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AN ELASTO-PLASTIC FOOTING MODEL FOR CIRCULAR FOOTINGS RESTING ON CARBONATE SAND AND SUBJECTED TO INCLINED LOAD

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ABSTRACT: There are currently several elastic-perfectly plastic footing models for the rational determination of bearing capacity of strip footings resting on silica sand and subjected to inclined load. The applicability of these footing models for determination of the bearing capacity of offshore circular footings resting on carbonate sand and subjected to inclined load is investigated. A more suitable elasto-plastic hardening footing model has been proposed for the rational determination of the bearing capacity of offshore circular footings resting on compressible carbonate sands and subjected to inclined load. The bearing capacities obtained for circular footings using different models and approaches, including finite element analysis, are compared with each other, as well as with available experimental data on model scale footings. It was observed that the proposed footing model based on elasto-plastic strain hardening assumption shows good agreement with experimental data. It also provides a rational framework for determination of the bearing capacity of circular footings resting on carbonate sand and subjected to inclned load.

KEYWORDS: Bearing pressure, circular footing, carbonate sand, footing model, elasto-plastic, inclined load.

INTRODUCTION

Oil platforms are often constructed in offshore areas. The foundations of these structures rest on the ocean bed, which frequently consist of highly compressible organic sediment layers, known as carbonate sands. These foundations are mostly circular in shape, and they are subjected to both horizontal and vertical loads due to the environmental effects on the superstructure. Three-dimensional finite element routines, using complex elasto-plastic constitutive laws, are necessary to correctly determine the load-displacement response of such foundations under inclined loads. However, in most practical situations, such analyses may neither be feasible nor practicable. Thus, alternative analytical methods are needed to determine the bearing capacity of circular foundations under inclined load. In this paper, such a new method is proposed for analytical determination of the load-displacement response of circular foundations, resting on carbonate sands and subjected to inclined loads.

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In this paper, the analytical methods currently available to determine the bearing capacity of footings subjected to inclined load is first discussed. The applicability of the classical bearing capacity analysis for footings on carbonate sand is then investigated. Next, the suitability of elastic-perfectly plastic models for determination of the bearing capacity of footings on carbonate sand is studied. Finally, an analytical method for determination of the bearing capacity of circular footings resting on carbonate sand and subjected to inclined load is proposed. The bearing capacities obtained for circular footings using different approaches, including finite element analysis, are compared with each other, as well as with available experimental data on model scale footings.

CLASSICAL ANALYSIS

The bearing capacity q_{av} of a shallow footing of any shape resting on a rigid perfectly plastic solid subjected to inclined load is usually described by the following equation:

$$q_{av} = cN_c\zeta_c\zeta_{ci} + \sigma'_v N_c N_q \zeta_q \zeta_{qi} + \frac{1}{2}\gamma' B N_\gamma \zeta_\gamma \zeta_{\gamma i} \tag{1}$$

In equation (1), c is the cohesion, σ'_v is the effective surcharge pressure, B is the footing width and γ' is the effective unit weight of the soil. N_c, N_q, N_γ are the usual bearing capacity factors, ζ_c, ζ_q, ζ_γ are correction factors for foundation shape and ζ_{ci}, ζ_{qi}, $\zeta_{\gamma i}$ are correction factor for load inclination. Hansen (1970) and Meyerhoff (1953, 1963) proposed empirical expressions for the inclination and shape factors. For compressible soils, Terzaghi (1943) proposed a reduction of the cohesion c and friction angle ϕ to c^* and ϕ^* as follows:

$$c^* = 0.67c \tag{2}$$

$$\phi^* = \tan^{-1}(0.67\tan\phi) \tag{3}$$

These were used to compute the reduced value of cohesion and friction angle for carbonate sand. These are given in Table 1.

Table 1. Mohr-Coulomb strength parameters for carbonate sand

Carbonate sand	c (kPa)	ϕ (degrees)	c^* (kPa)	ϕ^* (degrees)
Cemented (Yeoh, 1996)	400.0	23.0	266.7	15.8
Uncemented (Huang, 1994)	0.0	39.0	0.0	28.4

The bearing capacity of a model-scale footing (Pan, 1999) was computed using equation (1) and the shape and inclination factors proposed by Hansen (1970) and Meyerhoff (1963). The computed bearing capacities were compared with experimental data. These comparisons are presented in Tables 2 and 3.

