection of slabs, none provides the designer with a single approach to readily quantify the slab deflection for any span ratio and support condition. The paper briefly reviews the methods available for the calculation of slab deflection and describes the development and verification of a Finite Element model used for the purpose of calculation of slab deflection. The model has been verified against the ACI moment coefficient method. A companion paper describes the development and verification of coefficient method for calculation of slab deflection.

KEYWORDS: Deflection, edge supported slab, coefficient method, strip moment, ACI method

INTRODUCTION

Edge-supported slabs are typically thin relative to their span, and may show large deflections even though strength requirements are met, unless certain limitations are imposed in the design to prevent this. The paper briefly reviews the methods presently available for computing the slab deflection. It can be observed from the review that although claimed to be simple, these are complicated in nature and require the use of either extensive calculations or use of complicated finite element or finite difference packages, which is rather difficult to use from the view point of a practising engineer. The present paper then describes the development and verification of a Finite Element model used to simulate slab response. ANSYS, a general purpose Finite Element software, has been employed for this purpose. Shell elements have been used to develop the slab model. Verification has been conducted by comparing moments against the moments obtained by ACI moment coefficient method for various support cases. Since results from the software are obtained in terms of moment per element width, methods have been formulated to convert this moment into strip moments as per ACI method.

ACI method for deflection calculation of edge supported slabs

The simplest approach to deflection control is to impose a minimum thickness-span ratio. According to ACI Code (1995) calculation of deflection

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is not required provided that the slab thickness is not less than a certain quantity. This quantity depends on: presence or absence of drop panels, ratio of flexural stiffness of beam section to flexural stiffness of a width of slab bounded laterally by centrelines of adjacent panels (if any) on each side of the beam, ratio of clear spans etc.

Simson's Method

This method (described by Ugral, 1981) is applicable only for simply supported rectangular plates under various loading. Due to complicated mathematical equations this is difficult for day to day use.

Levy's solution for rectangular plate

The Levy's method (described by Ugral, 1981) is applicable to the bending of rectangular plates with particular boundary conditions on the two opposite sides and arbitrary conditions of support on the remaining. Complexities in mathematical derivations are larger than the previously stated method.

The Strip Method

In strip method (described by Ugral, 1981), the plate is assumed to be divided into two systems of strips at right angles to one another, each regarded as functioning as a beam. The method permits qualitative analysis of the plate behaviour very easily but is less adequate, in general, obtaining accurate quantitative results.

Chang and Hwang method

Chang and Hwang (1996) developed an algebraic equation for estimating the mid-panel deflections of the two-way slab systems subjected to uniform gravity loads. The proposed method has initially been derived from the differential equation of plate and then calibrated using the results of finite-element analysis. The semi empirical equation for slab centre deflection is:

$$\Delta_s = K_a K_b K_c \frac{(w_v + i w_s) l^4_{n1}}{D} \tag{1}$$

Obtaining various components of the above equation requires extensive calculations.

Polak method

Polak (1996) presented a procedure for calculating deflections of reinforced concrete two-way slabs using Serendipity-plate-bending element (eight-noded element with 24 degrees of freedom). The method involves the use of computer programming for obtaining the deflection.

Vanderbilt method

On the basis of "exact" solutions and test data from five multiple panel structures, Vanderbilt, Sozen and Siess (1965) developed an approximate method (using finite difference equations) to predict deflections of floor slab systems with or without beams. A physical analogy, for which deflections can be determined on the basis of two-dimensional methods of analysis, was proposed as a substitute for three-dimensional structure.

Controlling slab deflection

Besides the computation of slab deflection several methods (Chang et al 1996, Gilbert 1985, Grossman 1981, Rangan 1982) have been proposed for controlling slab deflection.

MODELLING SLAB RESPONSE

This section describes the finite element model for simulating slab behaviour. ANSYS, a general purpose Finite Element software has been employed for this purpose. The developed model uses shell elements with four nodes for modelling the slab. The developed model can simulate all the possible support and load conditions that usually occur in normal building slabs. To estimate the number of elements in each direction of the slab, studies have been performed to select the number of elements that gives a constant moment with increasing number of elements. Moments extracted from the numerical model have been compared with the moments obtained from ACI moment coefficient method. Necessary conversions of nodal moments for comparison with the ACI method have also been described.

