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A DESIGN RATIONALE FOR CIRCULAR SILOS BASED ON FINITE ELEMENT ANALYSIS

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ABSTRACT: The current practice of silo analysis is based on several assumptions and idealisations. Analysis of silos by finite element methods and comparison of the results with corresponding values obtained from conventional methods reveal that the conventional methods cannot predict all the stress resultants required for the silo design. Traditional methods also cannot efficiently incorporate effects of lateral loads due to wind. On the other hand, axisymmetric shell finite elements can easily analyse circular silos to determine all the design forces and moments for various load combinations including wind forces. The authors have carried out an extensive parametric study based on finite element analysis of the structure. The investigation revealed the need for incorporating various other stress resultants, hitherto neglected, in the design process. On the basis of the parametric study, a design rationale has been proposed. The proposed procedure gives all possible forces and moments required for the design of the silo and considers various support systems and loading conditions, including the wind effect. The proposed method will be able to reduce the amount of rigorous calculation required in the conventional methods. The method also considers the variation of the design parameters along the height of the silo and results in an economic design.

KEY WORDS: Silo, finite element methods, meridional force/moment, circumferential force/moment

INTRODUCTION

Bins (silos and bunkers) have been used for ages to store coal, cement, food grains and other granular materials. Reinforced concrete silos have almost replaced the steel storage bins because of some advantages. Concrete silos and bunkers may be single or multiple and of various plans. The most common shape is circular, as the circular vertical wall is under tension with no bending moment because of uniform lateral pressure of the material stored inside. A flat bottom may create problem during unloading of the storage material; the conical hopper, offering a self-cleaning mechanism, is therefore used extensively as the outlet. Vertical wall and conical hopper of the silo may be monolithically constructed and supported on columns or continuous circular vertical wall. In some cases the vertical wall and the conical hopper may be supported separately. This paper deals with circular silos with conical hoppers and three different types of support systems (Fig. 1).

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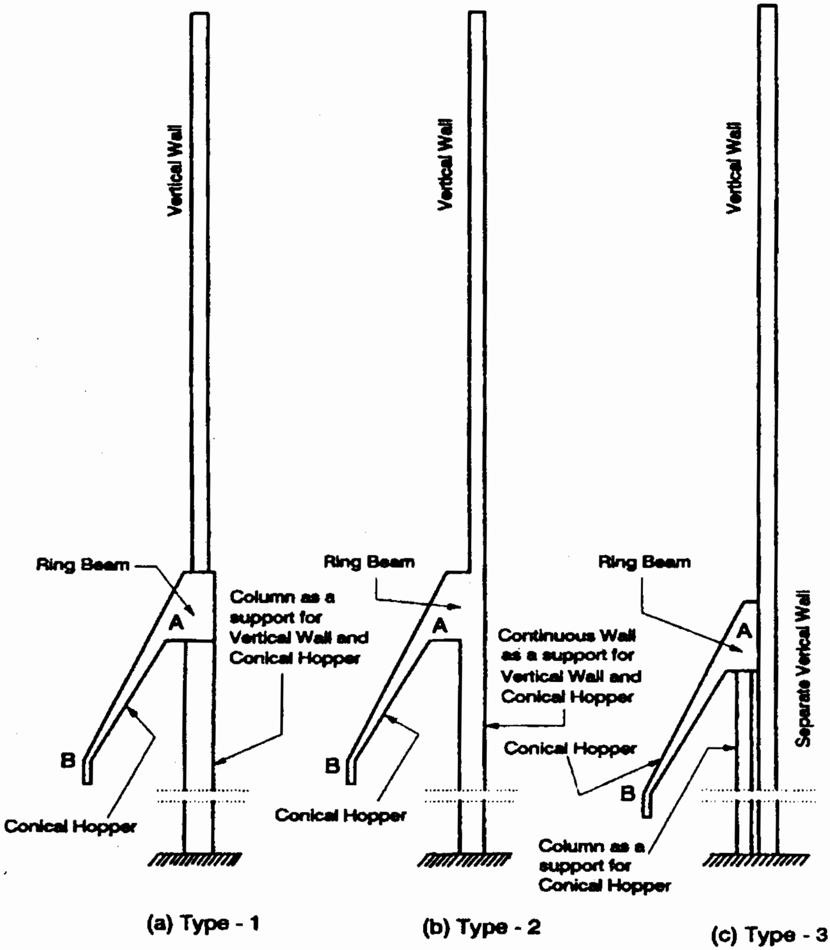

Fig. 1. Types of silos depending on ring beam support
(a) Ring beam supported by columns,
(b) Ring beam supported by monolithic continuous wall, and
(c) Ring beam supported by separate columns.

CONVENTIONAL METHODS VS FINITE ELEMENT METHODS

Circular silos are generally analysed as single silos. Interaction among silos of a group, if present, is then considered and necessary modifications are made. Analytical methods normally give the static pressures only. The design pressure can be estimated by modifying the computed static pressure to account for material flow, eccentric discharge and other conditions. Methods available to compute the static pressure are due to Janssen (1895), Airy (1897), and Reimbert and Reimbert (1976). All these methods can deal with only axisymmetric loading. Silos, being tall structures, are subject to lateral loads induced by wind and earthquake. The conventional methods cannot incorporate the effect of lateral loads in the design process properly. The traditional approach cannot predict any type of moments at all, but moments may have important localized effect on behaviour of the structure.

With this background, Alauddin (1994), Alauddin and Ahmad (1995), and Ahmad et al. (2001) modeled and analyzed a circular silo by the finite element method using axisymmetric shell elements (Ahmad et al., 1968) and compared the results found from finite element method and Janssen's method. This study revealed that the assumption that the vertical wall and conical hopper are separate structures and are subjected to membrane action only is not valid. Negative and positive meridional moments occur at the bottom of the vertical wall because of the partial fixity provided by the ring beam. Meridional moments, both positive and negative, developed in the conical hopper are also appreciable. Finite element analysis has shown that considerable negative hoop force develops at the bottom of the vertical wall due to the stored material pressure. The location of maximum hoop force has also been found to be different from that found from conventional approach. The finite element method can also deal efficiently with wind and earthquake forces. Wind forces induce both tensile and compressive meridional forces and circumferential moment in the vertical wall. Conventional methods of analysis are completely unable to predict such moments.

PARAMETRIC STUDY

With a view to understanding the behaviour of circular silo, an extensive parametric study has been performed considering various geometric properties of the structure (Fig. 2) and properties of the material stored. Three different support conditions as stated above have been considered. The input parameters and their range of variation are shown in Table 1. Four stress resultants: meridional force, hoop force, meridional moment and circumferential moment have been taken as the output parameters. Again, these four resultants have been considered separately for the vertical wall and the conical hopper. Since silo is an elevated structure, the values of these stress resultants vary along the height. Evidently, force and moment at any location is a function of its height and maximum value. Therefore, the parametric study has considered the sensitivity of the maximum stress resultants. Three load cases have been considered:

1. Self weight, considered axisymmetric,
2. Stored material pressure, considered axisymmetric, and
3. Wind load, considered non-symmetric.