Table 2. Comparison with data for footing on artificially cemented carbonate sand

Inclination Method	0° q_{av} (MPa)	10° q_{av} (MPa)	20° q_{av} (MPa)	30° q_{av} (MPa)	10° $\left(\frac{q_{av}}{q_o}\right)$	20° $\left(\frac{q_{av}}{q_o}\right)$	30° $\left(\frac{q_{av}}{q_o}\right)$
Hansen	4.51	3.40	2.53	1.85	0.75	0.56	0.41
Meyerhoff	4.42	3.54	2.84	2.27	0.80	0.64	0.51
Experiment	6.33	6.17	4.26	3.25	0.97	0.67	0.52

Table 3. Comparison with data for footing on uncemented carbonate sand

Inclination Method	0° q_{av} (MPa)	10° q_{av} (MPa)	20° q_{av} (MPa)	30° q_{av} (MPa)	10° $\left(\frac{q_{av}}{q_o}\right)$	20° $\left(\frac{q_{av}}{q_o}\right)$	30° $\left(\frac{q_{av}}{q_o}\right)$
Hansen	1.27	0.96	0.68	0.40	0.76	0.54	0.32
Meyerhoff	1.09	0.86	0.68	0.54	0.79	0.62	0.49
Experiment	4.15	2.59	1.91	0.82	0.62	0.46	0.19

q_{av} is traction (pressure) under inclined load and q_o is the bearing capacity (pressure) under vertical load. For the model-scale footing, the traction mobilised at 10% resultant displacement (normalized by the footing diameter) was adopted as the nominal bearing capacity of the footing. It was observed that the conventional bearing capacity equations predict significantly lower bearing resistance for footings on uncemented and artificially cemented carbonate sand.

PLASTICITY ANALYSIS

In recent times, researchers such as Butterfield and Gottardi (1994), Gottardi and Butterfield (1994, 1993), Nova and Montrasio (1991) and Georgiadis and Butterfield (1988) have used a new plasticity based approach to determine the bearing capacity of footings subjected to inclined load. Their proposed method is described in this section.

A strip footing of width B subjected to an inclined load Q at zero eccentricity is considered. The resultant applied load Q, with inclination θ, is assumed to cause bearing capacity failure of the footing. Q is transformed into its statically equivalent components of vertical load V and horizontal load H. The interaction diagram for the failure locus of the footing is represented by a non-linear equation, expressed as follows:

$$F(V,H,V_M) = \frac{H}{mV_M} - \frac{V}{V_M}\left(1-\frac{V}{V_M}\right)^{\beta} = 0 \qquad (4)$$

Georgiadis and Butterfield (1988) proposed equation (4) for footings on silica sand. In equation (4), V_M is the bearing capacity under purely vertical load, and m and β are model parameters; m is the tangent of the soil-footing interface friction angle and is defined by the following equation:

$$H = mV \qquad (5)$$

The resultant bearing resistance for inclined loading of footings was computed using equation (4) for $\beta = 1.0$ and $\beta = 0.95$. This is the range of values for β proposed by the authors. Bearing capacity was defined as the traction mobilised at a resultant displacement of 10% of the footing diameter. The results obtained were compared with experimental data of the model scale footing. The results are presented in Table 4.

Table 4. Comparison of plasticity analysis with test data

Method \ Inclination	10° $\frac{q_{av}}{q_o}$	20° $\frac{q_{av}}{q_o}$	30° $\frac{q_{av}}{q_o}$
Georgiadis and Butterfield (1988)	0.79	0.59	0.33
Nova and Montrasio (1991)	0.78	0.56	0.31
Experimental data (Cemented sand)	0.97	0.67	0.52
Experimental data (Uncemented sand)	0.62	0.46	0.19

In Table 4, q_{av} is the bearing resistance (traction) under inclined load and q_o is the bearing capacity under vertical load. It was observed that the change in traction with load inclination predicted by plasticity model does not compare well with the experimental data for model-scale footings resting on uncemented and artificially cemented carbonate sand.