Numerical model of edge supported slab

This section describes the technique used for modelling edge-supported slab. Elements used to represent the slab; boundary conditions; load application and material properties have been separately described.

Figure 1 shows the Finite Element mesh of the slab. Four noded shell elements have been used to represent the concrete slab. These elements have six degrees of freedom per node.

To represent the support condition proper boundary conditions have been used. When the support is fixed all the degrees of freedom (U_x, U_y, U_z, V_x, V_y, V_z) have been restrained (edges AB, AD, AC) and for simple support only the translational degrees of freedom (U_x, U_y, U_z) have been restrained (edge BC).

Load is applied to the surface of the elements as pressure load. To separate the self-weight from the external loading; a gravitational constant of –1 have been used so that the self-weight of each element is automatically calculated from the given thickness, element size and unit weight.

Material properties include modulus of elasticity of concrete (E_c = 20685 N/mm^2). Since serviceability deflection is considered this is sufficient because the response will be only linearly elastic. The model includes reinforcement in the in an indirect way (the shell elements

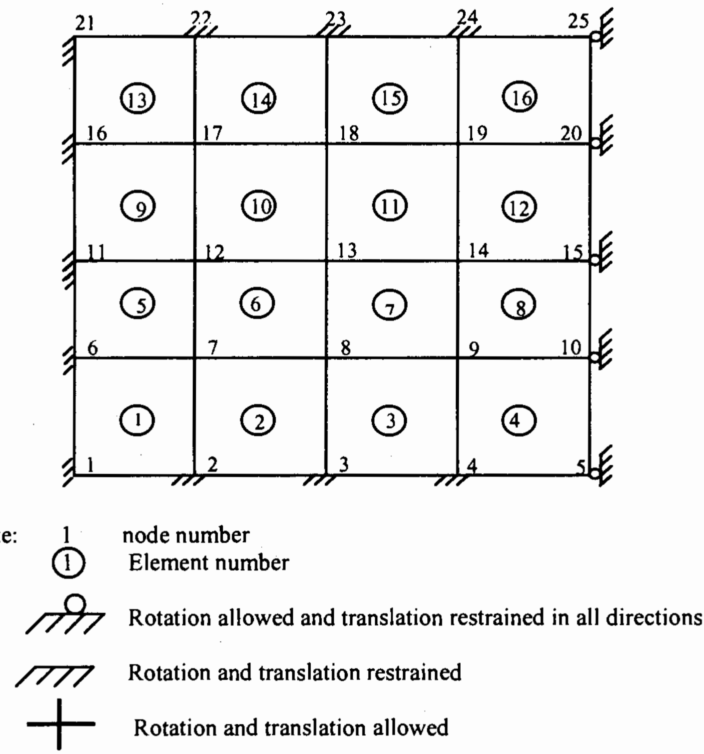


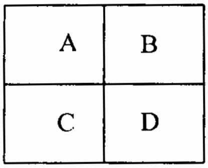
Fig 1. Typical Finite element simulation of slab (see Fig 3, case 9)

incorporated in ANSYS can not include reinforcement). To consider the effect of reinforcement at first the effective inertia of the slab for unit width is calculated considering transformed area of the reinforcement than the thickness is calculated for slab having unit width and same inertia. This thickness is used to represent a given reinforced slab. Increase in dead weight due to this is negligible as the unit weight of steel is higher than concrete.

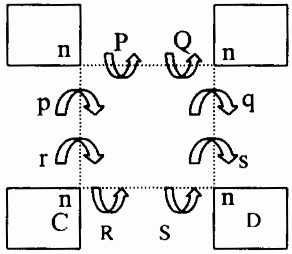
Interpretation of numerical results for comparison with ACI results

The ACI coefficient method considers inelastic redistribution for driving the coefficients. At the same time end fixity is not considered to be 100% for all load case. The method divides the slab into middle strip and edge strip; moment is considered to be constant at the middle strip and reduced to one third at the end of edge strip. Thus to compare the FE results with this method, FE results are to be converted accordingly.

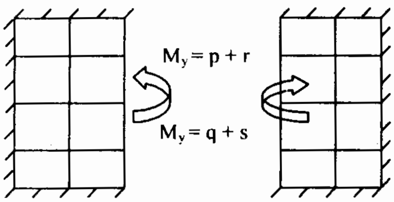
Results obtained from the model are moments at the node per element in the global direction as shown in Figure 2.



