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THREE DIMENSIONAL ELASTO-PLASTIC FINITE ELEMENT ANALYSIS OF OFFHORE CIRCULAR FOUNDATIONS RESTING ON CARBONATE SANDS AND SUBJECTED TO INCLINED LOADS

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ABSTRACT: A critical state constitutive model developed for carbonate sands was used to analyse the response of offshore circular foundations subjected to inclined loads. The geometry of the problem is axisymmetric while the load is non axisymmetric. The model was incorporated in a semi-analytical three dimensional finite element program. The response of circular foundations subjected to inclined load was analysed. The numerical results obtained for load-displacement response of circular foundations resting on both cemented and uncemented carbonate sands was compared with experimental laboratory data for prototype footings. While reasonable agreement was observed between numerical and experimental data at relatively smaller load inclinations, the agreement between numerical and experimental data at relatively larger load inclinations was not satisfactory.

KEYWORDS: Critical state, elasto-plastic, constitutive model, finite element, three-dimensional, offshore foundation, carbonate sand, inclined load.

INTRODUCTION

Highly compressible carbonate sands are encountered at the ocean bed in offshore areas or continental shelves in many regions of the world. The foundations of oil platforms are often constructed on these sands. In these cases, the superstructures are subjected not only to vertical loads, but also to horizontal wave forces which induce moments at the base. For rational design, it is necessary to develop appropriate tools to analyse the mechanical response of such foundations. This paper describes the tools developed by the authors to undertake such an analysis.

In this paper, the characteristics and the stress-strain response of carbonate sands under different stress paths is first described. A constitutive model developed to simulate the stress-strain response of carbonate sands is then presented. Next, a three-dimensional semi-analytical elasto-plastic finite element routine developed at the University of Sydney is discussed. Finally, the three dimensional finite element routine is used to generate the load-displacement response of axisymmetric footings subjected to inclined loads. The predicted

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response is compared with the experimental load-displacement data obtained from model-scale circular footings resting on carbonate sands and subjected to inclined loads. The results of all of the above analysis are presented in this paper.

CHARACTERISATION OF CARBONATE SANDS

The mechanical behaviour of carbonate sands have been extensively studied by various authors (Coop, 1990; Coop and Atkinson, 1993; Huang, 1994). Detailed microscopic investigations and triaxial study of carbonate sands obtained from the ocean bed of the North West Shelf of Australia were carried out at the University of Sydney.

Carbonate sands are formed from the remains of marine organisms through a variety of physico-chemical processes occurring over geological time periods. Frequently these sands exhibit strong cementation properties, which further complicate their mechanical behaviour. Cementation is usually a result of chemical deposition at the interparticle contacts.

Although carbonate sands show stiffer elastic behavior then silica sands at low pressures, they undergo significant plastic volumetric compression once certain threshold pressures are exceeded. Silica sands, on the other hand, show dilatational behaviour except at very high pressures. Carbonate sand particles exhibit high angularities resulting in high void ratios concurrently with high ultimate state friction angles (40 degrees or more. For soils, the ultimate or critical state is defined as a state of constant stress at which it undergoes continuous plastic shear at constant volume. For carbonate sands, the critical state is mobilized at shear strains as large as 50%. Silica sands reach the critical state at much lower strains. Thus a different approach from conventional sand models is required to predict the stress-strain response of carbonate sands. The details of triaxial stress-strain response for carbonate sands are available at (Huang, 1994, Lagioia and Nova, 1995).

STRESS-STRAIN MODEL FOR CARBONATE SANDS

A large variety of critical state models have been developed to predict the stress-strain response of silica sands. Although many of these models can satisfactorily predict the mechanical response of such sands, these are inadequate for predicting the plastic volumetric compression exhibited by carbonate sands at low pressures. Clay models, which generally follow the associated flow rule, also cannot predict plastic volumetric compression at lower pressures. The yield locus of clay models are significantly larger than that obtained for un-cemented or lightly cemented carbonate sands. Complicated models, such as the Molenkamp model, using large number of material parameters, may be used to curve fit the stress-strain response of carbonate sands. However, this would require complicated numerical simulations and most of the parameters would have to be assigned numbers which do not have any physical meaning.