FINITE ELEMENT ANALYSIS

Taiebat (1999) developed a semi-analytical three-dimensional finite element procedure for elasto-plastic solids. A critical state constitutive model named the SU2 (Islam, 2000) was incorporated in the semi-analytical three-dimensional finite element procedure named AFENA (Carter and Balaam, 1995). The stress-strain response of the carbonate sand was characterized by the constitutive model. The predicted traction q_{av} mobilised at 10% resultant displacement and its change with load inclination are compared with the experimental data and presented in Table 5.

It was observed that satisfactory predictions of the pressure-displacement response of circular footings subjected to inclined load are obtained using finite element analysis for footings on artificially cemented sand. However, the predictions of the inclined load response for footings on uncemented sand, are not entirely satisfactory.

Table 5a. Comparison of 3D finite element predictions with test data

Inclination Method	0° q_o (MPa)	10° $(q_{av})_{10\%}$ (MPa)	20° $(q_{av})_{20\%}$ (MPa)	30° $(q_{av})_{30\%}$ (MPa)
3D FE Cemented	5.72	5.49	4.75	3.77
Test data Cemented	6.33	6.17	4.26	3.25
3D FE Uncemented	4.25	4.10	3.60	2.91
Test Data Uncemented	4.15	2.59	1.91	0.82

Table 5b. Comparison of 3D finite element predictions with test data

Inclination Method	10° $\left(\frac{q_{av}}{q_o}\right)_{10\%}$	20° $\left(\frac{q_{av}}{q_o}\right)_{10\%}$	30° $\left(\frac{q_{av}}{q_o}\right)_{10\%}$
3D FE Cemented	0.96	0.83	0.66
Test data Cemented	0.97	0.67	0.52
3D FE Uncemented	0.96	0.85	0.69
Test Data Uncemented	0.62	0.46	0.19

ELASTO-PLASTIC FOOTING MODEL FOR CARBONATE SAND

General

No model has yet been proposed to predict the bearing response for inclined loading of cicular footings on carbonate sand. This is because limited experimental and numerical data is available for footings on carbonate sand subjected to inclined load. Footing models with perfect plasticity assumptions are unable to predict the behaviour of footings on carbonate sand subjected to inclined load. In this paper, a strain hardening footing model has been proposed to predict the response of circular footings on carbonate sand, subjected to inclined load.

The proposed model idealises the pressure-displacement curve at any inclination to be bilinear as shown in Fig. 1. The linear elastic (assumed analogous to the elastic rebound curve for soil) and linear plastic part (assumed analogous to the plastic consolidation curve for soil) of the idealised bilinear bearing pressure curve, shown in Fig. 1, may be defined by the following equations:

$$V_o' = N + K_{ve}v_e' \qquad (6)$$

$$V_o' = \Gamma + K_{vep}v' \qquad (7)$$

In equations (6) and (7), V_o', v_e' and v' are respectively the normalized bearing pressure (normalized by a reference pressure (q_{ref}), normalized elastic vertical displacement and normalized total vertical displacements of the footing subjected to vertical load. All footing displacements are normalized by the footing diameter. The stiffness K_{ve} is given by the slope of the linear elastic part and the stiffness K_{vep} is given by the slope of the linear plastic part, as shown in Fig. 1. The constant N in equation (6) is the pressure represented by the intercept of the unload-reload line with the ordinate or V_o' axis. The constant Γ in equation (7) is the pressure at the intersection of the linear plastic part of the bearing pressure curve with the ordinate or V_o' axis.

