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COEFFICIENT METHOD OF ANALYSIS FOR OCTAGONAL SLAB

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ABSTRACT: The paper describes the numerical analyses of octagonal slabs with few selected support conditions. Based on the availability of middle strip, numerical analyses described in this paper have been divided into two sets. In the first set, where middle strip exists, middle strip moments and slab centre deflections have been compared with that of rectangular slabs to identify the range of parameters when octagonal slab can be simplified as rectangular slab. A slab where middle strip does not exist belongs to the second set. Finally results of FE analyses have been utilised to formulate coefficient tables to compute the moment and deflection of octagonal slabs. Simplified method has also been described where octagonal slab can be analysed as a rectangular slab.

KEYWORDS: Finite element analysis, support cases, octagonal slabs, coefficient method, and deflection.

INTRODUCTION

In designing the rectangular slabs there are convenient design methods available to users to obtain the slab strip moments (ACI, 1963) and methods are also available to compute the deflection of such slabs (Ahmed and Chowdhury, 1999). Such methods are not available for analysis and design of octagonal slabs. From the support conditions rectangular slabs are classified into nine categories (ACI, 1963). This number is dependent on the number of supporting edges of the slab. Considering the same Octagonal slabs can be classified in to a large number of categories. Instead of going through all the cases the paper aims to identify a few number of selected cases that are most likely to occur in building structures. In addition to that it is of importance to identify cases where a octagonal slab can be simplified as a rectangular slab and when it must be analysed as a octagonal slab. The present paper identifies such cases and also presents a tabular form of coefficients to compute the design moments and deflection of octagonal slabs with few selected support conditions.

FE ANALYSES

Selection of parameters

To study the behaviour of octagonal slabs in addition to the span ratio (a/b); followings have been considered:

- Support conditions
- a_1/a and b_1/b ratios (see Figure 1 for definition of a_1, a, b_1 and b)

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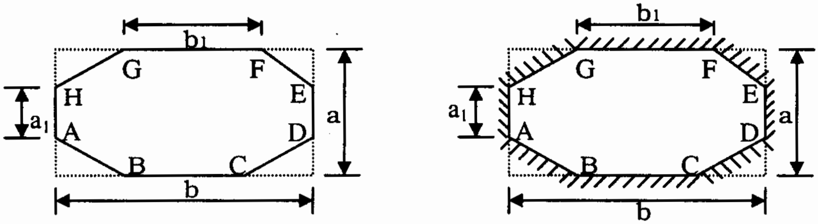
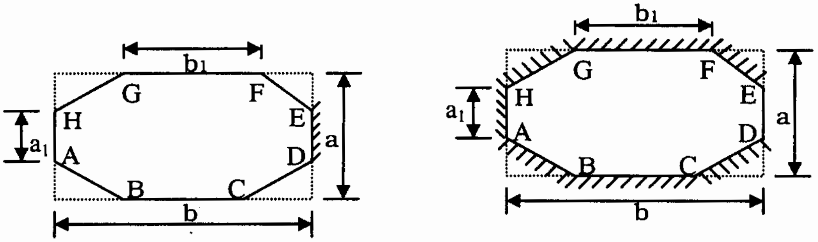
Support condition

It is known that there are nine possible support conditions for rectangular slabs. Number of support cases increase rapidly with increasing number of slab edges. Thus it became impossible to cover up the full set of possible cases in the present research scheme. Only four support cases are considered in the present paper (Case A means all edges simply supported; Case B means all edges fixed; Case C means only one edge fixed while others simply supported and Case D means only one edge simply supported while others fixed as shown in Figure 1).



Case 1 (all edges simply supported) *Case 2 (all edges are fixed)*



Case 3 (Except DE all edges are simply supported) *Case 4 (except DE all edges are fixed)*

Fig 1. Selected support condition

a_1/a and b_1/b ratio

As mentioned in the introduction two sets of analyses have been selected. To conduct each set of analyses a certain range of parameters has been selected as shown in Table 1.