The first author of this paper has proposed a critical state model (hereinafter called the SU2, Islam, 2000) patterned after the Modified Cam Clay (Roscoe and Burland, 1968) to realistically predict the stress-strain response of carbonate sands. The SU2 model incorporates the spacing ratio parameter in the Modified Cam Clay model thus generalizing it. The spacing ratio is a measure of the separation of the critical state line and the isotropic consolidation line in $e\text{-}\ln p'$ space, where e is the void ratio and p' is the mean isotropic stress. Carbonate sands generally have a spacing ratio of 5 or greater, which is assumed to a constant of 2 for the Modified Cam Clay model. The SU2 model uses the spacing ratio parameter to scale down the Modified Cam Clay yield locus in $q\text{-}p'$ stress space, where q is the deviator stress. This shifts the intersection of the critical state line with the yield locus further to the left of the preconsolidation pressure. This shift allows the proposed constitutive model to predict compressive plastic volumetric behaviour even at low isotropic pressures. The equation of the SU2 yield locus is given as follows:

$$\left(\frac{q}{Mp'}\right)^2 = \frac{\left(\frac{p'_o}{p'}\right) - 1}{r - 1} \tag{1}$$

where p'_o is the preconsolidation pressure, M is the critical state ratio and r is the spacing ratio. The spacing ratio is defined as below:

$$r = \frac{p'_o}{p'_{cs}} \tag{2}$$

In equation 2, p'_o is the mean pressure on the isotropic consolidation line and the p'_{cs} is the mean pressure on the critical state line, both drawn in $e\text{-}\ln p'$ space and measured along an elastic rebound line.

The critical state line represents the ultimate stress state at which there is continuous plastic shear at constant volume. On the other hand, the isotropic consolidation line represents isotropic stress states at which purely plastic volumetric compressive behaviour occurs at zero shear strain. These are identical to that of the Modified Cam Clay model.

The plastic potential function of the proposed model is identical to the Modified Cam Clay yield locus. The plastic potential function is this given by the following equation:

$$\left(\frac{q}{Mp'}\right)^2 = \left(\frac{p'_o}{p'}\right) - 1 \tag{3}$$

Thus the yield and plastic potential function of the proposed model are both elliptical. However, they are represented by ellipses of different sizes. Hence the model follows a non-associated flow rule, which is in greater agreement with the behaviour exhibited by carbonate sand. An infinite series of plastic volumetric strain hardening yield loci is assumed to exist in q- p' stress space. The yield locus and plastic potential function of the proposed model and the spacing ratio concept is depicted is illustrated in Figures 1, 2 and 3.

For elasticity, the elastic bulk modulus is assumed to vary with the isotropic pressure p'. The elastic shear modulus is determined from the elastic bulk modulus assuming a constant value of the elastic Poisson's ratio.

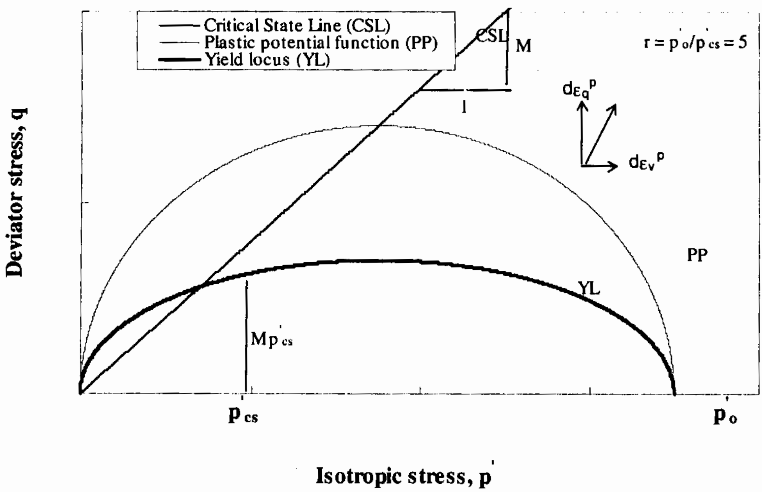

Fig 1. SU2 model yield locus, plastic potential function an spacing ratio

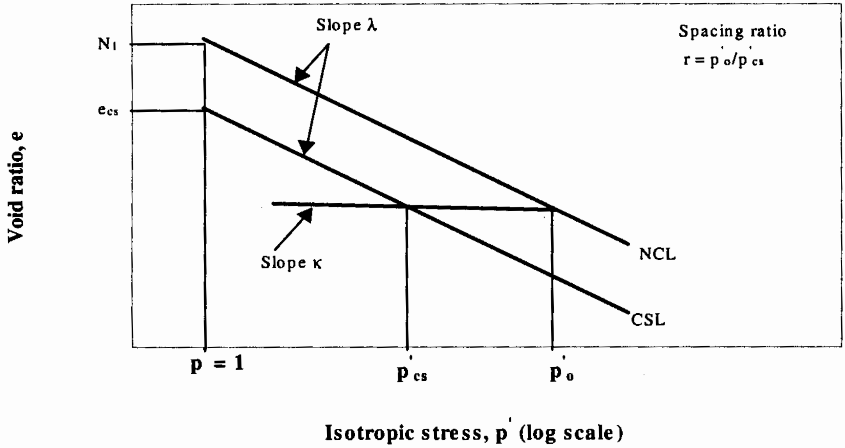

Fig 2. Illustration of the spacing ratio concept in the mean effective stress versus void ratio $(e - \ln p')$ *space*