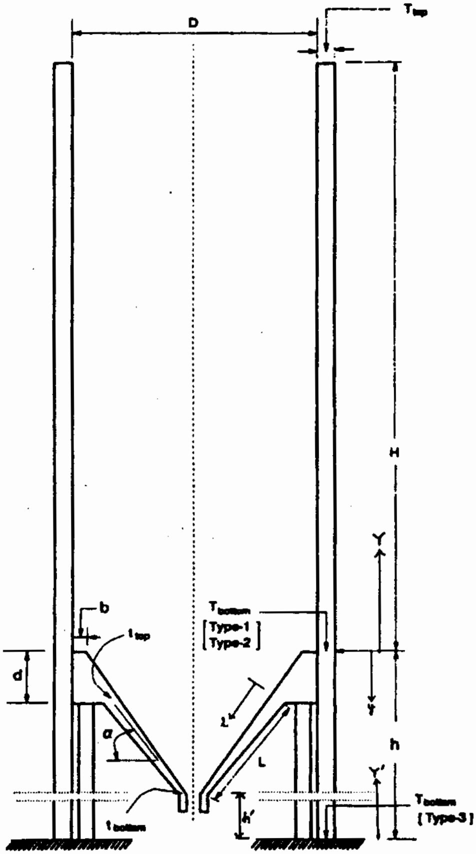

Fig. 2. Various dimensions of a circular silo

Table 1. Parameters Considered in the Parametric Study

Name of the parameters	Range
Height of vertical wall, H	40′ - 80′
Internal diameter of silo, D	10′ - 100′
Thickness of vertical wall at bottom, T_{bottom}	6″ - 13″
Thickness of conical hopper at top, t_{top}	6″ - 13″
Depth of ring beam, d	24″ - 96″
Inclination of conical hopper with horizontal, α	40° -75°
Height of hopper bottom (opening) above floor level, h′	8′ - 25′
Unit weight, γ	35 pcf-160 pcf
Angle of internal friction, ρ	15° -50°
Coefficient of wall friction, μ′	0.2 – 0.7

DESIGN RATIONALE

On the basis of the parametric study, it has been found that the meridional and circumferential moments have significant values and it is suggested that these be considered in design. Presence of negative hoop forces is also significant. The extensive parametric investigation based on finite element analysis has led to a number of equations to determine various stress resultants. These equations incorporate all stress resultants in the design of the silo.

The proposed design philosophy is very simple. At first the maximum of any of the stress resultants is found from the table. Then, to account for the vertical variation of the design forces and moments, charts are used to determine the value of the stress resultants at various heights. The design procedure follows the following four steps:

Determination of Type of Silo

The first step is to determine the type of silo. Depending on the support condition of the ring beam, the silo type is determined with the help of Fig 1. Two sets of design formulas and charts are provided: one for either of Type 1 or 2 and the other for Type 3. Only circular silos are under consideration.

Determination of Maximum Stress Resultants

Tables 2 to 9 give the maximum values of the four stress resultants for three different load cases and two different structural components (vertical wall and conical hopper). First four tables (Tables 2 to 5) refer to Type 1 or Type 2 silos, whereas Tables 6 to 9 refer to Type 3 silos. All the equations in the table (the exceptions are duly noted) are presented in the form:

Maximum stress resultant (per foot of the element) = $\pm k.f_1.f_2.f_3..f_4.f_5.....f_n$

Here, k is numeric constant and f_1, f_2, f_3........ f_n are multiplying factors, each corresponding to a definite input parameter. Each of k, and f_1, f_2, f_3,........ f_n is different for different stress resultants, structural components and load cases. A positive force sign means tension, while a negative sign refers to compression. A positive moment means water-retaining curvature. It must be remembered that k, f_1, f_2, f_3, ...f_n are empirical constants and refer to Imperial units only; i.e. units for forces are pound and that for moments are pound-feet. Concrete unit weight of 150 pcf has been assumed and the formulas are produced accordingly.

Table 2. Maximum Forces in the Vertical Wall due to Material Pressure for Type 1 and Type 2 Silos

Merid. Force	Hoop force (positive)		Hoop force (negative)	Meridional moment (positive)	Meridional moment (negative)	Circumferential moment (negative)
$F_{max} = 0.975R(\gamma H-0.8q_{max})$ $q_{max}=\gamma D[1-\exp(-4\mu' kH/D)]/(4\mu' k)$ $k=(1-\sin\rho)/(1+\sin\rho)$ R= hydraulic radius= D/4	$F_{max}=1.06p_{d,max}D/2$ $p_{d,max}=kq_{max}$	k	-1273	91.3	-362	-91
		f_1	$1+0.0071(H-39.6)^{0.827}$	$1+0.033(H-39.6)^{0.636}$	$1+0.0299(H-39.6)^{0.531}$; $H\leq 140$ $1+4.2686(H-39.6)^{-0.534}$; $H>140$	$1+0.0295(H-39.6)^{0.544}$ $H\leq 140$ $1+2.8482(H-39.6)^{-0.439}$; $H>140$
		f_2	$1+0.0583(D-10)^{1.916}$	$1+0.0574(D-10)^{1.739}$	$1+0.0785439(D-10)^{1.850}$	$1+0.0574(D-10)^{1.86}$
		f_3	$1-0.0226(\alpha-40)^{1.014}$	$1-0.0141(\alpha-40)^{0.961}$	$1-0.01943(\alpha-40)^{1.013}$	$1-0.0192(\alpha-40)^{1.012}$
		f_4	1	$1+0.0138(T_{top}-4)^{1.023}$	$1+0.0066(T_{top}-4)^{1.013}$	$1+0.0068(T_{top}-4)^{0.98923}$
		f_5	$1-0.0392(T_{bottom}-6)^{1.053}$	$1+0.1087(T_{bottom}-6)^{0.988}$	$1+0.15483(T_{bottom}-6)^{0.748}$	$1+0.1531(T_{bottom}-6)^{0.749}$
		f_6	$1-0.0243(d-24)^{0.732}$	$1-0.0111(d-24)^{0.744}$	$1-0.0053(d-24)^{0.986}$	$1-0.0047(d-24)$
		f_7	$1+0.0285(\gamma-35)$	$1+0.0286(\gamma-35)$	$1+0.0286(\gamma-35)$	$1+0.0285(\gamma-35)$
		f_8	$1+0.008(\rho-15)^{1.311}$	$1+0.0046(\rho-15)^{1.027}$	$1+0.007(\rho-15)^{1.181}$	$1+0.0069(\rho-15)^{1.168}$
		f_9	$1-0.8723(\mu'-0.2)^{0.511}$	$1-1.0073(\mu'-0.2)^{0.614}$	$1-1.0969(\mu'-0.2)^{0.585}$	$1-1.0908(\mu'-0.2)^{0.586}$
		f_{10}	1	1	$1+0.0124(t_{top}-6)^{0.492}$; $t_{top}\leq 9''$ $1.0187-0.0029(t_{top}-9)^{1.536}$; $t_{top}>9''$	$1+0.0125(t_{top}-6)^{0.481}$; $t_{top}\leq 9''$ $1.0189-0.0031(t_{top}-9)^{1.413}$; $t_{top}>9''$