For octagonal slabs belonging to set-1 middle strip moment and slab centre deflection to be compared with that of the rectangular slab. Using results of set-1 and that of set-2 coefficients can be obtained so that moment and deflection can be calculated by using these coefficients using simplified equations.

Table 1. Selected a_1/a and b_1/b ratios for set-1 and set-2

	Set-1		Set-2	
Span ratio (a/b)	a_1/a	b_1/b	a_1/a	b_1/b
0.50	0.50	0.50	0.00	0.00
	0.75	0.75	0.25	0.25
	0.90	0.90	0.40	0.40
0.75	0.50	0.50	0.00	0.00
	0.75	0.75	0.25	0.25
	0.90	0.90	0.40	0.40
1.0	0.50	0.50	0.00	0.00
	0.75	0.75	0.25	0.25
	0.90	0.90	0.40	0.40

COMPUTATION OF DESIGN MOMENTS FROM FE RESULTS

To compute the middle strip moments of octagonal slabs belonging to the first set (set-1) using FE results method described by Ahmed and Chowdhury (1999) have been followed. To calculate moments of octagonal slabs belonging to the second set (set-2) from the results of EF model, since middle strip (a/2 or b/2) is not available, different approach is essential that is described below.

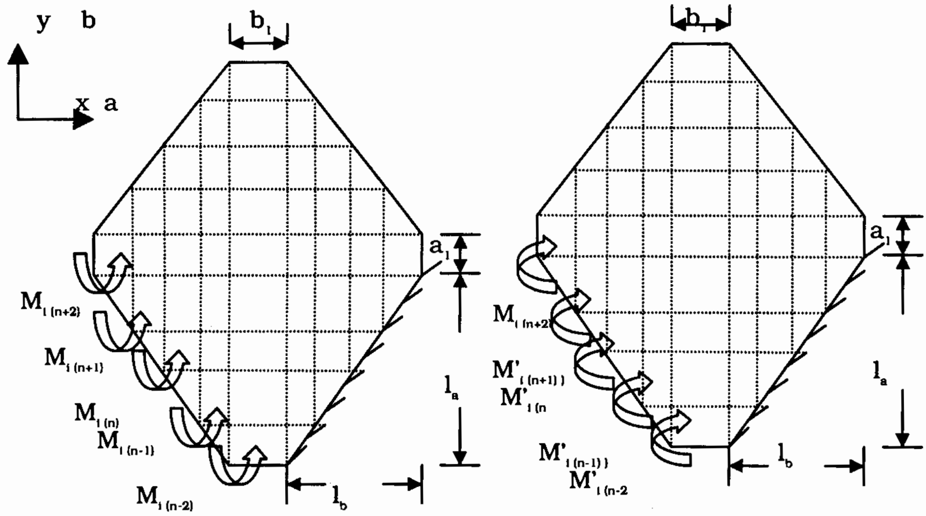
Figure 2 shows the procedure to calculate moment of inclined portion of octagonal slab. Moments of $M_{i(n-2)}$, $M_{i\,(n-1)}$, $M_{i(n)}$, $M_{i(n+1)}$, $M_{i(n+2)}$ and $M^3_{i(n-2)}$, $M^3_{i\,(n-1)}$, $M^3_{i(n)}$, $M^3_{i(n+1)}$, $M^3_{i(n+2)}$ of Figures 2(a), 2(b), 2(c), 2(d) are to be calculated using the method described by Ahmed and Chowdhury (1999). From Figure 2 (a), negative support moment in short direction (M_{ia}^-) can be obtained by averaging them over the horizontal portion (l_b) of the inclined part of octagonal slab, as the FE results are per element width. Thus approximately:

$$M_{ia}^{-} = \frac{M_{i(n-2)} + M_{i(n-1)} + M_{i(n)} + M_{i(n+1)} + M_{i(n+2)}}{l_b} \tag{1}$$

