Journal of Civil Engineering
The Institution of Engineers, Bangladesh
Vol. CE 27, No. 1, 1999

STUDY OF THE SEMI-RIGID PROPERTIES OF REINFORCED CONCRETE BEAM-COLUMN JOINTS

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ABSTRACT: A computational investigation is made to study the behavior of a beam to column reinforced concrete joint. Separate finite element modeling is done for concrete, steel reinforcement and the bond-slip mechanism between the two. Two dimensional plane stress and truss elements were used for concrete and steel respectively while special dimensionless link elements were used to simulate the bond slip behavior. A joint composed of a cantilever beam connected to the middle of a column is analyzed. A sensitivity analysis of different parameters related to joint design is studied. Based on the sensitivity analysis the semi-rigid characteristics of the joint is investigated. Such investigation will enable to formulate the semi-rigidity properties of typical RC joints in terms of moment and rotation and enable to analyze large scale RCC frames in a more realistic manner recognizing the semi-rigid characteristics of RC joints instead of conventional frame analysis using rigid connections. More realistic sway calculations will thus be possible for RC frames under various lateral loads such as wind or earthquake loading.

KEY WORDS: Concrete, joint, semi-rigid, joint stiffness, flexibility.

INTRODUCTION

Modern tendency in concrete construction is to use high strength or high performance concrete which ultimately reduces the member sizes and proportions. Such reduction in the cross section of structural components, specially in a framed structure, results in more flexible structural system for which the sway characteristics are becoming more and more important from the serviceability point of view. Correct and rational prediction of such sway characteristics of reinforced concrete (RC) framed structures eventually necessitates the knowledge of the semi-rigid characteristics of joints. In a RC framed structure, beam to column joints are perhaps among the most complicated yet one of the least understood components of a building system. Proper understanding of the joint characteristics is one of the most challenging fields among researchers.

It has long been recognized that the key to appreciating the effects of joint performance on the behavior of frames is the knowledge of the connection's moment-rotation (M-ϕ) characteristics. The primary distortion of a connection is the rotational deformation ϕ, caused by moment M. Methods have been proposed for calculating the M-ϕ relationship for

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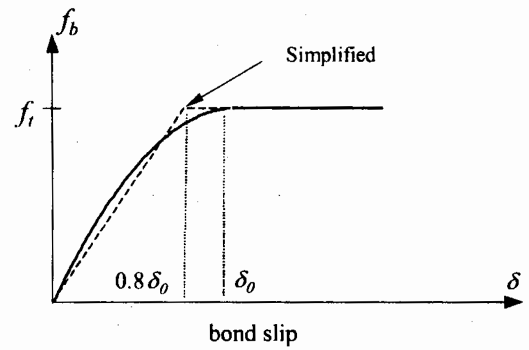

Fig 1. Bond stress vs slip relationship of Dorr (1980).

semirigid connections, but most *M-ϕ* curves must be determined experimentally. Initiative works in this line are the linear models proposed by Batho (1931, 1934, 1936), Rathbun (1936), Baker (1934). Later, the bi-linear models of Romstad and Subramanian (1970), Melchers and Kaur (1982), Lui and Chen (1983), piecewise linear models of Razzaq (1983), polynomial model of Fry and Morris (1975), cubic spline models of Cox (1972), Jones et. al. (1981, 1982), multi parameter exponential model of Chen and Lui (1983), all contributed to the development of our understanding of semi-rigid characteristics. However, it is to be mentioned that the mainstream development in this area is focused on the joint characteristics of steel structures due to the higher joint flexibility of steel structures.