Table 3. Maximum Forces in the Conical Hopper due to Material Pressure for Type 1 and Type 2 Silos

	Meridional force (positive)	Hoop force (positive)	Meridional moment (positive)	Meridional moment (negative)	Circumferential moment (positive)	Circumferential moment (negative)
k	1579.2	1999.5	82	-267.6	14	-41
f_1	$1+0.0332(H-39.6)^{0.491}$ $H\le 230'$ 1.437; $H>230'$	$1+0.0316(H-39.6)^{0.51}$; $H\le 180'$ 1.3934; $H>180'$	$1+0.0433(H-39.6)^{0.385}$ $H\le 210'$ 1.313; $H>210'$	$1+0.0265(H-39.6)^{0.684}$	$1+0.0255(H-39.6)^{0.521}$; $H\le 180'$ $1.2911-1.9\times10^{-5}(H-180)^{1.474}$; $H>180'$	$1+0.0266(H-39.6)^{0.757}$
f_2	$1+0.1677(D-10)^{1.517}$	$1+0.151984(D-10)^{1.581}$	$1+0.1416(D-10)^{1.733}$	$1+0.0411(D-10)^{1.858}$	$1+0.2266(D-10)^{1.756}$	$1+0.0243(D-10)^{1.913}$
f_3	$0.9569+0.0002(\|\alpha-55\|)^{2.435}$	$1-0.0123(\alpha-40)^{0.897}$	$1-0.0282(\alpha-40)^{0.91}$	$1-0.0439(\alpha-40)^{0.706}$	$1-0.0583(\alpha-40)^{0.755}$	$1-0.0386(\alpha-40)^{0.637}$
f_4	$1+0.0059(T_{top}-4)$	$1+0.0064(T_{top}-4)^{1.005}$	1	$1+0.0061(T_{top}-4)$	1	1
f_5	$1-0.0073(T_{bottom}-6)^{0.83}$	$1-0.01385(T_{bottom}-6)^{0.731}$	$1-0.0412(T_{bottom}-6)^{0.833}$ $T_{bottom}\le 8.5''$ 0.9116; $T_{bottom}>8.5''$	$1+0.2167(T_{bottom}-6)^{0.702}$	1	$1+0.4128(T_{bottom}-6)^{0.703}$
f_6	$1-0.00027(d-24)^{1.549}$	$1-0.0024(d-24)^{1.096}$	$1-0.032(d-24)^{0.634}$	$1-0.1044(d-24)^{0.353}$; $d\le 72''$ 0.5906; $d>72''$	$1-0.0344(d-24)^{0.651}$	$1-0.1236(d-24)^{0.31}$; $d\le 66''$ 0.6062; $d>66''$
f_7	$1+0.0286(\gamma-35)$	$1+0.0286(\gamma-35)$	$1+0.0286(\gamma-35)$	$1+0.0286(\gamma-35)$	$1+0.0284(\gamma-35)$	$1+0.0286(\gamma-35)$
f_8	$1+0.0082(\rho-15)^{1.219}$	$1+0.008(\rho-15)^{1.223}$	$1+0.0061(\rho-15)^{1.305}$	$1+0.0027(\rho-15)^{0.896}$; $\rho\le 40°$ $1.0416-9.1\times10^{-5}(\rho-15)^{2.127}$; $\rho>40°$	$1+0.0081(\rho-15)^{1.238}$	$1-0.00038(\rho-15)^{1.716}$
f_9	$1-1.0918(\mu'-0.2)^{0.578}$	$1-1.1156(\mu'-0.2)^{0.566}$	$1-1.1307(\mu'-0.2)^{0.539}$	$1-0.9078(\mu'-0.2)^{0.637}$	$1-1.1034(\mu'-0.2)^{0.544}$	$1-0.789(\mu'-0.2)^{0.712}$
f_{10}	$1-0.0061(t_{top}-6)^{0.9}$	$1-0.0186(t_{top}-6)^{0.764}$	$1+0.0848(t_{top}-6)^{1.16}$	$1+0.0637(t_{top}-6)^{0.515}$; $t_{top}\le 9.5''$ $1.105-0.0156(t_{top}-9.5)^{1.478}$; $t_{top}>9''$	$1+0.1328(t_{top}-6)^{1.058}$	$1.036-0.0259(\|t_{top}-7.5\|)^{1.68}$
f_{11}	1	$1-0.0046(t_{bottom}-4)^{0.882}$	$1+0.0506(t_{bottom}-4)^{0.991}$	$1-0.0139(t_{bottom}-4)^{0.998}$	1	$1-0.0139(t_{bottom}-4)^{0.998}$

Table 4. Maximum Forces in the Vertical Wall due to Wind for Type 1 and Type 2 Silos

	Meridional force (positive)	Meridional force (negative)	Hoop force (positive)	Meridional moment (positive)	Circumferential moment (positive)	Circumferential moment (negative)
k	3200	-6525	230	173	15.5	-10.2
f_1	$1+0.00973(H-39.6)^{0.622}$; $H\leq 160'$ $1+0.0037(H-39.6)^{1.301}$; $H>160'$	$1+0.14(H-39.6)^{0.478}$; $H\leq 110'$ $1+7.8\times10^{-6}(H-39.6)^{2.303}$; $H>110'$	$1+0.1287(H-39.6)^{0.808}$; $H\leq 110'$ $1+0.0105(H-39.6)^{1.3}$; $H>110'$	$1+0.0363(H-39.6)^{0.489}$	$1+0.0464(H-39.6)^{0.947}$; $H\leq 140'$ $4.285+0.0376(H-140)^{0.452}$; $H>140'$	$1+0.2336(H-39.6)^{0.604}$; $H\leq 160'$ 5.18; $H>160'$
f_2	$0.616+0.00023(\lvert D-20\rvert)^{2.381}$; $D\leq 60'$ $1.46+0.0187(D-60)^{0.686}$; $D>60'$	$0.3078+0.0632(\lvert D-30\rvert)^{0.633}$	$0.9296-0.0064(\lvert D-60\rvert)^{1.199}$	$0.7503+0.0163(\lvert D-20\rvert)^{1.118}$	$1+0.1481(D-10)^{1.251}$; $D\leq 50'$ $11.5375-0.00024(D-50)^{2.426}$ $D>50'$	$12.397-2.2\times10^{-6}(\lvert D-50\rvert)^{3.31}$
f_3	$1-0.075(T_{top}-4)^{0.681}$	$1-0.14(T_{top}-4)^{0.601}$	$1-0.1183(T_{top}-4)^{0.69}$	$1-0.0455(T_{top}-4)^{0.745}$	1; $T_{top}\leq 6''$ $1-0.0522(T_{top}-6)^{0.973}$; $T_{top}>6''$	1; $T_{top}\leq 6''$ $1+0.0534(T_{top}-6)^{0.995}$; $T_{top}>6''$
f_4	$1-0.0365(T_{bottom}-6)^{0.948}$	$1-0.0507(T_{bottom}-6)^{0.855}$	$1-0.1962(T_{bottom}-6)^{0.798}$	1	$0.8652+0.0079(\lvert T_{bottom}-9.5\rvert)^{2.748}$	$0.8663+0.0071(\lvert T_{bottom}-9.5\rvert)^{2.648}$
f_5	$1+0.1087(q-9.2)$	$1+0.1087(q-9.2)$	$1+0.1082(q-9.2)$	$1+0.1084(q-9.2)$	$1+0.1085(q-9.22)$	$1+0.1085(q-9.22)$