Similarly as shown in Figure 2 (b), support moment in long direction (M_{ib}^-) can be obtained by averaging them over the horizontal portion (l_a) of the inclined part of octagonal slab, as the FE results are per element width. Thus approximately:

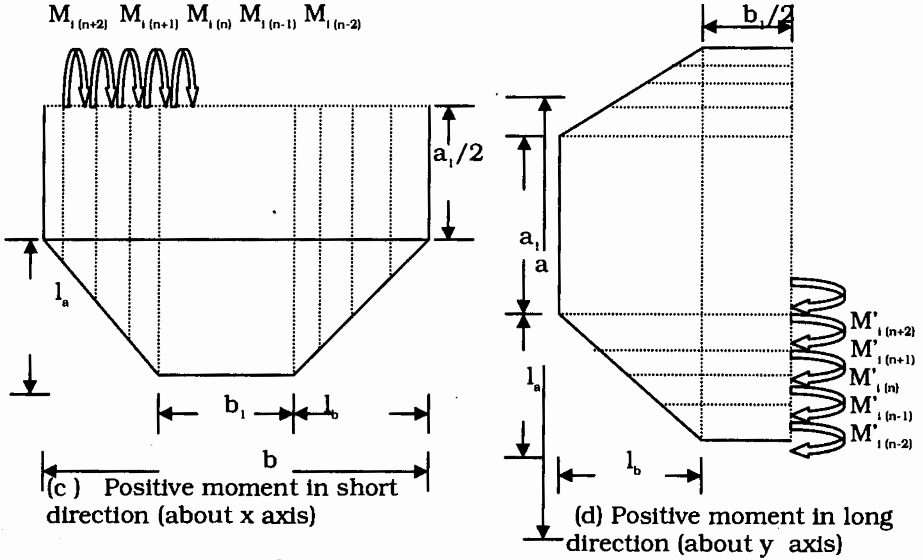
$$M_{ib}^{-} = \frac{M'_{i(n-2)} + M'_{i(n-1)} + M'_{i(n)} + M'_{i(n+1)} + M'_{i(n+2)}}{l_a} \tag{2}$$

Similarly from Figure 2 (c) approximately:



(a) Negative moment in short direction (about x axis)

(b) Negative moment in long direction (about y axis)



(c) Positive moment in short direction (about x axis)

(d) Positive moment in long direction (about y axis)

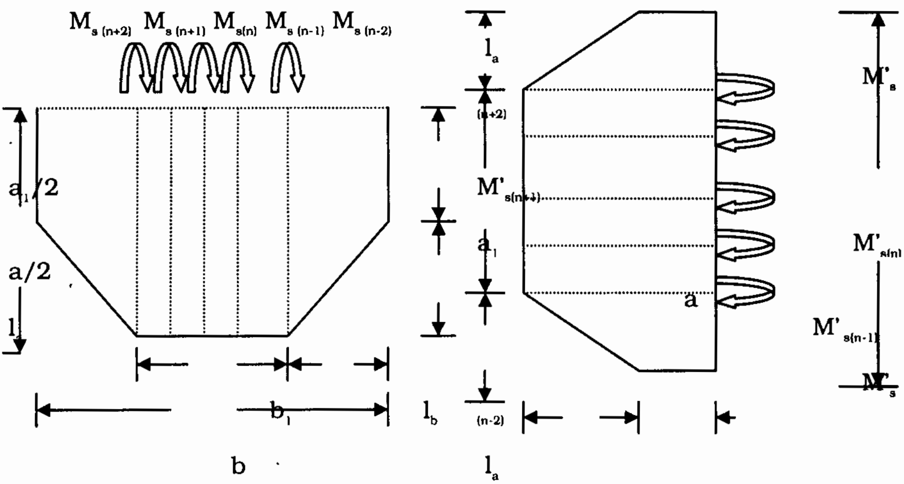
Fig 2. Calculation of moments in the inclined portion of octagonal slab

$$M_{ia}^{+}=\frac{M_{i(n-2)}+M_{i(n-1)}+M_{i(n)}+M_{i(n+1)}+M_{i(n+2)}}{l_b} \qquad (3)$$

Similarly from Figure 2 (d) approximately:

$$M_{ib}^{+}=\frac{M'_{i(n-2)}+M'_{i(n-1)}+M'_{i(n)}+M'_{i(n+1)}+M'_{i(n+2)}}{l_a} \qquad (4)$$

For straight portion of octagonal slab, positive moment in short direction can be obtained by averaging them over the straight portion (b_1).