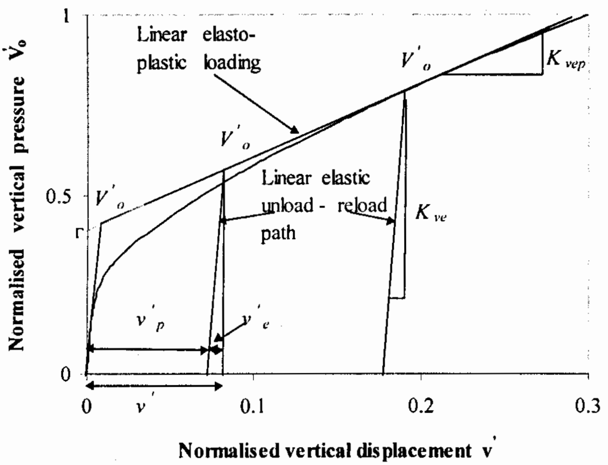

Fig 1. Idealised bilinear vertical pressure-displacement curve

Elastic Response

The initial elastic response of a footing on carbonate sand may be described by the following equations:

$$V_e' = K_{ve}v_e' \qquad (8)$$

$$H_e' = K_{he}h_e' \qquad (9)$$

V_e and H_e are respectively the elastic component of the applied vertical and horizontal loads (force), V_e' and H_e' are the corresponding tractions (pressure). The traction is normalized by reference pressure q_{ref}. Elastic footing displacements v_e and h_e are normalised by footing diameter B to give normalised displacements v_e' and h_e', respectively. K_{ve} and K_{he} are respectively the vertical and horizontal elastic stiffness of the footing.

The elastic load-displacement relation for a footing as given by Poulos and Davis (1974) are as follows:

$$V_e = \left[\frac{4GR}{1-\nu}\right] v_e \quad (10)$$

$$H_e = \left[\frac{32GR(1-\nu)}{7-8\nu}\right] h_e \quad (11)$$

In equations (10) and (11), V_e is the vertical and H_e is the horizontal elastic component of the inclined load, G is the elastic shear modulus of the soil, ν is the Poisson's ratio of the soil and R is the footing radius.

Using equations (8), (9), (10) and (11), K_{ve} and K_{he} may be obtained from the elastic shear modulus G and Poisson's ratio ν of the underlying soil as follows:

$$K_{ve} = \frac{8}{\pi}\frac{1}{q_{ref}}\frac{G}{1-\nu} \quad (12)$$

$$K_{he} = \frac{64G}{\pi}\frac{1}{q_{ref}}\frac{1-\nu}{7-8\nu} \quad (13)$$

Hardening Function

The hardening function defines the change in the vertical bearing pressure V_o' (the hardening parameter) with plastic vertical displacements v_p'. V_o' may be considered analogous to the preconsolidation pressure p_o', and v_p' analogous to plastic volumetric strains $d\varepsilon_v^p$, of a soil. dv_p' is the difference between the incremental total vertical displacement dv' and incremental elastic vertical displacement dv_e'. Thus,

$$dv_p' = dv' - dv_e' \quad (14)$$

Differentiating equation (8) and (9), and using the incremental relation (14), the incremental hardening function for the footing is derived as follows:

$$dV_o' = K_{vp} dv_p' \tag{15}$$

where

$$K_{vp} = \frac{K_{vep} K_{ve}}{K_{ve} - K_{vep}} \tag{16}$$

K_{vp} is the plastic stiffness of the footing under vertical load.

Yield Locus

The yield locus for a footing subjected to inclined load defines the limit of its elastic response in $H' - V'$ space. Two separate yield loci have been defined in the footing model. The first yield locus is defined by a cap function, which represents the contours of equal plastic resultant displacement (Fig. 2) or vertical displacement (Fig. 3) in $H' - V'$ space. The yield cap shown in Fig. 3 is used in the current model for simplicity. It is approximated by an equation given as follows:

$$H'^2 = m^2 (V_o' - aV') \tag{17}$$

$$m = \tan \phi' \tag{18}$$

H' and V' are the normalised applied horizontal traction and vertical pressure respectively, ϕ' is the friction angle of the soil and a is a model parameter. The intersection of the yield cap with the V' axis as shown in Fig. 2 and Fig. 3 represents the bearing pressure V_o' of the footing under vertical load.