Variation of Stress Resultants in Vertical Direction

To account for the variation of the stress resultants along vertical direction and to incorporate design economy Fig 3 to 23 are produced. With the maximum stress value from step 2, and appropriate curve from these figures, the designer gets the value of that stress resultant at various horizontal sections at different heights. Curves for only those resultants are produced which vary considerably in the vertical direction. In other cases, the maximum value is to be used all through.

Design Pressure

To get the design pressure, appropriate 'overpressure' factor is to be used with the stress resultants found from step 3. Suggestions made by the ACI-313-77 (1983) is widely used and recommended to determine the overpressure factor.

Table 5. Maximum Forces in the Vertical Wall and Conical Hopper due to Self-Weight for Type 1 and Type 2 Silos

	Vertical wall		Conical hopper	
	Meridional force	Hoop force (compressive)	Merid. Force	Hoop force (tensile)
k		-868		169.5
f_1	$\gamma_c H(T_{bottom}+T_{top})/24$ + DL roof	$1+0.0061(H-39.6)^{0.968}$	$W_g/(\pi D \sin\alpha)$	$1-0.00303(H-39.6)^{0.744}$
f_2		$1+0.0204(D-10)^{1.220}$		$1+0.1477(D-10)^{1.137}$
f_3		$1-0.0140(\alpha-40)^{0.794}$		$1-0.026(\alpha-40)^{0.914}$
f_4		$1+0.0393(T_{top}-4)^{1.002}$		$1+0.1034(t_{top}-6)^{0.986}$
f_5		$1+0.0145(T_{bottom}-6)^{0.811}$		$1+0.0288(t_{bottom}-4)^{1.137}$
f_6		$1+0.0375(t_{top}-6)^{0.972}$		$0.95+0.000363\{\lvert d-54\rvert\}^{1.65}$
f_7		$1+0.01609(t_{bottom}-4)^{1.004}$		1
f_8		$0.9638+0.000223\{\lvert d-36\rvert\}^{1.658}$		1

Table 6. Maximum Forces in the Vertical Wall and Conical Hopper due to Self-Weight for Type 3 Silos

	Vertical wall		Conical hopper	
	Meridional force	Hoop force (compressive)	Meridional force	Hoop force (tensile)
k		-890		168
f_1	$\gamma_c H(T_{bottom}+T_{top})/24$ + DL roof	$1+0.0126(H-39.6)^{0.99}$	$W_g/(\pi D \sin\alpha)$	1
f_2		$1+0.0099(D-10)^{0.918}$		$1+0.2711(D-10)^{1.126}$
f_3		$1+0.00083(\alpha-40)^{1.452}$		$1+8\times10\text{-}7(\alpha-40)^{3.38}$
f_4		$1+0.0737(T_{top}-4)$		$1+0.0898(t_{top}-6)^{0.867}$
f_5		$1+0.0797(T_{bottom}-6)^{0.996}$		$1+0.0213(t_{bottom}-4)^{1.039}$
f_6		$1+0.0375(t_{top}-6)^{0.972}$		$1-0.287(\{d-24\}^{0.11}$
f_7		$1+0.01609(t_{bottom}-4)^{1.004}$		1
f_8		1		1

Table 7. Maximum Forces in the Vertical Wall due to Material Pressure for Type 3 Silos

Meridional force	Hoop force (positive) in pressure zone		Hoop force (positive) below pressure zone	Hoop force (negative) below pressure zone	Meridional moment (positive) in pressure zone	Meridional moment (negative) below pressure zone
$F_{max} = 0.975R(\gamma H-0.8q_{max})$ $q_{max}=\gamma D[1-\exp(-4\mu' kH/D)]/(4\mu' k)$ $k=(1-\sin\rho)/(1+\sin\rho)$ R= hydraulic radius= D/4	$F_{max}=1.02p_{d,max}D/2$ $p_{d,max}=kq_{max}$	k	712	-305.5	30.5	-45.5
		f_1	$1+0.0499(H-39.6)^{0.618}$	$1+0.028(H-39.6)^{1.105}$	$1+0.0666(H-39.6)^{0.559}$	$1+0.0134(H-39.6)^{1.169}$
		f_2	$1+0.1519(D-10)^{1.329}$	$1+0.1783(D-10)^{0.744}$	$1+0.1802(D-10)^{1.292}$	$1+0.1773(D-10)^{0.822}$
		f_3	1	1	$1+0.0305(T_{top}-4)^{1.003}$	1
		f_4	1	1	$1+0.128(T_{bottom}-6)^{0.952}$	$1+0.1813(T_{bottom}-6)^{0.994}$
		f_5	$1+0.0286(\gamma-35)$	$1+0.0284(\gamma-35)$	$1+0.0285(\gamma-35)$	$1+0.0285(\gamma-35)$
		f_6	$1-0.000182(\rho-15)^{2.015}$	$1-0.004(\rho-15)^{1.252}$	$1-1.06\times10^{-4}(\rho-15)^{2.165}$	$1-0.004(\rho-15)^{1.254}$
		f_7	$1-1.0907(\mu'-0.2)^{0.707}$	$1+0.8143(\mu'-0.2)^{0.703}$	$1-1.123(\mu'-0.2)^{0.704}$	$1+6.6257(\mu'-0.2)^{2.08}$; $\mu'\le0.4$ $1+0.5591(\mu'-0.2)^{0.649}$; $\mu'>0.4$

Table 8. Maximum Forces in the Conical Hopper due to Material Pressure for Type 3 Silos