The American Concrete Institute report ACI 352R-91, suggests that *"The designer should consider the possible effect of joint rotations on cracking and deflection."* However, it does not give us any guideline whatsoever regarding the rotational characteristics of RC joints. Conventional analysis and design of reinforced concrete frameworks are usually carried out under the assumption that the connections joining the beams to the columns are fully rigid which implies that full slope continuity exists between the adjoining members and that the full (or a substantial percentage) of gravity moment is transferred from the beam to the column. Although such assumption drastically simplifies the analysis and design procedure, the validity of the assumption may be questionable in light of the fact that concrete, due to its greater compressibility and the presence of reinforcement with its inherent tensile and ductile characteristics may impart to the connection some degree of flexibility. The effects of rotational connection flexibility on multistory RC structure may be twofold, a) the joint rotation contributes to the overall frame deformations, in particular the frame sway under lateral loads. This reduction in frame stiffness will also affect the natural period of vibration and therefore, the dynamic response to earthquake motions, b) The joint rotation will affect the distribution of internal forces and moments in

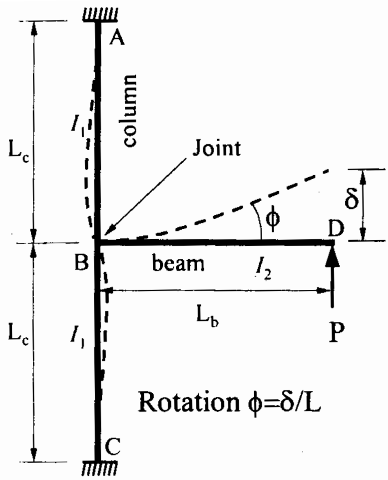

Fig 2. The beam-column structure

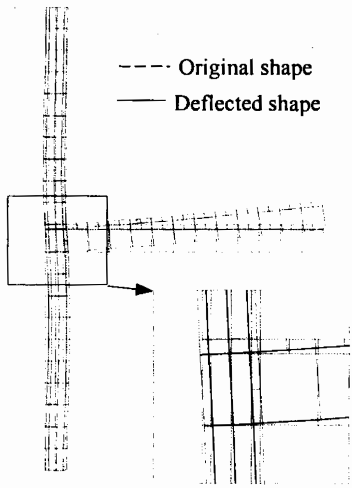

Fig 3. The finite element mesh with typical deflected shape

girders and columns. An analysis which neglects connection deformation may thus be unable to arrive at realistic predictions of stresses and deflections. In this paper, focus is given on understanding the semi-rigid characteristics of beam to column RC joint through numerical finite element simulation of a typical exterior joint. It has been shown that the rotational characteristics are dependent on a few joint parameters such as steel ratio, effective depth etc. Based on the study an attempt has been made to characterize the moment-rotation behavior of the joint in terms of these parameters.

NUMERICAL MODELING OF REINFORCED CONCRETE

When the overall macroscopic behavior of concrete is of primary interest we can model the reinforced concrete by finite element discretization where we use two or three dimensional elements for concrete and two or three dimensional truss elements or line elements for steel in such a way that the nodes where the steel elements are connected to the concrete element have the same degrees of freedom. The disadvantage of such modeling is that they cannot simulate the actual cracking phenomenon since perfect bond is maintained throughout the analyses. A more realistic model is to discretize the concrete and steel separately so that each components have completely independent degrees of freedom which are connected via special finite elements to simulate the bond-slip mechanism. Such modeling is more straightforward and we will use this approach in this paper.

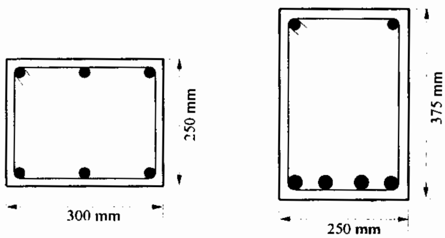

Fig 4. Beam and Column sections

Material Modeling

Since our goal is towards the development of the proper understanding of the rotational characteristics of RC joints under service load conditions, linear behavior of different component materials is assumed. Thus it is assume that the loading will be such that the stress level of both the materials will be within the elastic range.