The yield caps proposed in the footing model fits quite well with the yield locus computed from 3D finite element analysis. When the applied footing traction represented by (H', V') touches the yield cap yield, the pressure-displacement response of the footing changes from linear elastic to linear elasto-plastic. The cap expands isotropically in $H' - V'$ space as a result of plastic vertical displacements v'. For constant load inclinations, the cap will expand indefinitely with increasing traction. Thus the footing is predicted to settle continuously with applied traction. This is in agreement with experimentally observed behaviour for circular footings on carbonate sand.

A deviator yield/failure function fixed in $H' - V'$ space is postulated for the footing. It is approximated by the following equation:

$$H' = mV' \tag{19}$$

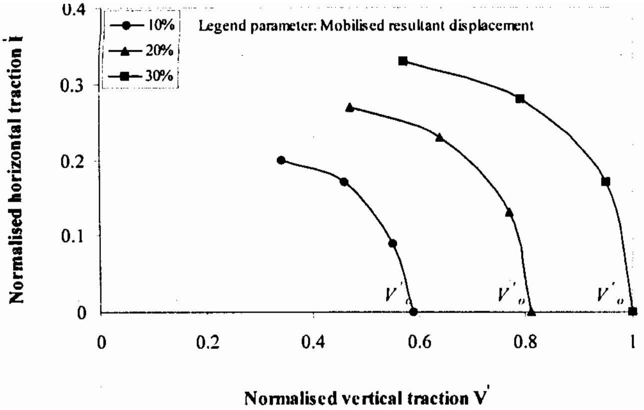

Fig. 2. Contours of equal plastic resultant displacement

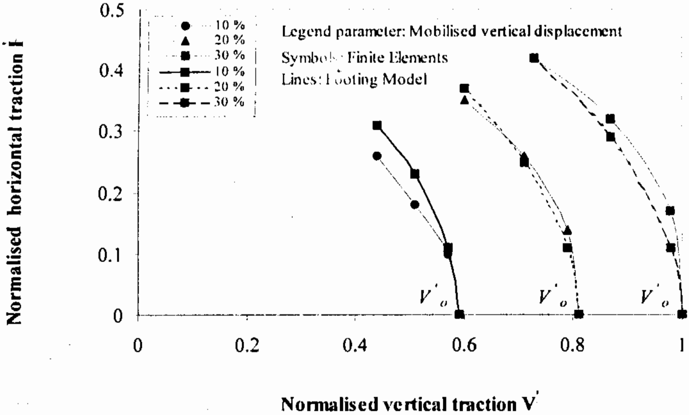

Fig. 3. Contours of equal plastic vertical displacement

Figure 4 shows the cap and deviator yield locus in $H'-V'$ space. It is assumed that infinite horizontal displacements occur when the footing pressure (H', V') touches the deviator yield locus given. The footing then fails by sliding along the soil-footing interface. Experimental results of Pan (1999) show that large horizontal displacements occur when footings on carbonate sand are subjected to load inclination close to the friction angle of the sand.

Stress-Dilatancy and Plastic Potential

The relative magnitudes of the incremental plastic vertical and incremental plastic horizontal displacements of a footing subjected to inclined load, may be termed as the dilatancy ratio. The 3D finite element results show that dilatancy ratio of a footing on carbonate sand subjected to inclined load, may be expressed as a function of load inclination. This function is termed as the stress-dilatancy function. The integration of this function gives the plastic potential function. The gradient of the plastic potential function is used in the strain-hardening model to determine the relative magnitudes of the incremental plastic vertical and horizontal displacement of the footing.