	Meridional force (positive)	Hoop force (positive)	Meridional moment (positive)	Meridional moment (negative)	Circumferential moment (positive)
k	341.5	955.6	55.3	-60.2	903.5
f_1	$1+0.0379(H-39.6)^{0.501}$; H≤ 230′ 1.5257; H> 230′	$1+0.0394(H-39.6)^{0.501}$; H≤ 210′ 1.517; H> 210′	$1+0.0428(H-39.6)^{0.468}$; H≤ 210′ 1.474; H> 210′	$1+0.0471(H-39.6)^{0.467}$; H≤ 210′ 1.5178; H> 210′	$1+0.0492(H-39.6)^{0.434}$; H≤ 210′ 1.4575; H>210′
f_2	$1+0.6539(D-10)^{1.556}$	$1+0.3162(D-10)^{1.654}$	$1+0.2268(D-10)^{2.207}$	$1+0.0101(D-10)^{2.153}$	$1+0.00062(D-10)^{2.491}$
f_3	$1-0.0038(\alpha-40)^{0.929}$; $\alpha \leq 65°$ 0.9244; $\alpha > 65°$	$1-0.0054(\alpha-40)^{1.099}$	$0.7508-0.0011(\lvert\alpha-50\rvert)^{1.791}$	$1-0.0383(\alpha-40)^{0.814}$; $\alpha\leq 70°$ 0.3897; $\alpha > 70°$	$1-0.0844(\alpha-40)^{0.488}$
f_4	$1-0.0022(d-24)^{1.223}$	$1-0.0212(d-24)^{0.773}$	$1-0.1803(d-24)^{0.408}$; d ≤ 72″ 0.1251; d > 72″	$1-0.1087(d-24)^{0.519}$; d ≤ 84″ 0.09; d > 84″	$1-0.1867(d-24)^{0.404}$; d ≤ 84″ 0.0238; > 84″
f_5	$1+0.0286(\gamma-35)$	$1+0.0286(\gamma-35)$	$1+0.0286(\gamma-35)$	$1+0.0287(\gamma-35)$	$1+0.0285(\gamma-35)$
f_6	$1+0.01193(\rho-15)^{1.242}$	$1+0.012(\rho-15)^{1.238}$	$1+0.0119(\rho-15)^{1.241}$	$1+0.0124(\rho-15)^{1.229}$	$1+0.0118(\rho-15)^{1.242}$
f_7	$1-1.1032(\mu'-0.2)^{0.585}$	$1-1.1233(\mu'-0.2)^{0.572}$	$1-1.1041(\mu'-0.2)^{0.583}$	$1-1.1408(\mu'-0.2)^{0.561}$	$1-1.106(\mu'-0.2)^{0.582}$
f_8	$1-0.00049(t_{top}-6)^{2.507}$	$1.0042-0.00049(\lvert t_{top}-7\rvert)^{2.954}$	$1+0.2698(t_{top}-6)^{0.77}$	$1+0.1421(t_{top}-6)^{0.683}$	$1+0.297(t_{top}-6)^{0.95}$
f_9	$1-0.0013(t_{bottom}-4)^{1.022}$	$1-0.005(t_{bottom}-4)^{1.02}$	$1+0.0154(t_{bottom}-4)^{0.986}$	$1+0.1153(t_{bottom}-4)^{1.193}$	1

Table 9. Maximum forces in the vertical wall due to wind for Type 3 silos

	Merid. force (positive)	Merid. force (neg)	Hoop force (positive)	Merid. moment (positive)	Merid. moment (neg)	Circum. moment (positive)	Circum. Moment (nege)
k	5900	-9600	1260	234	-336	29.4	-29
f_1	$1+0.0043(H-39.6)^{1.085}$; $H\leq 160'$ $1+0.00034(H-39.6)^{1.62}$; $H>160'$	1; $H\leq 140'$ $1+0.00386(H-140)^{1.245}$; $H>140'$	$1+0.00664(H-39.6)^{0.987}$; $H\leq 160'$ $1+0.00035(H-39.6)^{1.609}$; $H>160'$	$1+0.00042(H-39.6)^{1.362}$; $H\leq 180'$ $1+0.00018(H-39.6)^{1.534}$; $H>180'$	$1-0.0116(H-39.6)^{0.637}$	$1+0.0935(H-39.6)^{0.496}$; $H\leq 160'$ 1.94; $H>160'$	$1+0.0923(H-39.6)^{0.496}$; $H\leq 140'$ 1.92; $H>140'$
f_2	$0.635+0.0009(\lvert D-20\rvert)^{1.849}$; $D\leq 70'$ $1+0.0059(D-10)^{1.193}$; $D>70'$	$0.2688+0.0133(\lvert D-30\rvert)^{1.204}$	$0.6649+5\times10^{-5}(\lvert D-20\rvert)^{2.733}$; $D\leq 60'$ $0.6649+0.0425(D-20)^{0.849}$; $D>60'$	$1+0.0283(\lvert D-30\rvert)^{1.087}$	$0.459+0.00124(\lvert D-20\rvert)^{1.904}$	$1+1.4059(D-10)^{0.626}$;	$1+1.4778(D-10)^{0.636}$; $D\leq 60'$ $1+29.777(D-10)^{-0.133}$; $D>60'$
f_3	$1-0.0383(T_{top}-4)^{0.752}$	$1-0.0755(T_{top}-4)^{0.541}$	$1-0.0355(T_{top}-4)^{0.762}$	$1-0.0182(T_{top}-4)^{0.868}$	$1-0.0406(T_{top}-4)^{0.854}$	$0.9734+0.0085(\lvert T_{top}-5.5\rvert)^{1.886}$	$0.9732+0.00874(\lvert T_{top}-5.5\rvert)^{.868}$
f_4	$1-0.0405(T_{bottom}-6)^{0.891}$	$1-0.0398(T_{bottom}-6)^{0.748}$; $T_{bottom}\leq 11''$ 0.866; $T_{bottom}>11''$	$1-0.0411(T_{bottom}-6)^{0.895}$	$1+0.1658(T_{bottom}-6)^{0.936}$	$1+0.1532(T_{bottom}-6)^{0.809}$	$0.9061+0.0068(\lvert T_{bottom}-9\rvert)^{1.417}$	$0.9069-0.00496(T_{bottom}-9)^{3.078}$; $T_{bottom}\leq 11''$ 0.935; $T_{bottom}>11''$
f_5	$1+0.1087(q-9.22)$	$1+0.1086(q-9.2)$	$1+0.1085(q-9.22)$	$1+0.1084(q-9.22)$	$1+0.1089(q-9.2)$	$1+0.1087(q-9.2)$	$1+0.1087(q-9.2)$