Bond Stress - Slip Relationship

For analytical applications several linear and non-linear approximations of the bond stress - slip relationship are available. All the models describe the bond stress as a function of relative slip. But there is still debate on the amount of maximum slip and the corresponding bond stress level at which perfect slip occurs. It appears that with so many influencing factors like the amount of confinement, spacing of ribs in reinforcement, tensile strength of concrete etc. the relative slip alone is not enough to define the bond stress - slip relationship. In the present study, our aim is only to see the rotational characteristics of an RC joint under service load conditions and hence we will adopt a simpler bond stress - slip relationship derived from the simplification of Dorr's model (Amanat 1997). Dorr proposed one nonlinear function relating the bond stress with the tensile strength and relative slip as,

$$f_b = f_t\left[5(\delta/\delta_0) - 4.5(\delta/\delta_0)^2 + 1.4(\delta/\delta_0)^3\right], \quad 0 < \delta < \delta_0 \qquad (1)$$

where f_t is the tensile strength in MPa and δ_0 is the amount of slip at which perfect slip occurs which is usually taken as 0.6 mm. For $\delta > \delta_0$ the value of f_b is constant at $1.9 f_t$. We will use a simplified form of the same as shown in Fig.1

Finite Element modeling

Since plane frames are typically two-dimensional, the beams and columns are modeled using two-dimensional plane stress elements. These

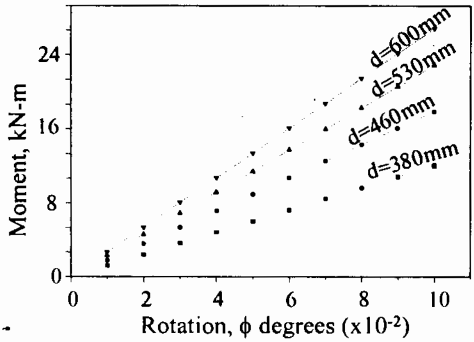

Fig 5a. M vs ϕ response for different beam depth

elements are the typical four or eight noded iso-parametric elements. The reinforcements are modeled using simple two dimensional truss elements.

The bond stress - slip relationship is simulated by the special contact elements. These contact elements are dimensionless bond link elements, connecting a single concrete node to a corresponding reinforcement node. This bond element is basically a simple spring placed in the direction of the reinforcement (parallel) which simulate the bond stress - slip relationship. The constitutive relation of this bond element is derived from the bond-slip relation of Fig.1.

The investigation was carried out by modeling a simple RC joint consisting a beam and two columns as shown in Fig.2. The study was confined to this simple prototype finite element model since further study of different kinds of other joints is beyond the scope of the present study.

The finite element mesh corresponding to the structure of Fig.2 is shown in Fig.3. While modeling the with plane stress elements the portion of concrete below the main reinforcement is neglected. The main reinforcement goes along the bottom corner nodes of the elements. The nodes connecting the reinforcements are separate from the nodes of concrete elements but have the same physical location. These pairs of nodes are connected by special dimensionless bond-link element which simulates the bond slip mechanism along the direction of reinforcement. Compatibility condition of zero relative displacement in the lateral direction is also enforced for these nodes. For the columns the reinforcement elements are directly attached to the mid-side nodes of the plane stress elements i.e. full bond is assumed.

Validation of the model

Before we begin our investigation of joint characteristics, it first necessary to verify the performance of the FE modeling by comparing the results with ordinary frame analysis. For this purpose columns are taken to be 300 mm × 250 mm size with 2.5% steel reinforcement as shown in

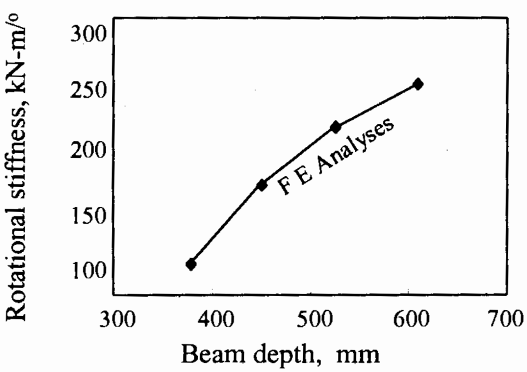

Fig 5b. Rotational stiffness vs beam depth

Fig.4(a). The beam has an overall depth of 375 mm and width of 250 mm reinforced with 4ϕ25mm bars as A_s and 2ϕ16 mm bars as A_s' as shown in Fig.4(b). The moment of inertia of the beam and column sections needed for the frame analysis are calculated according to the ACI recommended procedure. Young's modulus for the materials are for concrete E_c=20000 MPa and for steel E_s=2×10^5 MPa. The length L of the beam and columns is taken as 3000 mm. For the columns un-cracked transformed sections were used while for the beam cracked transformed section was used in calculating section modulus. An upward load of P=1 kN was applied at point D as shown in Fig.2.