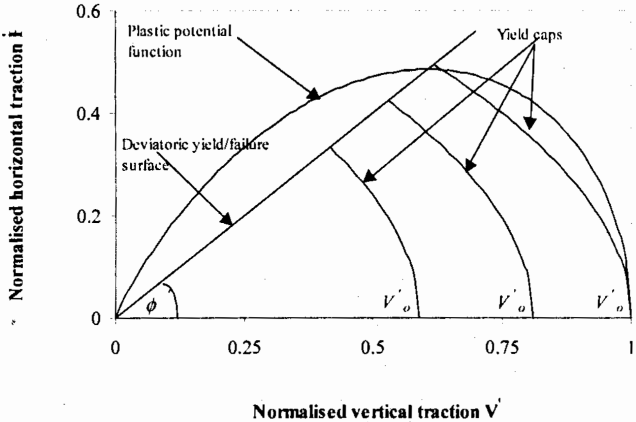

Fig. 4. Yield cap, failure locus and plastic potential function

The proposed stress-dilatancy function is as follows:

$$\frac{dv'_p}{dh'_p} = \frac{m^2 - n^2}{kn} \tag{20}$$

where

$$n = \frac{H'}{V'} = \tan\theta \tag{21}$$

$$m = \tan\phi' \tag{22}$$

ϕ' is the friction angle of the carbonate sand, θ is the angle of load inclination and k is a model parameter. Figure 5 plots the proposed stress-dilatancy function for k = 1 and compares it with predictions obtained for the model-scale footing from 3D finite element analysis. For k = 1, the stress-dilatancy relation can be integrated to obtain the equation for the plastic potential function as follows:

$$\left(\frac{H'}{mV'}\right)^2 + \ln\left(\frac{V'}{V'_o}\right)^2 = 0 \tag{23}$$

For $k \neq 1$, the following expression is obtained for the plastic potential function of the footing:

$$\ln\left\{1+k\left(\frac{H'}{mV'}\right)^2\right\}+\frac{2k}{k+1}\ln\left(\frac{V'}{V'_o}\right)=0 \qquad (24)$$

Figure 4 plots the plastic potential function given by equation (23).

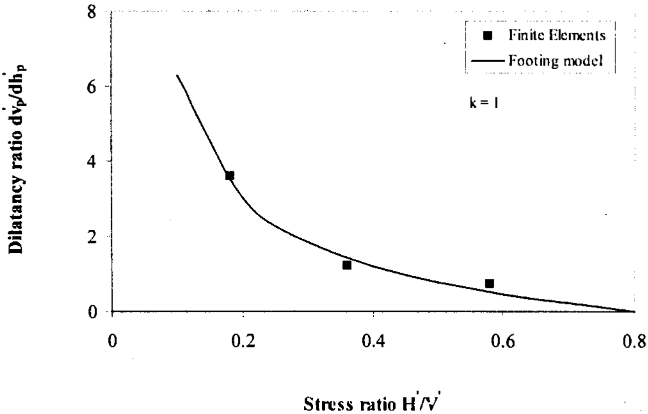

Fig. 5. Stress-dilatancy function

Plastic Hardening Modulus

The plastic hardening modulus defines the plastic stiffness of a circular footing resting on carbonate sand subjected to inclined load. It is given as follows:

$$H_{mod} = -\frac{\partial f}{\partial V'_o}\frac{\partial V'_o}{\partial v'_p}\frac{\partial g}{\partial V'_o} \qquad (25)$$

where, H_{mod} is the plastic hardening modulus, f is the yield function, g is the plastic potential function, V'_o is vertical bearing pressure. The value of $\frac{\partial V'_o}{\partial v'_p}$ may be obtained from equation (15), which in this case is equal to K_{vp}.

Incremental Plastic Displacement

The horizontal and vertical incremental plastic displacements of the footing for a given increment of applied horizontal and vertical traction (dH', dV') may be computed as follows:

$$dh'_p = d\lambda\frac{\partial g}{\partial H'} \qquad (26)$$

$$dv'_p = d\lambda\frac{\partial g}{\partial V'} \qquad (27)$$

where,

$$d\lambda = \frac{1}{H_{mod}}\left(\frac{\partial f}{\partial V'}dV' + \frac{\partial f}{\partial H'}dH'\right) \tag{28}$$

$d\lambda$ is a proportionality factor.

Load-Displacement Curve

The load-displacement curve for a circular footing subjected to inclined load may be computed using the simplified footing model. A value of 1.0 is assumed for the parameter *a*. Initial V_o' is the inflexion point of the bearing pressure curve under vertical load. Once initial V_o' and *a* are known, the initial cap yield locus, as defined by equation (17), is fixed in $H'-V'$ space. Given the friction angle of the underlying carbonate sand, the deviator yield locus given by equation (29), is also determined in $H'-V'$ space.