(a) Positive moment in short direction (about x axis)

(b) Positive moment in long direction (about y axis)

Fig 3. Calculation of moments of straight portion of octagonal slab

Thus from Figure 3 (a) approximately:

$$M_{sa}^{+}=\frac{M_{i(n-2)}+M_{i(n-1)}+M_{i(n)}+M_{i(n+1)}+M_{i(n+2)}}{b_1} \qquad (5)$$

From Figure 3 (b) approximately:

$$M_{sb}^{+}=\frac{M'_{i(n-2)}+M'_{i(n-1)}+M'_{i(n)}+M'_{i(n+1)}+M'_{i(n+2)}}{a_1} \qquad (6)$$

Similar method is applied for evaluation of the edge moments of straight portion.

Instead of computing moments, it is desirable to compute moment coefficients. It is possible to compute the coefficients for differential cases and span ratios as follows:

$$M^{-}_{ia,FE} = C^{-}_{ia,FE} W l^{2}_{ea} \tag{7}$$

$$C^{-}_{ia,FE} = \frac{M^{-}_{ia,FE}}{W l^{2}_{ea}} \tag{8}$$

Where: $l_{ea} = \sqrt{1/6\left(a_1^2 + 2l_{2a}^2 + 2l_{3a}^2 + a^2\right)}$ (see Figure 4)

W = Total uniformly distributed load

Similarly C_{ib}^{-}, C_{ia}^{+}, C_{ib}^{+} can be obtained.

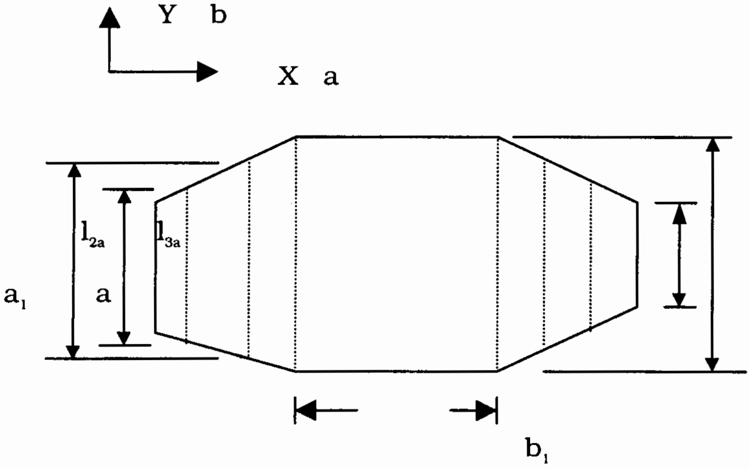


Fig 4. Calculation of equivalent length for calculating the moment of inclined portion of octagonal slab

For straight portion of octagonal slab:

$$M^{+}_{sa,FE} = C^{+}_{sa,FE} W a^{2} \tag{9}$$

$$C^{+}_{sa,FE} = \frac{M^{+}_{sa,FE}}{W a^{2}} \tag{10}$$

a = short span of slab

Similarly C_{sb}^{+}, C_{sa}^{-}, C_{sb}^{-} can be computed.