The horizontal reaction at top or bottom support calculated from ordinary frame analysis was 0.75 kN while the same from FE analysis using the mesh of Fig.3 gave a value of 0.7 kN giving a deviation of about 8%. The cause of this difference may be attributed to the fact that in FE analyses bond slip mechanism was incorporated via special bond-link elements which caused some redistribution of internal stresses when compared to the *fully-bonded* simplified frame analysis.

SENSITIVITY ANALYSIS

The primary purpose of our study is to investigate the M-ϕ relationship of the typical RC exterior joint for different conditions and to find out whether it is possible to characterize the joint's rotational characteristics in terms of different geometric and material parameters. Towards this achievement a limited sensitivity analysis was performed which attempts to establish the relative importance of these parameters on the behavior of RC joint. Rotations at the joint had been simulated by applying vertical displacements at the free end of the beam (see Fig.2). The rotation applied produced a certain amount of moment at the critical section of the joint. This moment was obtained by multiplying the vertical force at free end

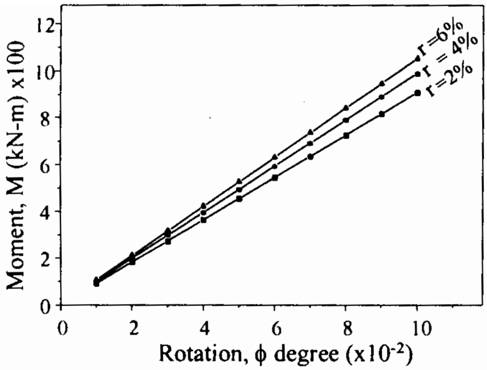

Fig 6a. M vs ϕ for different steel ratio

with the distance of the free end from the critical section. This vertical force was obtained as the reaction at the free end due to the applied displacement.

Joint behavior may be dependent on a large number of variables. Sensitivity analysis performed includes a selected number of parameters which have been considered to have likely effect on the joint rotational characteristics. The result of the present analysis are subjected to the limitations inherent in the scope of the range of parameters considered. The approach towards parametric study was that at a single instance only one parameter should be allowed to vary while all other parameters are fixed at the initial value. The parameters examined and the limits of those within the present study are summarized in the table 1.

Table 1. Initial values and range of parameters

Name of Parameter	Type	Range of Values
Beam depth	Geometric	375, 460, 540, 600
% of bottom reinforcement of beam	Material	2%, 3%, 4%, 5%, 6%;
% of column reinforcement	Material	2%, 4%, 6%
No. of main reinforcing bars (A_s) (for 19mm, 22mm, 25mm bars)	Material	2, 4, 6, 8, 10 Nos.

Effect of beam depth

The influence of variation of beam depth on the M-ϕ relationship of the joint has been shown in Fig.5a. Judging the nature of the plot it is obvious that for a certain beam depth and hence for a fixed beam cross-section the