DETERMINATION OF MODEL PARAMETERS

The proposed model has only 6 model parameters. Three of the parameters namely the preconsolidation pressure p_o', the plastic compression parameter λ and the elastic rebound parameter κ are determined from loading-unloading curves obtained from an isotropic consolidation test. The critical state line is obtained by plotting ultimate stress states in q- p' stress space. The ultimate stress states are determined from a series of triaxial tests conducted at different cell pressures. The slope of the critical state line gives the critical state ratio *M*. The ratio of the pressures on the isotropic consolidation line and the critical state line drawn in *e-ln* p' space and measured along an elastic rebound line, gives the spacing ratio.

The elastic bulk modulus is obtained from the elastic rebound parameter. The elastic shear modulus is obtained from the elastic bulk modulus using the elastic Poisson's ratio. The elastic Poisson's ratio was indirectly determined from drained triaxial shear tests where the volume change with axial strain is recorded. A constant elastic Poisson's ratio is assumed in the model. All parameters of the proposed model may be determined from routine laboratory tests and all of them represent physically meaningful soil properties. The soil parameters determined for the carbonate sand used for the model-scale footing tests subjected to inclined load are shown in Tables 1 and 2.

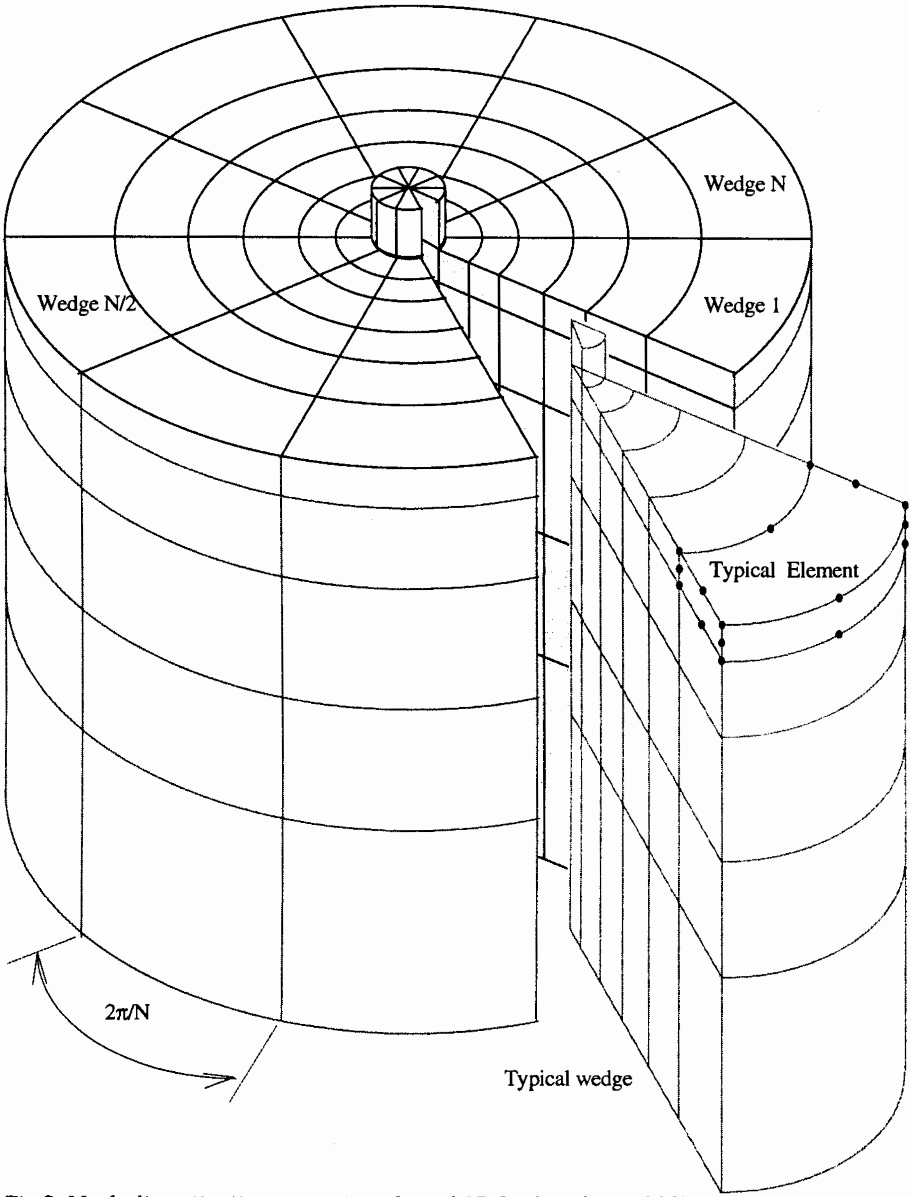

Fig 3. Mesh discretisation in semi-analytical 3D FE (Taiebat, 1999)