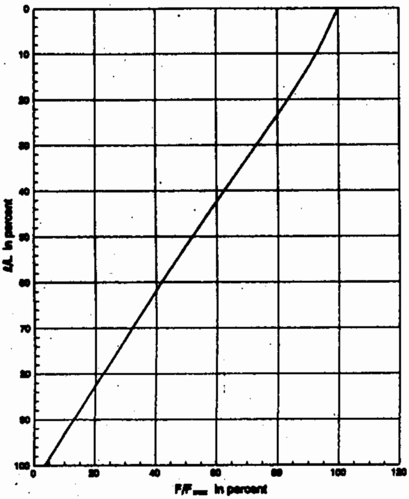

Fig 3 Positive (tensile Meridional Force in conical hopper due to self weight (Type 1 & Type 2)

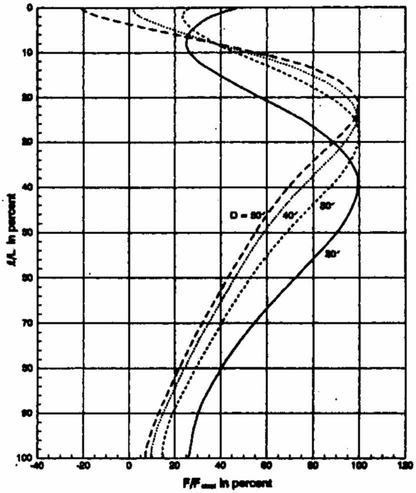

Fig 4. Hoop Force in conical hopper due to self weight (Type 1 & Type 2)

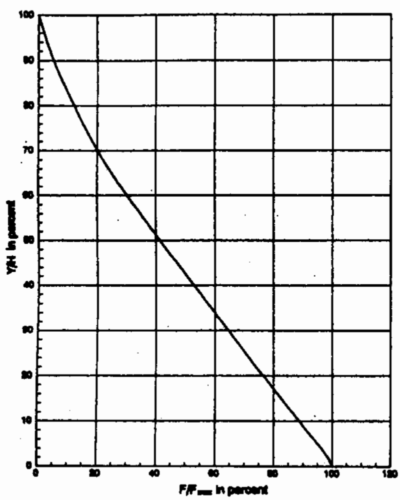

Fig 5. Meridional Force in vertical wall due to stored material pressure (Type 1 & Type 2)

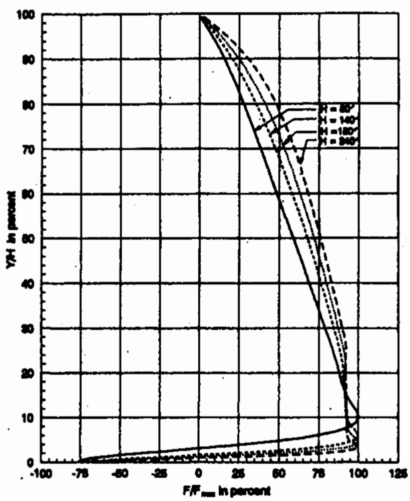

Fig 6. Hoop Force in vertical wall due to stored material pressure (Type 1 & Type 2)

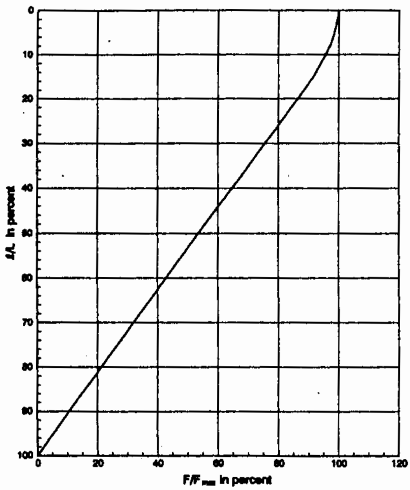

Fig 7 Positive (tensile) Meridional Force in conical hopper due to stored material pressure (Type 1 & Type 2)

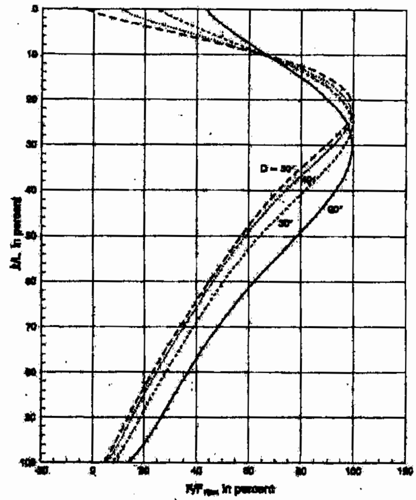

Fig 8. Hoop Force in conical hopper due to material pressure (Type 1 & Type 2)

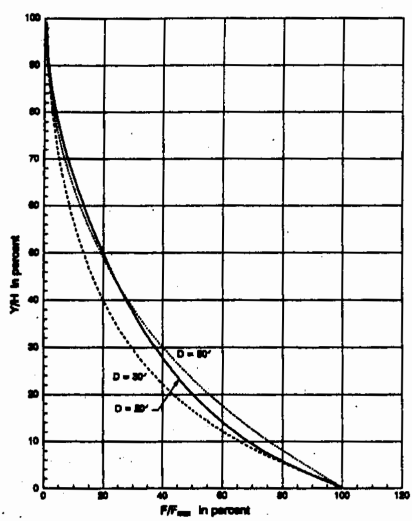

Fig 9. Positive (tensile) Meridional Force in vertical wall due to wind pressure (Type 1 & Type 2)

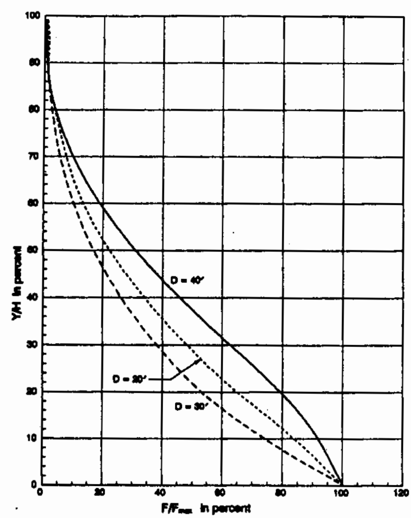

Fig 10. Negative (Comp.) Meridional Force in vertical wall due to wind pressure (Type 1 & Type 2)

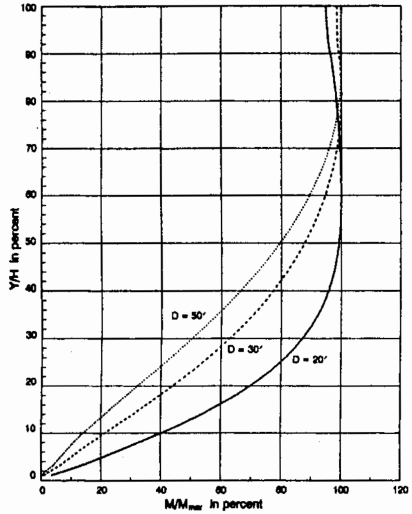

Fig 11. Positive Circumferential Moment in vertical wall due to wind pressure (Type 1 & Type 2)