Verification of FE model for few selected cases

Mesh required for the study mentioned above demands a finer mesh. Before conducting the planned study of the work reported herein; it is essential to develop meshes that can incorporate all the mentioned a_1/a, b_1/b ratios. Although four noded shell elements can be used in most parts of the models; in some regions it is essential to use triangular elements as well. For the purpose of checking the accuracy of introducing triangular elements, some verification is essential. This is conducted by comparing described by Ahmed and Chowdhury (1999) with results of mesh described above for cases $a_1/a = b_1/b = 0.00$ and $a_1/a = b_1/b = 0.75$ for various a/b ratios, as these a_1/a and b_1/b in the two later cases closely represents or nearly represents rectangular slab system. Thus these should produce same or very close displacement at the centre when subjected to same load. Besides this the moments should be also same or very close and the moment coefficients can also be used for the purpose of verification. The results compared well and are described by Chowdhury (2000).

RESULTS OF FE ANALYSES

Effect of a_1/a and b_1/b ratio on middle strip moment

Figure 5 shows the variation of positive moment coefficient ratio in short and long direction with b_1/b ratio for different a_1/a ratios for span ratio (a/b) 0.50. Moment coefficient ratio for different selected support cases and a/b ratios has been obtained by dividing the middle strip moment coefficient (positive or negative) in short or long direction of octagonal slab of given a_1/a, b_1/b ratio by the same of rectangular slab in the respective direction; both having same a/b ratio and support condition. It has been observed from the results of analyses (Chowdhury, 2000) that the moment coefficient ratio (positive and negative) is either constant or increases with a_1/a and b_1/b ratio for all support conditions. It indicates that middle strip moment of octagonal slab is either equal or increases with a_1/a and b_1/b ratio than that of rectangular slab for any support condition. This is happening because load distribution is either constant or increases with a_1/a and b_1/b ratio. It can also be seen that the ratio of positive moment coefficient in long direction is either constant or decreases with a_1/a and b_1/b ratio for all support conditions of figures of variation of C_b^+ as load distribution is either constant or decreases. It can be observed from the variation of moment coefficient ratio (positive and negative), maximum increase in long direction positive and negative moment is 30% and 15% respectively and maximum decrease in long and short direction negative moment is 60% and 25% respectively, with respect to rectangular slab.

Effect of a_1/a and b_1/b ratio on moment of straight portion of the octagonal slab

Figure 6 show the variation of negative moment with b_1/b ratio for selected support cases, a_1/a ratio, and a/b ratio of 0.50. Analyses conducted by Chowdhury (2000) shows that variation of C_{sb}^+ for different span ratios shows that C_{sb}^+ increases gradually when b_1/b ratio varies from 0.00 to 0.25 but after that it does not change considering all the selected support cases and a/b ratios. Variation of C_{sa}^+ for different span ratios shows that C_{sa}^+ is almost constant with b_1/b ratio. Variation of C_{sa}^- ,C_{sb}^- for different span ratios, shows that C_{sa}^- ,C_{sb}^- increases sharply with b_1/b ratio considering all the selected support cases. Variation of C_{sa}^+, C_{sb}^+, C_{sa}^-, and C_{sb}^- with a_1/a ratio increases. This is occurred due to the fact that all the moment coefficients increases as load distribution of the straight portion increases with a_1/a and b_1/b ratio. It is not possible to obtain the variation of C_{sa}^+, C_{sa}^- for cases A, B, C and D of b_1/b ratio = 0.00 with a_1/a ratio as the straight portion of octagonal slab is not available. For the same reasons the variation of C_{sb}^+, C_{sb}^- for cases A, B, C and D of a_1/a = 0.0 with b_1/b ratio is not possible to obtain. Results presented by Chowdhury (2000) can be utilised for computing the moment of straight portion of octagonal slab by the help of equation 9.