Table 1. Constitutive model parameters for artificially cemented carbonate sand

ϕ' (deg)	λ	κ	ν	p_o' (MPa)	r
39	0.213	0.005	0.2	3.5	5.0

Table 2. Constitutive model parameters for un-cemented carbonate sand

ϕ' (deg)	κ	λ	ν	p_o' (MPa)	r
39	0.005	0.213	0.2	3.0	5.0

MODEL-SCALE FOOTING TEST

Experimental studies of a model-scale footing on carbonate sand subjected to inclined load at zero eccentricity were carried out (Pan, 1999). The experiments and the results obtained are briefly described in this section.

The model steel footing was 25mm in diameter and 20mm in thickness. The footing was placed on the top of a cylindrical specimen of carbonate sand. The sand cylinder was 250mm in diameter and 175mm in thickness. The carbonate sand was subjected to a uniform isotropic pressure of 50 kPa. Vertical and horizontal loads at zero eccentricity were gradually applied on the footing.

The change of traction with load inclination was measured. Traction is defined here as the total load applied to the model footing divided by its plan area. The results obtained for artificially cemented and un-cemented carbonate sands are presented below in Tables 3 and 4.

Table 3. Experimental data for footing on artificially cemented carbonate sand

Inclination (deg)	$(q_{av})_{10\%}$ (MPa)	$(q_{av})_{20\%}$ (MPa)	$(q_{av})_{30\%}$ (MPa)	$\left(\frac{q_{av}}{q_o}\right)_{10\%}$	$\left(\frac{q_{av}}{q_o}\right)_{20\%}$	$\left(\frac{q_{av}}{q_o}\right)_{30\%}$
0	6.33	8.28	9.49	1.0	1.0	1.0
10	6.17	NA*	NA*	0.97	NA*	NA*
20	4.26	6.58	7.81	0.67	0.79	0.82
30	3.25	5.20	6.38	0.52	0.63	0.67

*NA implies not available

In Tables 3 and 4, q_{av} is the traction mobilised at displacements of 10%, 20% and 30% of the footing diameter, under inclined load. q_o is the bearing pressure under purely vertical load, mobilised at 10%, 20% and 30% vertical displacement relative to the footing diameter.

Table 4. Experimental data for footing on un-cemented carbonate sand

Inclination (deg)	$(q_{av})_{10\%}$ (MPa)	$(q_{av})_{20\%}$ (MPa)	$(q_{av})_{30\%}$ (MPa)	$\left(\frac{q_{av}}{q_o}\right)_{10\%}$	$\left(\frac{q_{av}}{q_o}\right)_{20\%}$	$\left(\frac{q_{av}}{q_o}\right)_{30\%}$
0	4.15	5.88	7.15	1.0	1.0	1.0
10	2.59	4.23	5.44	0.62	0.72	0.76
20	1.91	3.13	3.92	0.46	0.53	0.55
30	0.82	1.33	2.02	0.19	0.23	0.28

THREE-DIMENSIONAL ELASTO-PLASTIC FINITE ELEMENTS

In many practical problem the applied loading is three-dimensional while the geometry is axisymmetric. This is especially true for a circular footing subjected to inclined load. In such cases, a semi-analytical finite element approach is useful and efficient. (Lai and Booker, 1991) and (Runneson and Booker, 1982, 1983) used the discrete Fourier series approach to analyse the non-linear behaviour of axisymmetric solids under true three-dimensional loading conditions. (Taiebat, 1999) extended this approach and developed a semi-analytical finite element procedure for a three-dimensional elasto-plastic solid. The computation time for such an analysis has been shown to be less than 5% of the computation time required for a full three-dimensional finite element analysis. The bearing response of the model-scale circular footings under inclined load was simulated using the proposed constitutive model, which was implemented in the semi-analytical 3D FE procedure.

In this method, the field quantities are represented as discrete Fourier series. The axisymmetric body is divided into N identical wedges as shown in Figure 3. The body then exhibits a polar periodicity with period N. The field quantities are written in terms of Fourier coefficients as:

$$\left(u_r, u_z, u_\theta, q, f\right)_j = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \left(U_r, U_z, U_\theta, Q, F\right)_k e^{ijk\alpha} \tag{4}$$

In the above equation $\left(u_r, u_z, u_\theta\right)_j$ denote the nodal displacements of wedge j, q_j are the excess pore pressures at nodes on wedge j and f_j are nodal forces applied to wedge j. $\left(U_r, U_z, U_\theta, Q, F\right)_k$ are the kth Fourier coefficients of nodal displacements, porewater pressures and external applied load. The inverse relations are defined by:

$$\left(U_r, U_z, U_\theta, Q, F\right)_K = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} \left(u_r, u_z, u_\theta, q, f\right)_j e^{-ijk\alpha} \tag{5}$$

The details of the derivation are available in (Taiebat, 1999).