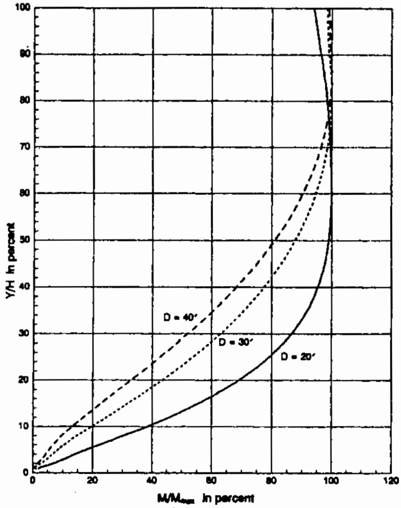

Fig 12. Negative Circumferential Moment in vertical wall due to wind pressure (Type 1 & Type 2)

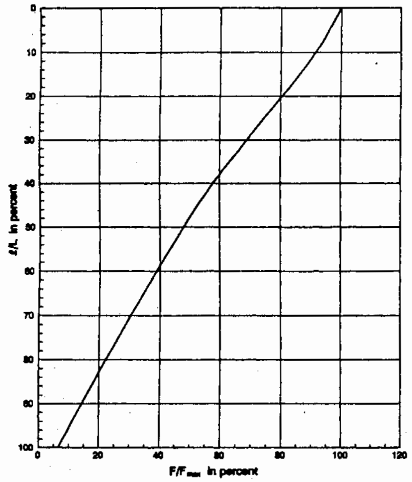

Fig 13. Positive (tensile) Meridional Force in conical hopper due to self weight (Type 3)

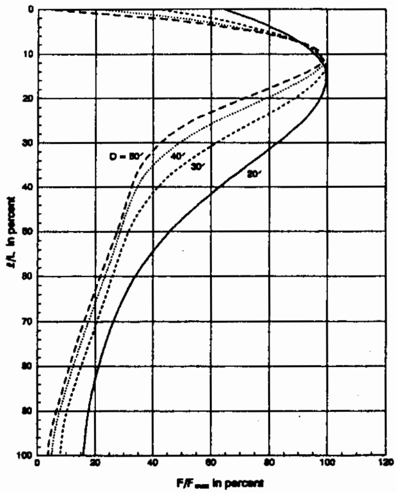

Fig 14. Hoop Force in conical hopper due to self weight (Type 3)

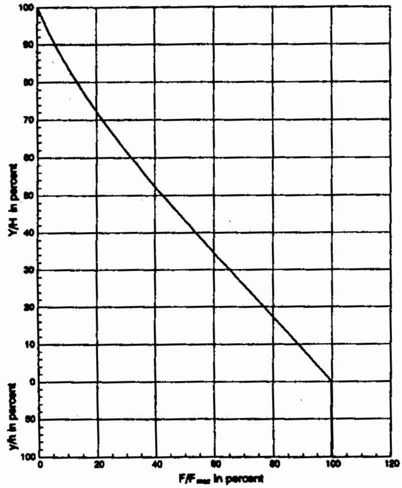

Fig 15. Meridional Force in vertical wall due to material pressure (Type 3)

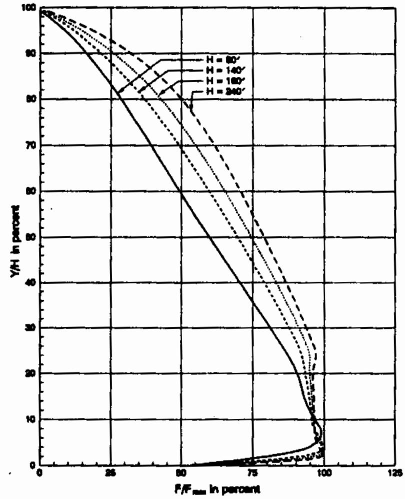

Fig 16. Hoop Force in vertical wall in pressure zone due to stored material pressure (Type 3)

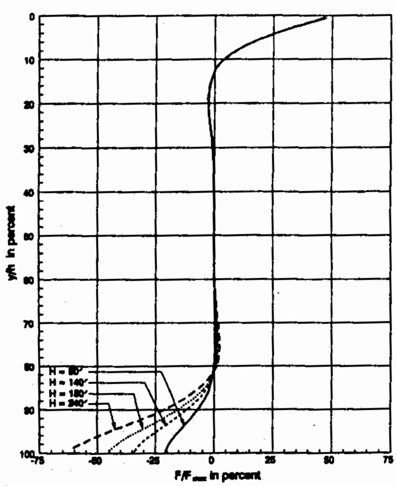

Fig 17. Hoop Force in vertical wall below pressure zone due to stored material pressure (Type 3)

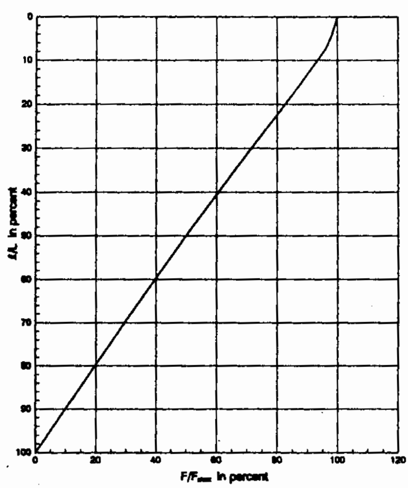

Fig 18. Positive (tensile) Meridional Force in conical hopper due to stored material pressure (Type 3)

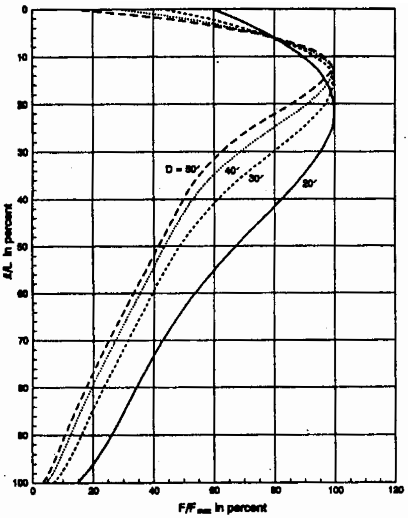

Fig 19. Hoop Force in conical hopper due to stored material pressure (Type 3)

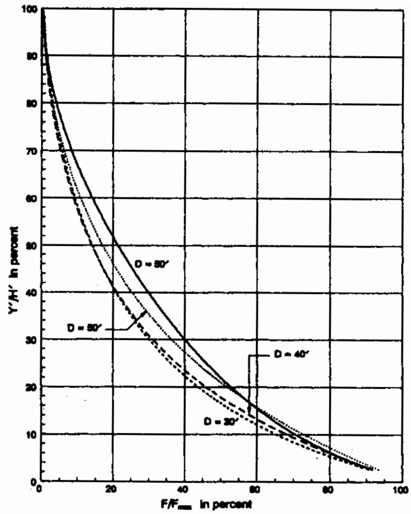

Fig 20. Positive (tensile) Meridional Force in vertical wall due to wind pressure (Type 3)