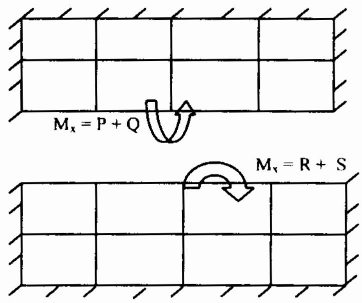
2(a) Four elements connected to a node



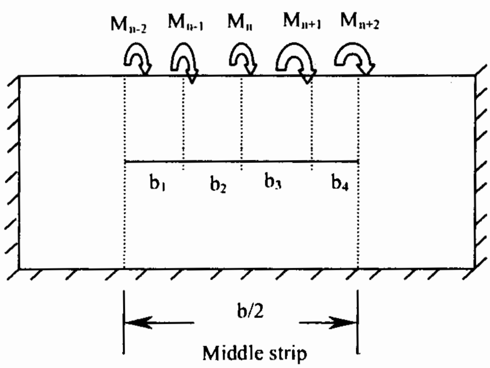
2(b) Moments acting on the elements at the common node



2(c) Moment M_y on the slab at node n



2(d) Moment M_x on the slab at node n



2(e) Calculation of middle strip moment from nodal moment

Fig 2. Conversion of nodal moments to slab internal moment

Figure 2(a) shows the common node "n" associated with four elements A, B, C and D. As the loading on the elements are only pressure and no concentrated external moment acts at node "n" and the rotational degrees of freedom are not restrained, the summation of nodal solution from all the attached elements will result into zero moment at node "n". Thus to obtain slab middle strip moment, element solution as shown in Figure 2(b) are to be transformed. Moments P, Q, R, S are about x-axis and p, q, r, s are about y-axis and per element (for elements A, B, C and D respectively). To obtain M_x, M_y in slab these elements can be assembled as shown in Figures 2(c) and 2(d) respectively. Once these moments are obtained for nodes along a slab section it becomes essential to find out the middle strip moment as shown in Figure 2(e).

Moment's M_{n-2}, M_{n-1}, M_n, M_{n+1} and M_{n+2} are to be calculated using method shown in Figure 2(d), for respective nodes. As the ACI method assumes constant moment along the middle strip; for comparing FE results with ACI method it can be obtained by averaging them over the middle strip. Thus:

$$M_a(+) = \frac{\frac{1}{2}M_{n-2} + M_{n-1} + M_n + M_{n+1} + \frac{1}{2}M_{n+2}}{\frac{b}{2}}$$

$$= \frac{2}{b}\left[2(M_{n-2} + M_{n-1} + M_n + M_{n+1} + M_{n+2}) - (M_{n-2} + M_{n+2})\right] \quad (2)$$

and

$$M_b(+) = \frac{2}{a}\left[2(M'_{n-2} + M'_{n-1} + M'_n + M'_{n+1} + M'_{n+2}) - (M'_{n-2} + M'_{n+2})\right] \quad (3)$$

Similar methods are applied for evaluating the edge moments. It should be noted that equations 2 and 3 considers only the average moments while ignores the inelastic redistribution i.e., the fixed supports are assumed to have full fixity always, thus FE results interpreted using these equations must show certain amount of variation for slabs having fixed support at one or more edges. Since the moments are assumed to be parabolic and no inelastic redistribution is considered, thus it is expected that the numerical moments will be larger than ACI moments in such cases. Instead of comparing moments; it is desirable to compare the coefficients. It is possible to compute the coefficients for different support cases and span ratios as follows:

$$M^+_{a,FE} = C^+_{a,FE} . wl_a^2 ; \quad (4)$$

$$M^+_{b,FE} = C^+_{b,FE} . wl_b^2 ; \quad (5)$$

thus:

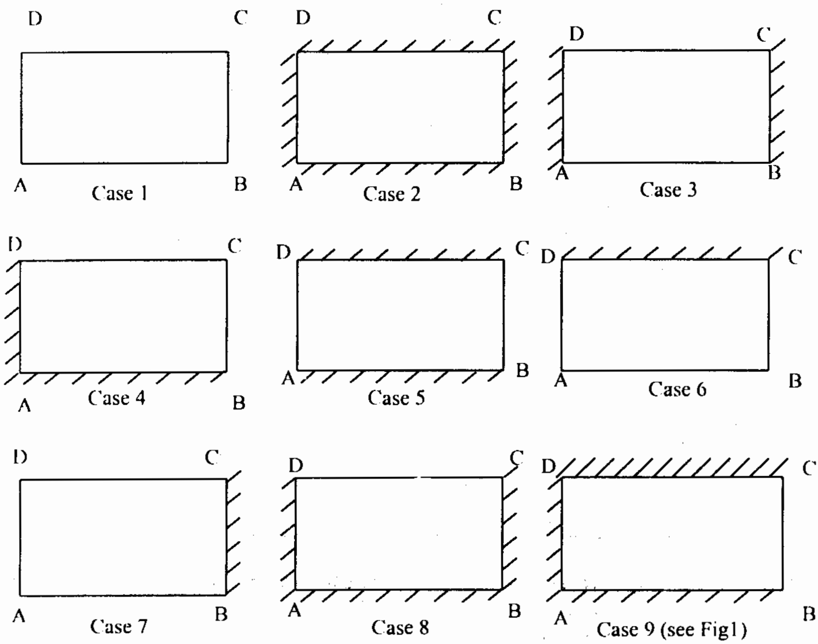
$$C^+_{a,FE} = \frac{M^+_{a,FE}}{wl_a^2} \quad (6)$$

$$C^+_{b,FE} = \frac{M^+_{b,FE}}{wl_b^2} \quad (7)$$

Where $M^{+}_{a,FE}$ and $M^{+}_{b,FE}$ can be obtained from FE results using the method just illustrated through using equations 2 and 3. Use of equations 2,3,6 and 7 thus allows the computation of $C^{+}_{a,FE}$, $C^{+}_{b,FE}$, $C^{-}_{a,FE}$, $C^{-}_{b,FE}$.

Selection of problems for verification of FE model

The coefficient method for calculating slab moments classifies edge-supported slabs into nine categories as shown in Figure 3 (cross-hatched indicate fixed, no hatch indicate simple supported). Besides support condition to estimate the coefficients the span ratio is used (11 span ratios are present in the Tables from 0.50 to 1.0 with in increment of 0.05). This produce a total of 9x4x11 = 396 possible combinations for verification. To reduce the total number of problems for verification span ratio will be varied from 0.50, 0.60, 0.75, 0.90, 1.0, this produces a total of 9x4x5 = 180 cases for verification.



Note: —— Simply supported

/////// Clamped edge

Fig 3. Nine different support conditions

Optimum mesh for the slab model

Before comparing results obtained from the FE model with ACI moments, it is essential to select the mesh that provides sufficiently accurate results at the same time consists of minimum number of nodes and elements.

For this purpose nodal moments are extracted at the centre of span and edge in both directions for all nine cases and span ratios 1.0, 0.90, 0.75, 0.60 and 0.50. The moments have been plotted against the mesh fineness and the optimum mesh has been selected (for which all the nodal moments become constant). Table 1 shows the optimum mesh for different span ratios and covering all support cases, complete sets of graphical results are shown elsewhere by Chowdhury (1999).

Selection of different parameters

Table 2 lists the different parameters with their magnitude, used to develop the FE model. Besides these the actual span that has been used ranges from 3.05m (10 ft) to 6.10m (20 ft). For various span ratios, smaller span (3.05m) has been kept constant and the larger span has been changed.

Table 1 Optimum mesh for different span ratio

Span ratio	Number of divisions in shorter direction	Number of divisions in longer direction
1.00	8	8
0.90	10	10
0.75	8	10
0.60	6	10
0.50	6	12

Table 2 Input parameters for the model

Modulus of elasticity of concrete (E_c)	Thickness	Unit weight	Live load	Poisson's ratio(υ)
20685 N/mm² (3x10⁶ psi)	≥89 mm $\geq \frac{Perimeter}{180}$	23.55×10^{-6} N/mm³ (150 pcf)	1.916×10^{-3} N/mm² (40 psf)	0.18

Comparison of coefficients for middle strip moments with ACI moment coefficients

Results obtained for all cases with different span ratios from the numerical model have been used to compute the moment coefficients. Figures 4 to 7 show the variation of $C_a(+)$; $C_b(+)$; $C_a(-)$; $C_b(-)$ with different span ratio and cases. Complete set of results is presented elsewhere by Chowdhury (1999). From the above Figures it can be seen that for $C_a(+)$; $C_b(+)$; $C_a(-)$; $C_b(-)$ compared well with the ACI coefficients having insignificant difference in few cases. The cause of variation between Finite element results and ACI coefficients is due to the reason that in ACI method, inelastic redistribution is considered which is not considered in Finite Element analysis.