SIMULATING FOOTING RESPONSE WITH STRESS-STRAIN MODEL

The circular model footing was simulated as a rigid elastic material. The elastic modulus of the footing was assumed to be several orders of magnitude higher than the underlying sand. It was assumed that the rigid footing, having a perfectly rough base, was resting at the top surface of a weightless cylindrical soil sample (Figure 3). The stress-strain behaviour of the soil was simulated incorporating the proposed constitutive model in a finite element program named AFENA (Carter and Balaam, 1995). The footing was loaded by a uniform traction applied incrementally at its surface. The vertical traction was applied at the top nodes of the footing element, while the lateral traction was applied to the nodes at the bottom. This ensured the gradual application of an inclined load on the footing centre at zero eccentricity, i.e., with no moment about the centre of the footing-soil interface.

The vertical boundary of the cylindrical soil sample was assumed to be perfectly smooth. However, it was constrained from moving in the lateral and circumferential directions. Except for the area in contact with the footing, the top boundary of the soil sample was subjected to uniform vertical surcharge of 50 kPa. The bottom boundary was constrained from moving in the vertical, lateral and circumferential directions.

All analyses simulated fully drained conditions. An initial isotropic stress of 50 kPa was generated throughout the soil layer. This is consistent with the conditions applied in the experiments. Equivalent nodal forces were applied at the boundaries to maintain equilibrium. The finite element mesh constructed to simulate the 3D geometry of the problem is described in the next section.

The three dimensional geometry of the cylindrical soil-footing domain was generated using 12 identical cylindrical wedges (Figure. 3). Elements consisting of 20 node cubes (Figure. 4), with quadratic interpolation functions for displacements, were used to discretise each wedge. Figure 5 shows the angle of inclination of the applied load. Figure 6 represents a diametrical cross section through the cylindrical domain. A graded mesh was used with progressively coarser element size being constructed laterally away and vertically downward from the footing. 2x2x2 reduced Gaussian integration was used for all calculations.

CALIBRATION OF SU2 MODEL IN 3D FINITE ELEMENT

The purpose of this section is to validate the results obtained using the SU2 implementation of the 3D finite element procedure with those of corresponding 2D finite element analyses. Once the SU2 implementation in 3D finite element has been validated, it may then be used to predict the response of model-scale footings under inclined load.