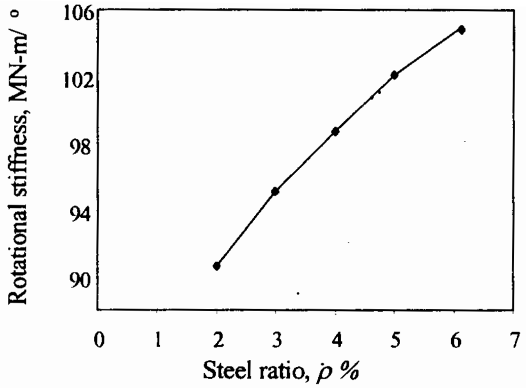

Fig 6b. Rotational stiffness vs steel ratio

M-ϕ relationship is linear. It can be explained by mentioning that in our analysis material properties of both concrete and reinforcement have been taken to be within the elastic range. Also the plot shows that M-ϕ curve gets steeper with the increase in beam depth. This is expected because the moment capacity and hence the rotational stiffness of a joint increases with the increase of effective depth of the beam which is nothing but an increase in overall beam depth. From the Fig.5a it observed that for a fixed rotation the moment has not increased uniformly although the beam depth was varied in an uniform manner. This insinuates that the relation between rotational stiffness of the joint against beam depth would be non-linear. Figure 5b supports this notion quite well. We can see that the stiffness increases with the increase in depth of beam but in a non-linear manner. The rate of increase of stiffness decreases as the beam depth gets higher. This is quite opposite to the common understanding that the stiffness of a beam increases according to the third power of depth. Such behavior can be attributed to the fact that with the increase in depth the moment arm corresponding to internal bending stress increases and the steel ratio is decreased correspondingly. Such increase in moment arm mobilizes the bond slip mechanism faster which eventually decreases the rate of moment development.

Effect of Variation of Percentage of Beam Reinforcement

As explained in the previous section, due to the inherent linearity of material properties the M-ϕ relationship for a certain percentage of beam reinforcement was also found to be linear. Figure 6a corroborates this fact. In this case also we observe an increase in moment for a particular rotation following an increase in percentage of beam reinforcement. Figure 6a also infers a non-linearity of the rotational stiffness when expressed against percentage of beam reinforcement. This is deduced from the fact that in the M-ϕ relationship in Fig.6a moment increase in a slightly non-

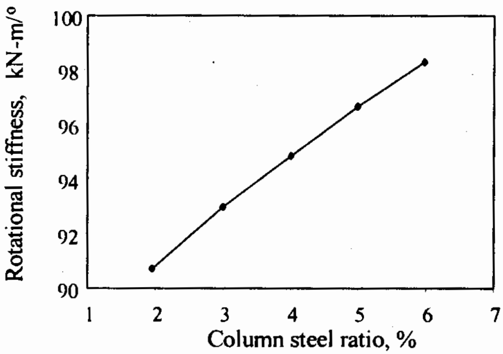

Fig 7. Rotational stiffness vs column steel ratio

uniform manner even though ρ is varied uniformly. Figure 6b supports this behavior by showing a slight curvilinear relation between rotational stiffness and percentage of beam reinforcement. But this non-linearity is not much pronounced as in the case of beam depth.

Effect of Variation of Column Reinforcement

Effect of column steel ratio on the rotational characteristics of the RC joint is shown in Fig.7. Effect of this variation is least pronounced on the rotational characteristics of the RC joint when compared to other parameters. Figure 7 shows the M-ϕ relationship for various percentage of column reinforcements. The almost linear M-ϕ relationship is evident. It is observed that the increase in moment for a certain ϕ is very small for increase in percentage of column reinforcement. The curve of rotational stiffness vs. percentage of column reinforcement, therefore, is almost a straight line as shown in Fig.7. This characteristics can be explained by mentioning that in our analysis moment was produced mainly at beam-joint interface and hence the columns were less affected.

Effect of Number of Bars of Different Bar Sizes

The effect has been expressed graphically in Fig.8. This is nothing but another representation of the variation of rotational stiffness with the variation of percentage of beam reinforcement. It is observed that for a particular bar size rotational stiffness increases with the increase in number of bottom bars. It this justified because increased bottom reinforcement entails higher moment capacity. The same reason is also applied for the fact that stiffness increases with an increase in bar size. However, the rate of increase of stiffness is not in direct proportion to the increase of number of bars. Due to the presence of bond-slip action we observe some non-linearity in the stiffness response of Fig.8.