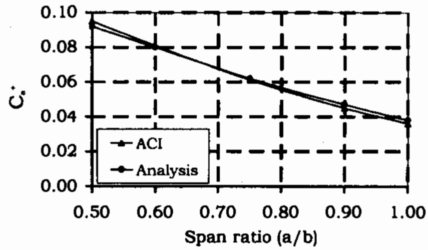


Fig 4(a). Variation of C_a (+) for case 1

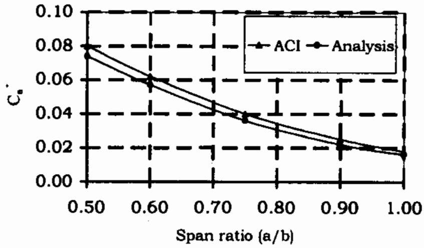


Fig 4(b). Variation of C_a (+) for case 3

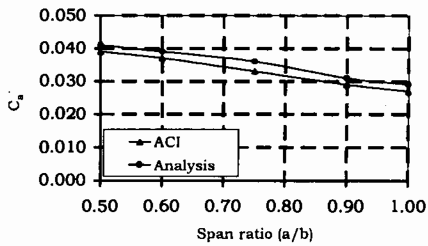


Fig 4(c). Variation of C_a (+) for case 5

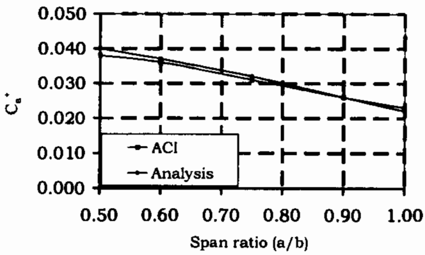


Fig 4(d). Variation of C_a (+) for case 9

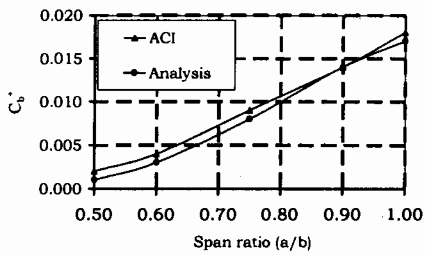


Fig 5(a). Variation of C_b (+) for case 2

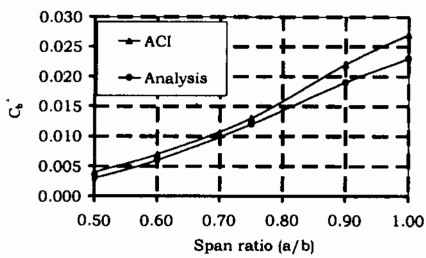


Fig 5(b). Variation of C_b (+) for case 4

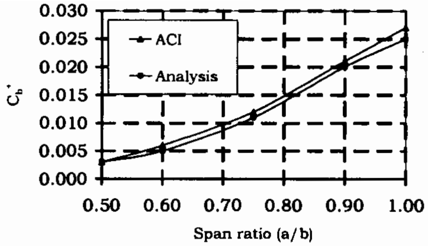


Fig 5(c). Variation of C_b (+) for case 6

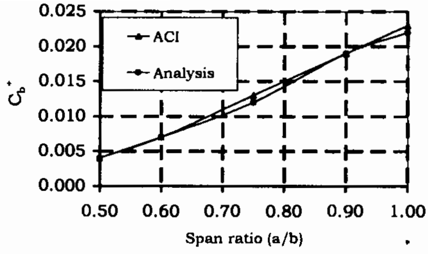


Fig 5(d). Variation of C_b (+) for case 8

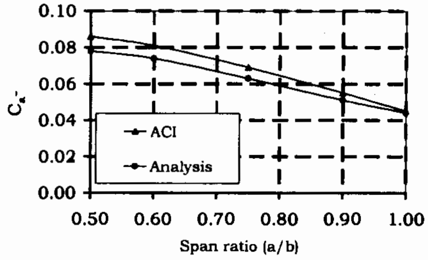


Fig 6(a). Variation of C_a (-) for case 2

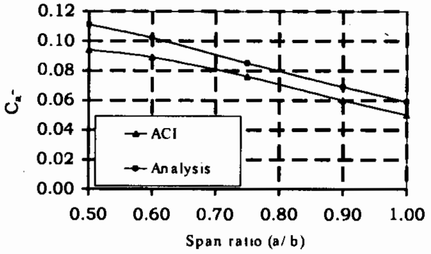


Fig 6(b). Variation of C_a (-) for case 4

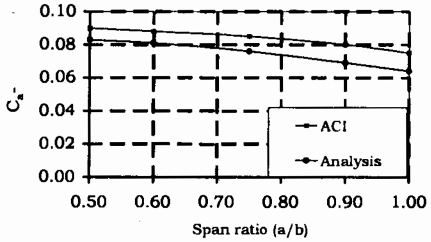


Fig 6(c). Variation of C_a (-) for case 5

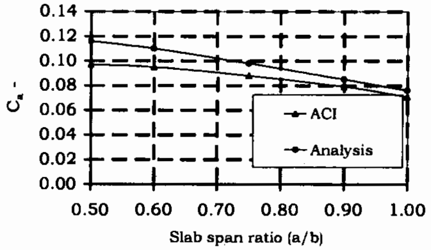


Fig 6(d). Variation of C_a (-) for case 6

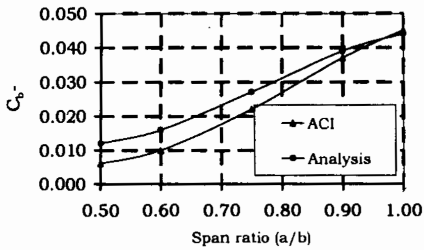


Fig 7(a). Variation of C_b (-) for case 2

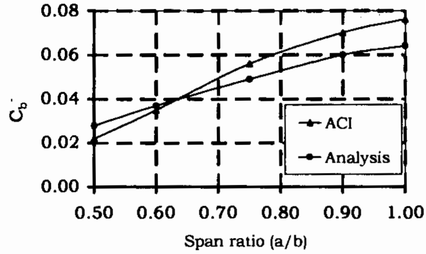


Fig 7(b). Variation of C_b (-) for case 3

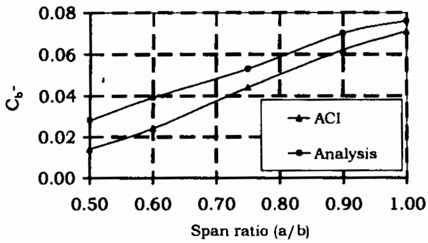


Fig 7(c). Variation of C_b (-) for case 7

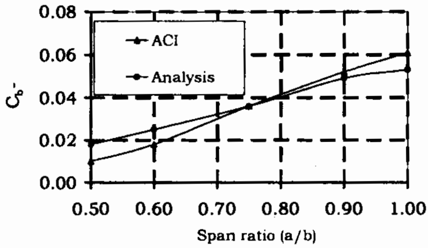


Fig 7(d). Variation of C_b (-) for case 8

CONCLUSIONS

A finite element model for edge supported slabs has been described in this paper that has been verified using ACI moment coefficient method. Method has been formulated for converting the FE nodal moments into ACI strip moments. The numerical model developed has been tested for mesh sensitivity and the optimum mesh has been determined for all the span ratios covering all the nine cases. Using the results obtained from these optimum mesh positive and negative moments in both short and long directions has been determined. Equations are derived for converting the FE moments and deflections into moment coefficient and deflection coefficient. Results obtained from the FE model have been compared graphically with ACI moment coefficients that demonstrated the accuracy of the model developed. It has been observed from the comparison of results of FE analyses and the ACI coefficients that quite satisfactory results are obtained using the developed model and thus concluded that it can be used for parametric study to gain insight of slab behaviour. A companion paper (Ahmed and Chowdhury, 1999) will describe the parametric study and the development of a coefficient method for calculating slab deflection.

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