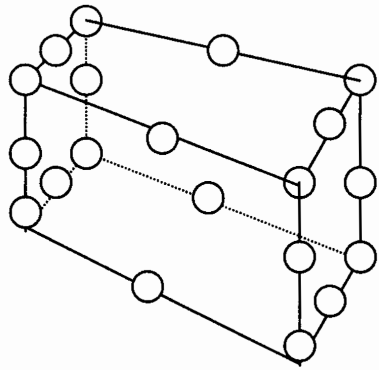

Fig 4. 20 -node cubic element used in 3D FE mesh

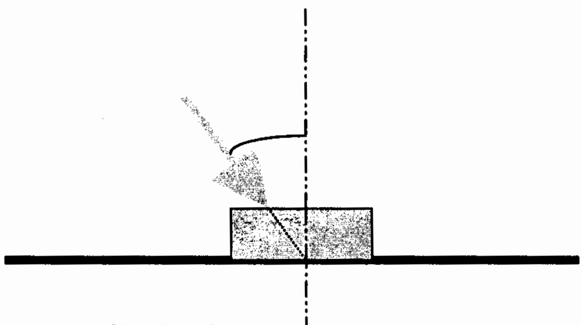

Fig 5. Angle of inclination of load applied at zero eccentricity

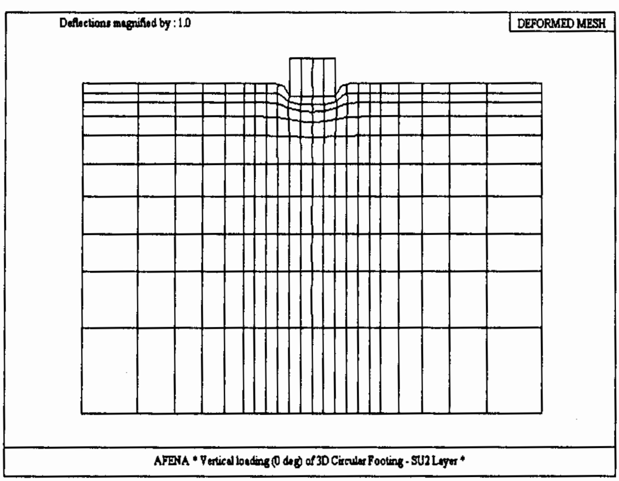

Fig 6. A diametrical section view of mesh used in 3D FE

Single element SU2 model simulations of triaxial tests were generated using the 3D semi-analytical finite element program. The predictions were compared with the corresponding simulations of 2D finite element analysis. Close agreement between the results obtained using the two methods (not shown). The 2D and 3D finite element programs were used to generate the SU2 model simulations of the pressure-displacement curve for a circular footing subjected to vertical load. The pressure-displacement curves obtained were observed to be nearly identical (not shown). This indicates that the SU2 model implementation is working properly in the 3D finite element program.

PREDICTING FOOTING RESPONSE TO INCLINED LOADS

Figures 7 and 8 show comparisons of the finite element predictions of the response of the model-scale footing with experimental data. It is observed that the SU2 model predicts reasonably well the change in traction with load inclination for footings on cemented carbonate sand. However, the predictions of the change in traction with load inclination for the model-scale footing resting on un-cemented sand are not entirely satisfactory. Figures 9 and 10 show the mobilised traction normalised by a reference pressure to make it dimensionless. The normalising pressure used in this case is the bearing resistance mobilised at a displacement of 30% of the footing diameter under vertical load. The change in traction with load inclination predicted by the SU2 model appears to be in good agreement with the experimental data for cemented carbonate sand. The agreement with the data for uncemented carbonate sand is not entirely satisfactory.

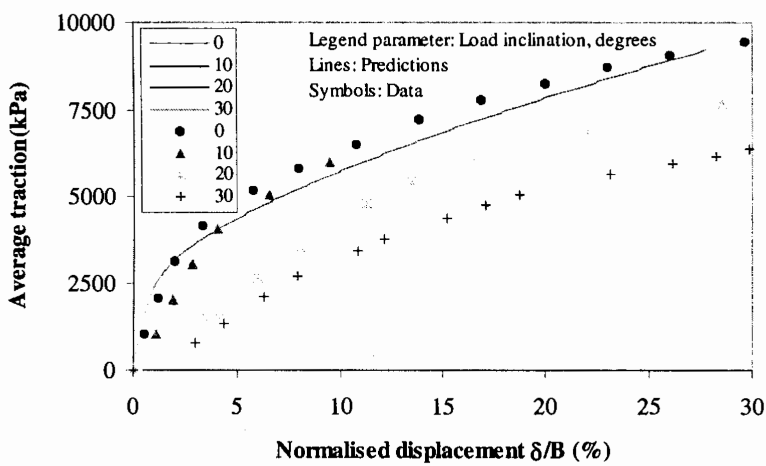

Fig 7. Comparison of 3D FE predictions with experimental data for footing on artificially cemented carbonate sand

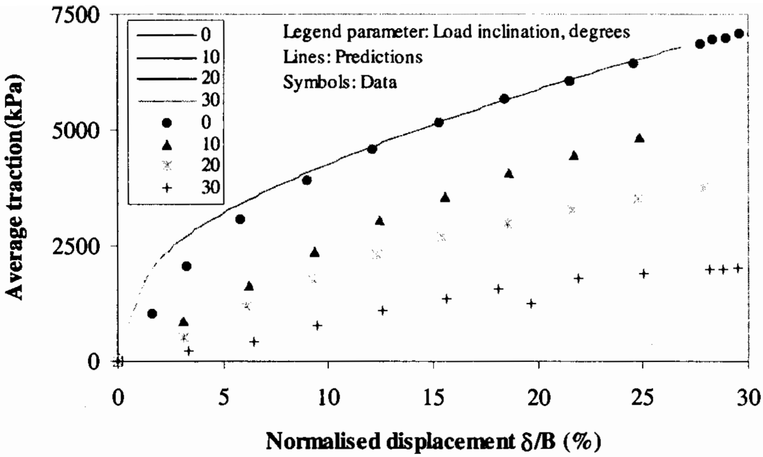

Fig 8. Comparison of 3D FE predictions with experimental data for footing on Un-cemented carbonate sand