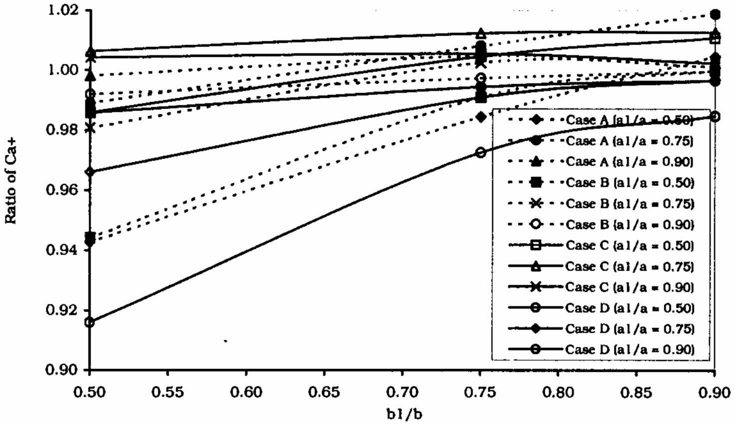


Fig 5. Variation of C_a^+ ratio with b_1/b ratio for a/b ratio 0.50

Effect of a_1/a and b_1/b ratio on moment of inclined portion of the octagonal slab

Analyses shows that (Figure 7), variation of (C_{ia}^+) for different span in short direction is either remain constant or slightly increases with b_1/b ratio, whereas coefficient of positive moment of inclined portion of octagonal slab (C_{ib}^+) in long direction decreases sharply (Chowdhury, 2000) with b_1/b ratio. Variation of (C_{ia}^-) shows that (Chowdhury, 2000) coefficient of negative moment of inclined portion of octagonal slab (C_{ia}^-) in short direction increases sharply with b_1/b ratio whereas coefficient of negative moment of inclined portion of octagonal slab (C_{ib}^-) in long direction decreasing slightly with b_1/b ratio. Variation of C_{ia}^+, C_{ia}^-, C_{ib}^+, C_{ib}^- for different span ratios shows that C_{ia}^+, C_{ia}^-, C_{ib}^+ decreases with a_1/a

ratio whereas coefficient of negative moment of inclined portion of octagonal slab (C_{lb}^-) in long direction increases with a_1/a ratio. . Results presented by Chowdhury (2000) can be utilised for computing the moment of inclined portion of octagonal slab by the help of equation 7.

