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EFFECT OF PARTICLE CHARACTERISTICS ON THE STRENGTH AND VOLUME CHANGE BEHAVIOUR OF SAND

S. J. M. Yasin[1] and A. M. M. Safiullah[1]

ABSTRACT: Granular soils, from the river beds of the Teesta, Jamuna and Meghna were collected from Rangpur, Tangail and Dhaka, respectively. A sample of Dhaka sand was also collected from an excavated pond in Dhaka city. The particle characteristics of these sands were determined using sphericity, elongation and flakiness as shape parameters. The shear strength and volume change behaviour were obtained from drained triaxial compression tests. It was observed that for any given relative density, the angle of internal friction increased with increasing elongation and flakiness i.e. decreasing sphericity of the particles and the higher the relative density the greater the effect of shape of the particles on the angle of internal friction. It was also found that sands with flakey and elongated particles underwent relatively small change in volume during shear compared to sands with spherical particles.

KEYWORDS: Particle shape parameter, particle size, triaxial test, strength parameter, critical void ratio.

INTRODUCTION

Discrete nature of particles is one of the most important characteristics of granular soils. Some of the important engineering properties of sand, such as dilatancy in the process of shear deformation, have been believed to occur due to this discrete nature of constituting grains and their configuration characteristics. Attempts have been made, for example by Rowe (1962, 1973) and Horne (1965), to describe the stress deformation behaviour of granular soils using the theories of particulate mechanics. These theories are based on elastic distortion of particles, and consider the sliding and rolling of particles in regular packing of equal sized spheres. In real granular soils the shape of particles are irregular, distribution of sizes differs greatly from that of uniform spheres, and packing is far from regular. The behaviour of a granular soil layer under stress is a function of many factors, such as the size and shape, gradation, mineralogical composition, surface properties, arrangement, adsorption complexes etc. of the particles. Thus the study of the physical properties of granular particles and their configuration relations are of great importance in the understanding of the stress-strain characteristics of deposits of granular media. Pioneering works have been performed by Hutchison and Townsend (1961) on gradation and density, by Kolbuszewski and Frederick (1963) on shape and size, by Kirkpatrick (1965) on size and

1 Department of Civil Engineering, BUET, Dhaka-1000, Bangladesh.

gradation, and by Koerner (1970) on shape and gradation. These works pointed out that characterization of the strength-deformation behaviour of sands and gravels cannot be done in terms of density or relative density alone, as had once been thought and stressed the need to put more insight on particle shape, arrangement and stress history.

This paper presents the findings of a study on the effect of particle shape and size on the strength parameters, the volume change characteristics during shear, critical void ratio and limiting densities (i.e. maximum and minimum densitiy) of granular deposits collected from four different locations in Bangladesh.

TEST MATERIALS

Soil samples were collected from four different locations in Bangladesh (Fig. 1). These locations are believed to represent different depositional environments and hence the samples collected from these places are expected to have different grain characteristics. Three samples representing recent deposits were collected from the riverbed of the Teesta, Jamuna and Meghna. One sample was collected from older deposit lying under the Pliestocene Madhupur deposit located in Dhaka. The Teesta sand was collected from Dalia, Rangpur near the cutoff wall of the Teesta Barrage from a depth of about 15-20 ft. The Meghna sand was collected from the riverbed near Meghna ghat, Dhaka, about 500 ft upstream of the Meghna bridge. The Jamuna sand was collected from the riverbank at Bhuyanpur, Tangail, near the Jamuna Bridge. The Dhaka sand was collected from a depth of about 40-50 ft from a dry excavated pond at Rajarbagh, Dhaka.

The grain size distributions of the insitu soils are shown in Fig. 2. The fineness modulus (F.M.), grain diameters d_{10}, d_{30}, and d_{60}, uniformity coefficient c_u ($=d_{60}/d_{10}$), coefficient of curvature, c_c [$=d_{30}^2/(d_{60}.d_{10})$] and specific gravity of these soils are presented in Table 1. The Teesta sand has the largest grains compared to other sands whereas the Jamuna sand has the finest grains. This is indicated by the fineness modulus of the soils. The Teesta sand contains very small amount of fines (material passing #200 sieve, opening 0.074mm) which is 0.17% by weight, whereas the Dhaka sand contains the largest amount of this fraction (6.62%). The other two sands, i.e., the Meghna and the Jamuna sands, contain fines fraction of 0.63 % and 5.48 %, respectively.