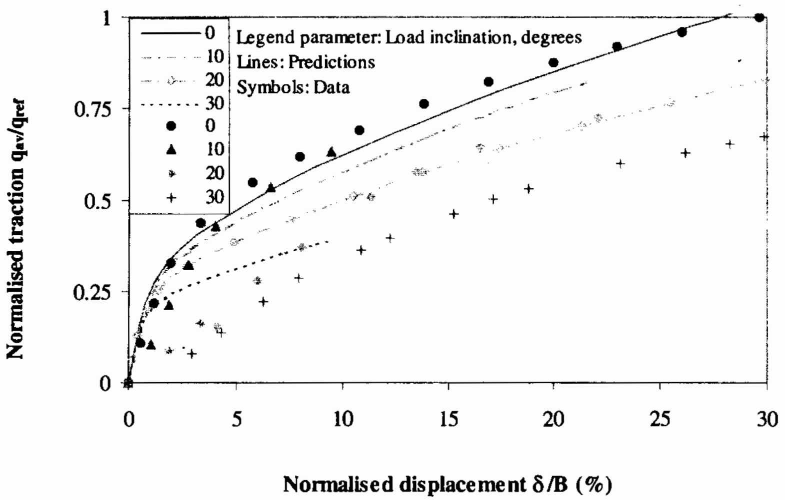

Fig 9. Comparison of predictions of change of footing traction with load inclination with data for footing on artificially cemented carbonate sand

The SU2 model predictions of the traction q_{av} mobilised at 10% resultant displacement and its change with load inclination are compared with the experimental data for the model-scale footing resting on uncemented and artificially cemented carbonate sand. The comparisons are given in Table 5.

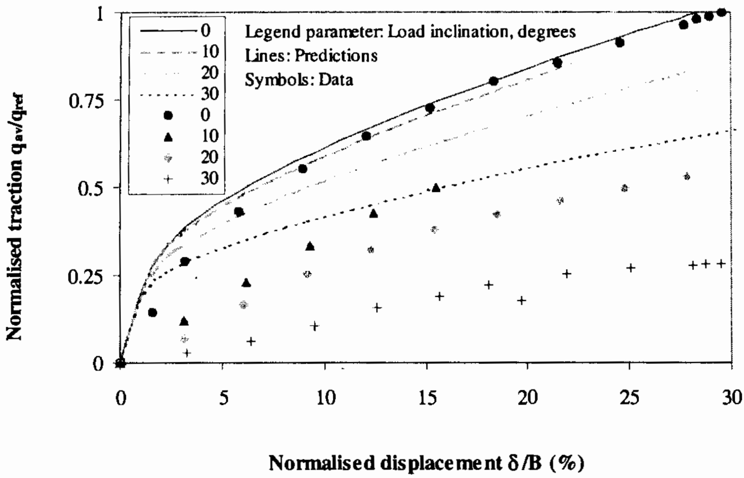

Fig 10. Comparison of predictions of change of footing traction with load inclination with data for footing on un-cemented carbonate sand

Table 5. Comparison of SU2 model predictions with test data

Inclination / Method	0 deg q_o (MPa)	10 deg q_{av} (MPa)	20 deg q_{av} (MPa)	30 deg q_{av} (MPa)	10 deg q_{av}/q_o	20 deg q_{av}/q_o	30 deg q_{av}/q_o
3D FE, SU2 Cemented	5.72	5.49	4.75	3.77	0.96	0.83	0.66
Test data Cemented	6.33	6.17	4.26	3.25	0.97	0.67	0.52
3D FE, SU2 Uncemented	4.25	4.10	3.60	2.91	0.96	0.85	0.69
Test Data Uncemented	4.15	2.59	1.91	0.82	0.62	0.46	0.19

EFFECT OF LOAD INCLINATION ON SOIL DOMAIN

The 3D finite element predictions of the distribution of displacement and plastic zones in the carbonate sand beneath the footing are now considered. These distributions help to understand what happens in the soil immediately below and around the footing when it is subjected to inclined loads.

Figure 11 shows that for a footing subjected to vertical load, the largest displacements occur in a small zone immediately beneath the footing. The displacements are aligned essentially in the vertically downward direction. Figure 12 shows that for a footing subjected to inclined load, the largest displacements are still concentrated in a small zone immediately beneath the footing. However, the displacements are aligned essentially in the direction of applied load in the plane of load application. The displacement vectors show some small outward movement of the soil immediately ahead of the footing at larger load inclinations. This is consistent with experimentally observed behaviour. It was also observed that when inclined load is applied, the plastic zone in the plane of load application immediately below the footing is shifted in the direction of the applied load (not shown). The effects in the soil domain predicted by SU2 model appear to be logical and consistent with intuitive reasoning.