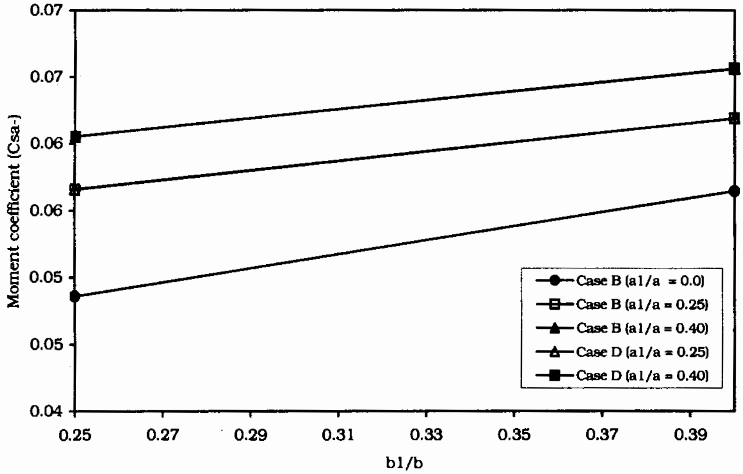


Fig 6. Variation of C_{sa}^- with b_1/b ratio for a/b ratio 0.50

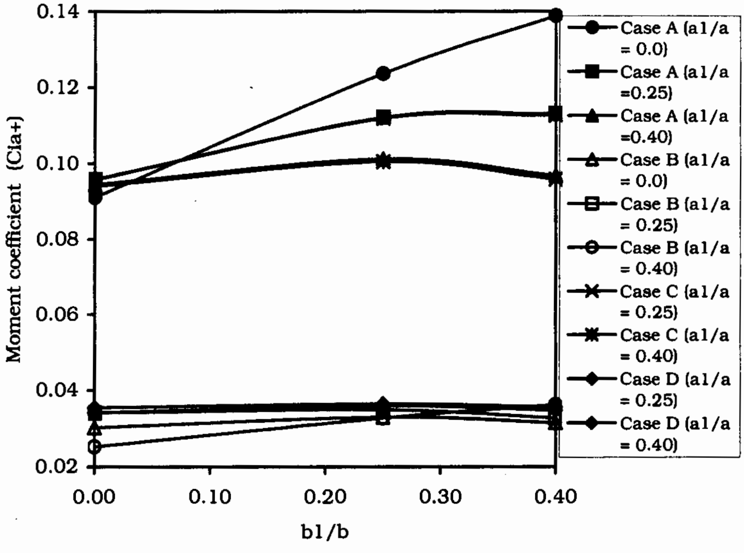


Fig 7. Variation of C_{ia}^+ with b_1/b ratio for a/b ratio 0.50

Effect of a_1/a and b_1/b ratio on deflection

Figure 8 shows the variation of deflection ratio with b_1/b ratio for different a_1/a ratios, selected support cases and 0.50 span ratio. Here deflection ratio has been defined for different selected support cases and span ratios, by dividing the slab centre deflection of octagonal slab of given a_1/a, b_1/b ratio by the same of rectangular slab considering same span ratio, support cases. Considering deflection of numerical analysis set-1, slab can be analysed as rectangular slab since maximum 10% variation of deflection have been observed. Slab centre deflection of octagonal slab set-1 can be directly computed knowing the slab centre deflection of rectangular slab using results presented by Chowdhury (2000). Figure 22 shows the variation of deflection with b_1/b ratio for different a_1/a ratios, selected support cases and 0.50 span ratio. From results presented by Chowdhury (2000) for set-1 and set-2 it can be seen that deflection ratio increases with a_1/a and b_1/b ratio as load distribution increases with a_1/a and b_1/b ratio. It is not possible to obtain the variation of deflection for cases C and D of a_1/a ratio = 0.00 with b_1/b ratio, as the straight portion of octagonal slab is not available.

DESIGN CONSIDERATIONS

Tables 2(a) and 2(b) lists the range of parameters when the positive and negative moments in long direction are significantly higher than that of rectangular slabs respectively obtained from the FE analyses. Tables 2(c) and 2(d) lists the range of parameters when negative moments in long and short direction are significantly lower than that of rectangular slabs respectively.

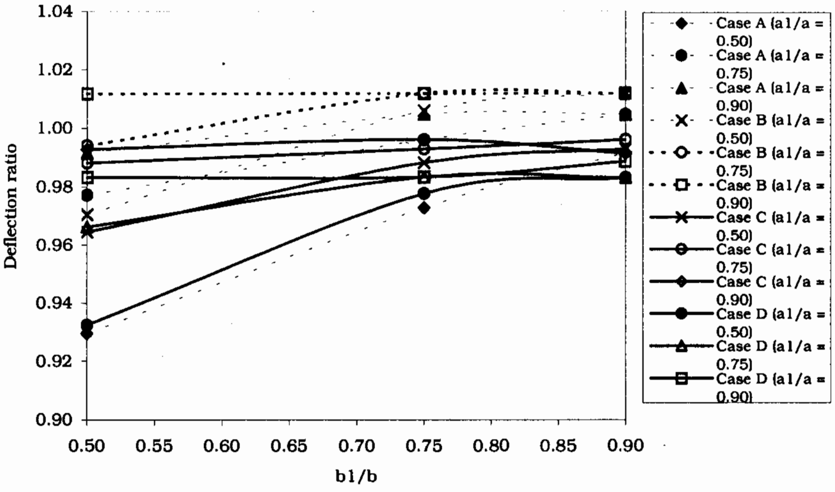


Fig 8. Variation of deflection ratio with b_1/b ratio for a/b ratio 0.50

Table 2(a) Range of parameters when positive moments in long direction are significantly greater than that of rectangular slab

Support case	a_1/a	b_1/b	Span ratio (a/b)
A	≤ 0.50	0.50 to 0.70	≤ 0.50
B	≤ 0.50	0.50 to 0.90	0.50 to 0.75
D	0.50 to 0.75	0.50 to 0.90	0.50 to 0.75