For stress states (H',V') on the yield cap, the magnitudes of the incremental plastic horizontal and vertical displacements of the footing are given by equations (26) and (27). The rate of hardening of the cap with plastic vertical displacements of the footing is defined by the hardening function given by equation (15). K_{vp} in equation (15) is determined from equation (16).

The incremental total vertical and horizontal displacements of the footing may now be computed for any given increment of vertical and horizontal traction as follows:

$$dh' = dh_e' + dh_p' \tag{29}$$

$$dv' = dv_e' + dv_p' \tag{30}$$

where, dh_e' and dh_p' are the incremental elastic and plastic horizontal displacements, and dv_e' and dv_p' are the incremental elastic and plastic vertical displacements, respectively, of the footing. The integration of applied incremental traction and corresponding incremental displacements gives the bilinear bearing pressure curve mobilized by a circular footing resting on carbonate sand and subjected to inclined load.

PREDICTIONS OF FOOTING MODEL

The bilinear pressure-displacement curves under inclined load for a model scale footing, 25mm in diameter, and resting on cemented sand (Pan, 1999), as well as that of a 25m diameter surface circular footing on normally consolidated sand were predicted, using the simple footing model. The model parameters selected for each case are given in Tables 6 and 7.

Table 6. Footing model parameters for model-scale footing

ϕ' (deg)	G (MPa)	ν	V'_o	K_{vep}	a	k	q_{ref} MPa
39.0	75.0	0.2	0.43	2.04	1.0	1.0	9.5

Table 7. Footing model parameters for 25m diameter footing

ϕ' (deg)	G (MPa)	ν	V'_o	K_{vep}	a	k	q_{ref} MPa
40.0	37.5	0.2	0.0	9.15	1.0	1.0	250

Figures 6 and 7 compare the pressure-displacement curve predicted by the footing model with the results of 3D finite element analysis. Reasonable agreement is observed. Figure 8 compares the predicted horizontal and vertical displacements with the results of 3D finite element analysis. Again a reasonable fit is obtained. Figure 9 compares the predicted load-displacement curves of a 25m diameter footing with results of 3D finite element analysis. The agreement is again satisfactory.

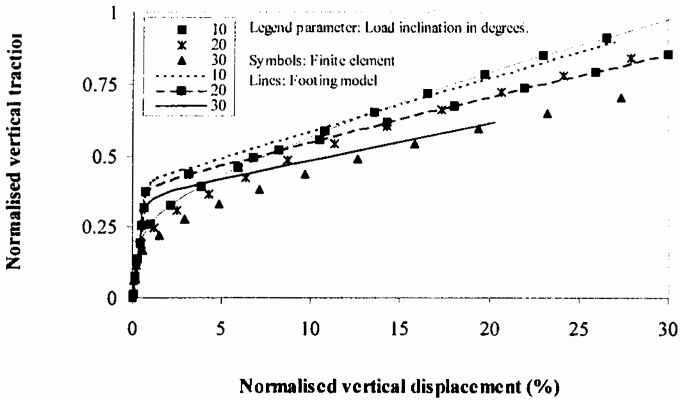

Fig. 6. Model prediction of vertical traction

DISCUSSIONS AND LIMITATIONS OF STUDY

Elasto-plastic models proposed for inclined loading of footings on silica sand were modified and an elasto-plastic footing model was proposed. The model was used to predict the response of circular footings on carbonate sand subjected to inclined load. It was observed that such a model could be used to fit the pressure-displacement curves of circular footings on carbonate sand subjected to inclined load at zero eccentricity. However, the simplified model is unable to predict the strongly non-linear elasto-plastic response even at small footing displacements. Elasto-plasticity needs to be introduced within the yield locus of the footing model to predict more accurately the inclined pressure-displacement response of circular footings on carbonate sand.