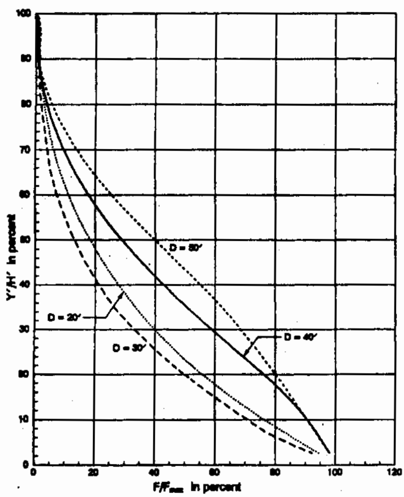

Fig 21. Negative (comp) Meridional Force in vertical wall due to wind pressure (Type 3)

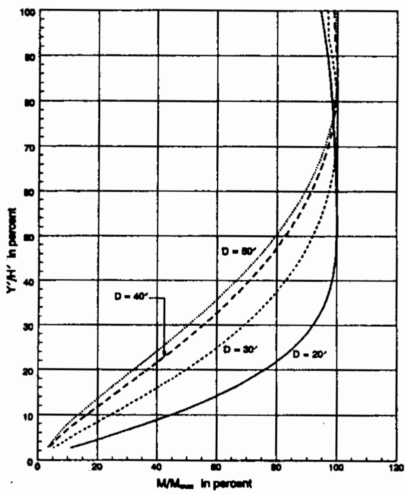

Fig 22. Positive Circumferential Moment in vertical wall due to wind pressure (Type 3)

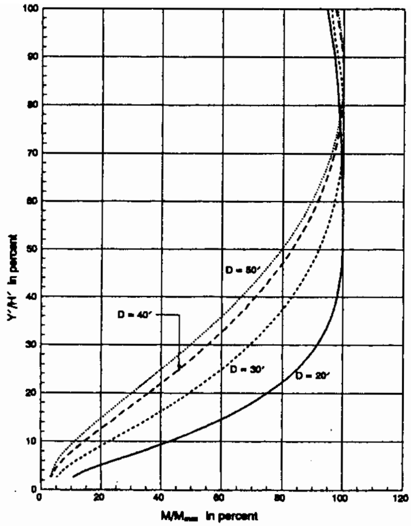

Fig 23. Negative Circumferential Moment is vertical wall due to wind pressure (Type 3)

VALIDITY OF THE PROPOSED GUIDELINE

The proposed equations to determine the forces and moments in a silo cover a wide range of geometric parameters and material properties. However, the equations are valid only for circular shaped concrete silos. They can handle three separate load cases i.e., self-weight, material pressure and wind load. The equations are empirical in nature and valid only for Imperial system of units. The acceptability of the proposed equations has been checked by comparing results from these equations with those from the direct finite element analysis. Few such comparisons are presented in tables 10 to 13.

Table 10. Data for five examples

No.	H	D	α	T_{top}	T_{bottom}	t_{top}	t_{bottom}	γ	ρ	μ'	d	q
	ft	ft	°	"	"	"	"	pcf	°		"	psf
1	100	30	60	5	10	8	5	60	30	0.40	48	36.86
2	130	30	60	6	11	9	7	50	35	0.50	42	50.18
3	130	25	55	7	9	9	6	80	40	0.45	36	25.60
4	180	35	65	7	12	11	4	70	30	0.50	54	57.60
5	160	40	60	6	9	10	5	65	25	0.60	60	43.26

Table 11. Maximum Forces due to Self Weight (Type 1 & Type 2)

No.	Vertical Wall				Conical Hopper			
	Meridional force		Hoop force		Meridional force		Hoop force	
	FE*	PE#	FE*	PE#	FE*	PE#	FE*	PE#
1	-10168	10375	-2014	-2018	1320	1224	595	613
2	-14605	14812	-2525	-2489	1582	1485	678	705
3	-13565	13843	-2245	-2240	1110	1074	591	620
4	-22440	22531	-3355	-3475	2188	2032	743	728
5	-16377	16313	-3455	-3600	2117	1962	1002	1009

* Finite element analysis # Proposed equation solution

Table 12. Maximum Forces and Moments due to Wind Load in vertical Wall (Type 1 &, Type 2)

No.	Sign	Meridional force		Hoop force		Meridional moment		Circumferential moment	
		FE*	PE#	FE*	PE#	FE*	PE#	FE*	PE#
1	+ve	16827	15452	602	824	807	808	1363	1274
	-ve	11396	11508	810	-	564	-	1597	1651
2	+ve	22106	22309	627	760	942	1114	2371	2353
	-ve	11525	8618	1117	-	610	-	2765	2763
3	+ve	9773	10944	534	560	412	486	936	912
	-ve	4084	6949	548	-	230	-	1090	1446
4	+ve	33692	33910	948	909	1226	1482	3887	4108
	-ve	15192	18592	1589	-	895	-	4529	4076
5	+ve	25449	31253	1689	2035	1308	1254	1704	3193
	-ve	17826	18076	1437	-	960	-	2002	2681

* Finite element analysis # Proposed equation solution

CONCLUSION

Conventional methods available for the analysis of silos fail to predict some of the localized stress resultants, which may have significant effect on the design of the structure. It has been found that moments and negative hoop forces, hitherto ignored, should be considered in the design process. An elaborate but simple design procedure based on extensive parametric investigation by the finite element method has been suggested.

Table 13. Maximum Forces and Moments due to Stored Material Pressure in Vertical Wall in Pressure Zone (Type 3)

No.	Sign	Meridional force		Hoop force		Meridional moment		Circumferential moment	
		FE*	PE#	FE*	PE#	FE*	PE#	FE*	PE#
1	+ve	0.0	0.0	24755	25315	793	787	88	-
	-ve	27891	28162	0.0	0.0	56	-	86	-
2	+ve	0.0	0.0	18739	18985	690	631	57	-
	-ve	33191	33731	0.0	0.0	40	-	91	-
3	+ve	0.0	0.0	22519	22656	728	680	57	-
	-ve	41835	42757	0.0	0.0	55	-	108	-
4	+ve	0.0	0.0	38678	38710	1555	1473	128	-
	-ve	83699	85338	0.0	0.0	128	-	223	-
5	+ve	0.0	0.0	39945	39813	1235	1032	153	-
	-ve	81983	83086	0.0	0.0	56	-	120	-

* Finite element analysis # Proposed equation solution

Proposed equations and charts will give all significant stress resultants at various locations along the height of the structure, together with the location and value of maximum stress resultants. This will incorporate economy in design by allowing curtailment of reinforcements. The tables and charts can easily be incorporated into a computer coding, relieving the designer from rigorous calculations or cumbersome computer modeling of the structure.

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