Table 2(b). Range of parameters when negative moments in long direction are significantly greater than that of rectangular slabs

Support case	a_1/a	b_1/b	Span ratio (a/b)
C	≤ 0.50	0.70 to 0.90	0.50 to 0.75

Table 2(c). Range of parameters when negative moments in long direction are significantly lower than that of rectangular slab

Support case	a_1/a	b_1/b	Span ratio (a/b)
B	0.50 to 0.90	0.50 to 0.90	0.50 to 0.75
	≤ 0.50	0.50 to 0.90	≤ 1.0
D	0.50 to 0.90	0.50 to 0.90	≤ 0.50
	≤ 0.50	0.50 to 0.90	0.75 to 1.0

Table 2(d). Range of parameters when negative moments in short direction are significantly lower than that of rectangular slab

Support case	a_1/a	b_1/b	Span ratio (a/b)
B	0.50	0.50 to 0.75	0.75 to 1.0
D	0.50	0.50 to 0.90	0.50 to 1.0

Octagonal slab with range of parameters shown in Tables 2a, 2b, 2c and 2d must be analysed as octagonal slab. Table 3 lists the range of parameters when slab can be analysed as rectangular slab with maximum variation in moment within 10%.

Middle strip moment of such type of octagonal slab can be directly computed knowing the middle strip moment of rectangular slab, if the slabs satisfy the above limits.

Finally using results of FE analyses, moment coefficient tables have been formed (Chowdhury, 2000) so that middle strip moments and deflection can be computed directly.

Table 3. Range of parameters when octagonal slab can be simplified as rectangular slab

Support case	a_1/a	b_1/b	Span ratio (a/b)
A	≤0.50	0.71 to 0.90	≤0.50
	0.75 to 0.90	0.50 to 0.90	≤0.50
	0.50 to 0.90	0.50 to 0.90	0.75 to 1.0
B	0.75 to 0.90	0.50 to 0.90	≤1.0
C	≤0.50	0.50 to 0.69	0.50 to 0.75
	0.75 to 0.90	0.50 to 0.90	0.50 to 0.75
	0.50 to 0.90	0.50 to 0.90	≤1.0
D	0.75 to 0.90	0.50 to 0.90	0.75 to 1.0

CONCLUSSION

The paper described the finite element analysis of octagonal slabs with four support cases are considered (all edges simply supported; all edges fixed; only one edge fixed while others simply supported; only one edge simply supported while others fixed). Equations have been formulated to obtain strip moments for such slabs from ANSYS results that are in terms of moment at a node per element. From the results of FE analysis it has been possible to identify when it is possible to analyse a slab as a rectangular slab. Besides this a set of coefficient tables have been prepared using results of FE analyses, that can be used to analyse only octagonal slab with mentioned support conditions.

REFERENCES

ACI Publication 318-63 (1963), "Building Code Requirements for reinforced Concrete" American Concrete Institute, Detroit.

Ahmed, B. and Chowdhury, S. R. (1999) "*Simplified calculation method for serviceability deflection of edge supported slabs*; Part 1: Modelling Slab Response" Journal of the Civil Engineering Division, IEB, Bangladesh, Vol. CE 27, No. 1, June1999, pp. 29-40.

Ahmed, B. and Chowdhury, S. R. (1999) "*Simplified calculation method for serviceability deflection of edge supported slabs*; Part 2: Coefficient method" Journal of the Civil Engineering Division, The Institution of Engineers, Bangladesh, Vol. CE 27, No. 1, June1999, pp. 41-57.

Chowdhury, S. R. (2000) "Numerical modelling of edge supported slab response", M.Sc. thesis, Department of Civil Engineering, BUET, Dhaka-1000, Bangladesh.

ANSYS 5.3, Operations Guide, SASIP Inc, 201 Johnson Road, Houston, PA 15342-1300.

ANSYS 5.4, Operations Guide, SASIP Inc, 201 Johnson Road, Houston, PA 15342-1300.