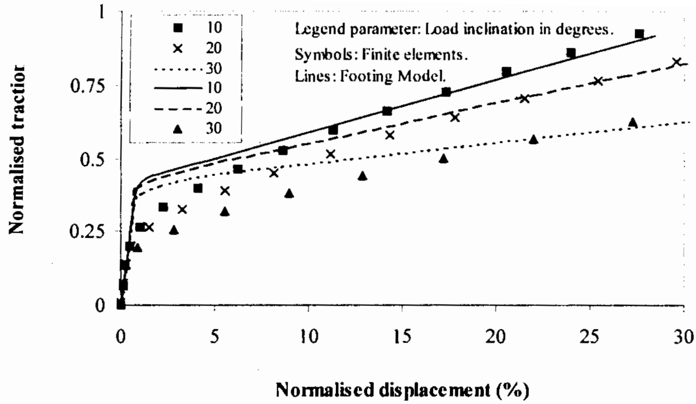

Fig. 7. Model prediction of resultant traction

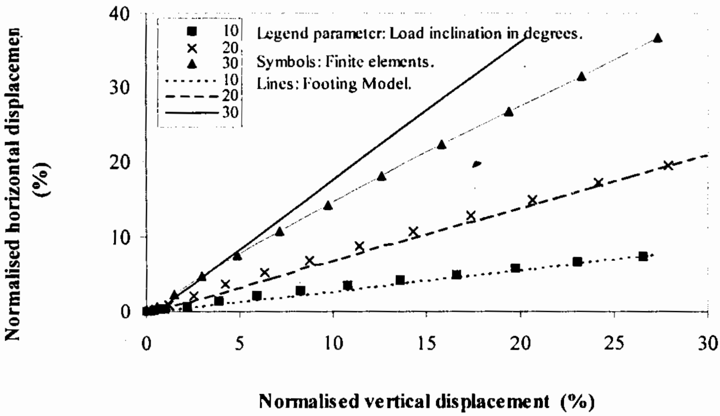

Fig. 8. Model predictions of displacements

CONCLUSION

The assumption that the soil is a rigid perfectly plastic material is not valid for footings resting on carbonate sand. Adequate experimental and field data for bearing capacity of footings on carbonate sand and subjected to inclined load is also not available. Thus the use of classical bearing capacity equations, with empirical reduction factors, for foundations resting on carbonate sands and subjected to inclined loads, is questionable.

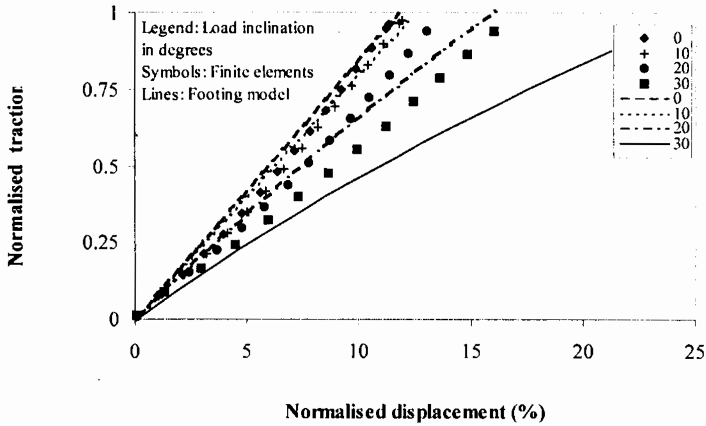

Fig. 9. Model predictions of resultant traction for 25m-diameter footing

Elastic-perfectly plastic footing models provide a rational basis for determination of the bearing capacity of foundations subjected to inclined loads. However, due to their elatstic-perfectly plastic assumption, these models are also not valid for determining the bearing capacity of footings resting on carbonate sand. The footing model proposed in this study, with an elasto-plastic strain hardening assumption, provides a rational framework for such a determination. The model is relatively simple, and shows good agreement with experimental data, as well as with the results of finite element analysis. However, more experimental and numerical studies need to be carried out, before such a framework can be used with confidence for analysis and design of offshore circular foundations resting on carbonate sands and subjected to inclined load.

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