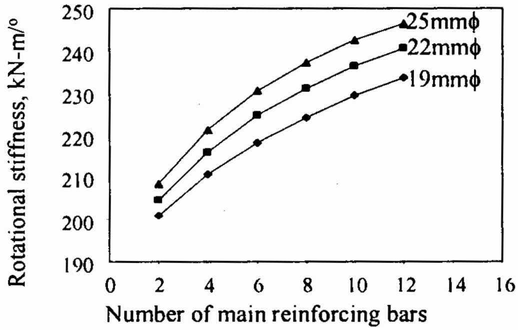

Fig 8. Rotational stiffness vs number of reinforcing bars

CONCLUSIONS

The rotational characteristics of a typical exterior RC joint is studied within the limited scope of the present investigation. The study shows the successful application of *micro-modeling* where each component of reinforced concrete is modeled individually. Such modeling enables us to gain in-depth understanding about the complex RC joint characteristics in a relatively straightforward way. The study reveals that increase of beam depth does increases the rotational stiffness but in a progressively decreasing manner which is quite contradictory to the common understanding. Beam steel ratio also produces similar effects. Although column reinforcement have some influence on the rotational characteristics of the beam, it is not much significant when compared to the other parameters.

The study presented in this paper clearly shows that the results of conventional frame analyses method may not be the representative of the actual behavior of a RC frame. The study also shows that proper analytical modeling of joint rotational characteristics can be developed based on the *micro-model* discussed in this paper. Such modeling will eventually lead to more realistic and safer design of RC frames.

REFERENCES

ACI-ASCE Committee 352 (1991), "Recommendations for Design of Beam-Column Joints in Monolithic Reinforced Concrete Structures," ACI Committee Report ACI 352R-91, pp.352R-1.

Amanat K.M. (1997), Gradient Plasticity Modeling for the Fracture and Strain Localization of Early Age Concrete, Ph.D. thesis, Dept. of Civil Engg., Nagoya University, Japan.

Baker J.F. (1934), "A Note on the Effective Length of a Pillar Forming Part of a Continuous Member in a Building Frame," 2nd Report, Steel Structure Research Committee, Dept. of Scientific and Industrial Research, pp.13-44, HMSO, London.

Batho C. (1931), "Investigation on Beam and Stanchion Connections," 1st Report, Steel Structure Research Committee, Dept. of Scientific and Industrial Research, Vol 1-2, pp.61-137, HMSO, London.

Batho C. and Lash S.D. (1936), "Further Investigation on Beam and Stanchion Connections Encased in Concrete," Final Report, Steel Structure Research Committee, Dept. of Scientific and Industrial Research, pp.276-363, HMSO, London.

Batho C. and Rowan H.C. (1934), "Investigations on Beam and Stanchion Connections," 2nd Report, Steel Structure Research Committee, Dept. of Scientific and Industrial Research, pp.92, HMSO, London.

Chen W.F. and Kishi N. (1989), "Semi-rigid steel Beam-to-column Connections: Data Base and Modeling," American Soc. of Civ. Eng. (ASCE), Journal of Structural Engineering, Vol. 115, No. 2, pp.105-119.

Cox M.G. (1972), "The Numerical evaluation of B-splines," J. Inst. Math. Applic, Vol. 10, pp.134-149, 1972.

Frye M.J. and Morris G.A. (1975), "Analysis of Flexibly Connected Steel Frames," Can. J. civil Engg., Vol. 2, pp.280-291.

Jones S.W., Kirby P.A. and Nethercot D.A. (1982), "Columns with Semi-rigid Joints," American Soc. of Civ. Eng. (ASCE), Journal of Structural Engineering, Vol. 108, pp.361-372.

Lui E.M. and Chen W.F. (1983), "Strength of H-Columns with End Restraints," J. Inst. Struct. Engrs., Vol. 61B, 17-26.

Melchars R.E. and Kaur D. (1982), "Behaviour of Frames with Flexible Joints," Proc. 8th Australian Conf. Mech. Of Structural Matrials, Newcastle, Australia, pp.27.1-27.5.

Rathbun J.C. (1982), "Elastic Properties of Riveted Connections," American Soc. of Civ. Eng. (ASCE), Journal of Transportation Engineering, Vol. 101, pp.524-563.

Razzaq Z. (1983), "End Restraint Effect on Steel Column Strength," American Soc. of Civ. Eng. (ASCE), Journal of Structural Engineering, Vol. 109, No. 3, pp.314-334.

Romstad K.M. and Subramanian C.V. (1970), "Analysis of Frames with Partial Connection Rigidity," American Soc. of Civ. Eng. (ASCE), Journal of Structural Engineering, Vol. 100, pp.2283-2300.