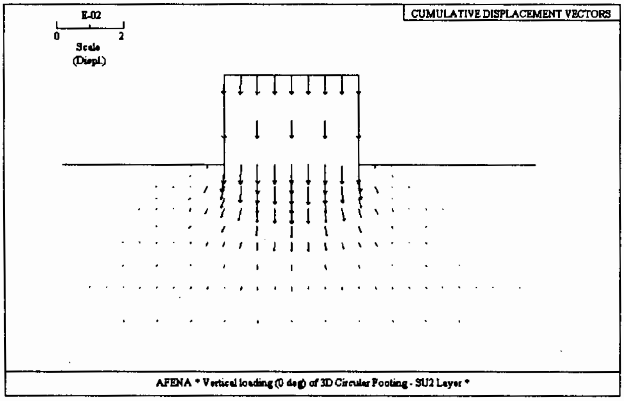

Fig 11. Cumulative nodal displacement vectors under vertical load

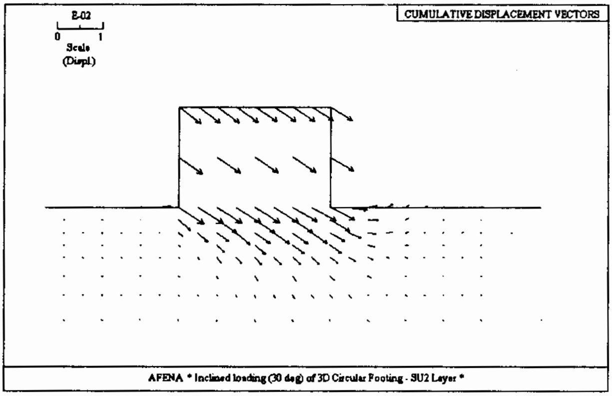

Fig 12. Cumulative nodal displacement vectors under inclined load

DISCUSSION AND LIMITATIONS OF STUDY

Finite element models generally assume continuum behaviour for the domain. However in the prototype experimental footings, the soil exhibited cracks at the pull-side or the side from which the inclined load was directed away from during the experiment. This was particularly true at higher load inclinations. This substantially lowered the mobilized load of the experimental footing at any displacement for inclined loads. If cracks could be simulated in the semi-analytical elasto-plastic finite element routine, it is quite likely that substantial improvements could occur in the quantitative agreement between the predicted response and the experimental data. Cracks have been successfully simulated by numerically incorporating shear band effects in finite elements. However, the use of shear bands in finite elements is quite complex even for simple problems. At this stage, it is difficult to visualize the use of shear bands in a complex three dimensional elasto-plastic finite element routine using a critical state soil model. It is also important that the soil-footing interface behaviour under inclined loading may be significantly different than that predicted by the SU2 model. This may be taken into consideration by incorporating interface elements in the finite element routine. However, interface elements have been frequently observed to give rise to numerical instabilities.

A second approach to improve the quantitative agreement between the predicted finite element response and the experimental footing data would be to attribute zero stiffness in an element once it exhibits tensile stresses. This is a crude mechanism used for simulating a tensile crack in a finite element domain problem. The effects of incorporating these approaches on the load-displacement response of footings could be investigated in the future.

CONCLUSION

A semi-analytical three-dimensional elasto-plastic finite element routine was used to predict the load-displacement response of axisymmetric footings on carbonate sands subjected to inclined loads. A constitutive model developed by the first author was used to simulate the stress-strain behaviour of carbonate sands. It was observed that the predicted load-displacement response of circular footings on carbonate sands subjected to inclined loads was in excellent qualitative agreement with the experimental data obtained for prototype footings. The mechanism of deformation under the footing obtained from finite element analysis agrees with intuitive reasoning. However, the qualitative agreement of the predicted load-displacement response of the footing with experimental data, particularly at larger load inclinations, was not as expected. At larger load inclinations, the predicted load response at any displacement was generally higher then that observed in experiments. This was found to be true for footings resting both on cemented and un-cemented carbonate sands. More studies need to be undertaken to investigate the cause of lack of quantitative agreement of the predicted response with the experimental data.

There are few three dimensional finite element studies of axisymmetric footings subjected to inclined loads. This is particularly true where the underlain soil response is characterised using a critical state model. This paper describes the tools and mechanisms developed for undertaking such a study. This study could be easily extended to investigate the behaviour of circular footing subjected to moments in addition to inclined loads. Such a situation is more likely for footings in offshore conditions. The analyses of circular footings resting on compressible medium subjected to combined loads are necessary for the development of rational design criteria for such footings. Such a development will significantly contribute to the economic and rational design of offshore oil platforms and their foundations.

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NOTATIONS

λ	plastic compression
κ	elastic rebound parameter
ν	Poisson's ratio
ϕ'	friction angle
e	void ratio
M	critical state ratio
p'	mean isotropic stress
p'_o	pre-consolidation pressure
p'_{cs}	mean pressure on the critical state line
q	deviator stress
q_o	bearing pressure under purely vertical load
q_{av}	traction mobilised
r	spacing ratio
$(u_r, u_z, u_\theta)_j$	nodal displacements of wedge j
$(U_r, U_z, U_\theta, Q, F)_k$	nodal forces applied to wedge j