The uniformity coefficient varies from 2.56 for Dhaka sand to 1.88 for both Meghna and Jamuna sands. The Teesta sand has a uniformity coefficient of 2.46. The coefficient of curvature of these sands ranges from 1.01 for Jamuna sand to 1.56 for Dhaka sand. According to Unified Soil Classification System, all of these four soils fall in group SP (poorly graded sands with little or no fines). The effective grain diameters are 0.24, 0.16, 0.09 and 0.08 mm for Teesta, Meghna, Dhaka and Jamuna sands, respectively. The specific gravity of all four types of sand are nearly the same and ranges from 2.66 to 2.70.

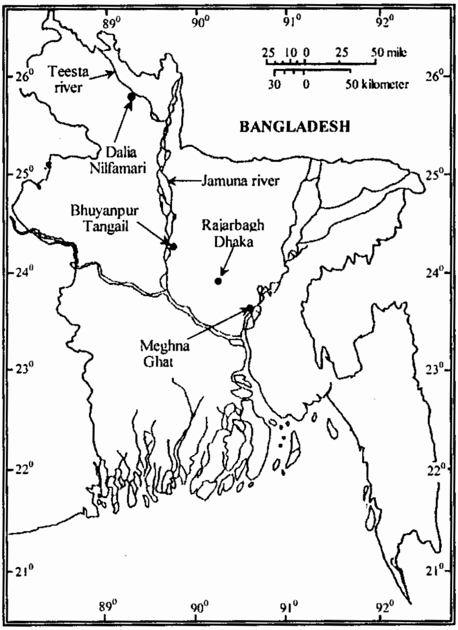

Fig. 1. Place of collection of soil samples

Table 1. Sampling location and grain properties of the sands

Sand type	Teesta sand	Meghna sand	Dhaka sand	Jamuna sand
Location	Teesta Barrage site, Rangpur	Meghna Bridge site, Dhaka	Rajarbagh, Dhaka	Jamuna at Bhuyanpur, Tangail
Approximate depth from ground surface	15-20 ft	River bed	40-50 ft	River bank
Specific gravity	2.67	2.7	2.66	2.69
Effective grain diameter, d_{10} mm	0.24	0.16	0.09	0.08
d_{30} mm	0.40	0.21	0.18	0.11
d_{60} mm	0.59	0.30	0.23	0.15
Fineness Modulus	2.40	1.35	0.98	0.40
Uniformity coefficient, $c_u = d_{60}/d_{10}$	2.46	1.88	2.56	1.88
Coefficient of curvature, $c_c = (d_{30})^2/(d_{60} \times d_{10})$	1.12	0.92	1.56	1.01

Due to limitations of the available apparatus, it was not possible to determine the particle characteristics of soil particles finer than #100 sieve (opening 0.149 mm). So, determination of limiting densities and triaxial test were performed on samples from which the fraction finer than #100 sieve was removed. Fig. 2 also shows the gradation curves of the four sands excluding the fraction finer than #100 sieve.

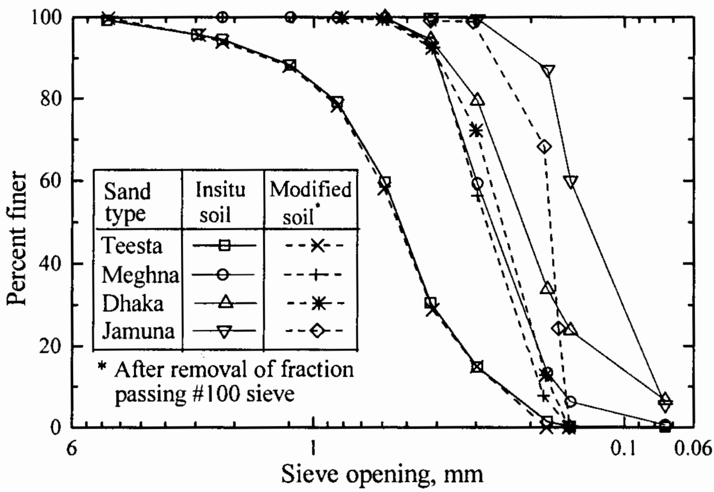

Fig. 2. Grain size distributions of the in-situ soils and those of samples excluding materials finer than # 100 sieve

The grain size distribution curves for the insitu soils are compared with the grain size distribution range provided by Morgan and McIntire (1959) for pleistocene and recent sediments of the Bengal Basin as shown in Figs. 3a and 3b. The soils studied are of coarser variety than those reported by Morgan and McIntire.

From visual inspection it was found that Teesta, Meghna and Jamuna river sands are composed of light gray opaque particles. Also, they contain small amount of white translucent particles. The particles of Dhaka sand are brown in colour and opaque. The specific gravity of these sands are approximately the same as quartz (silica). Comparison of X-ray diffractometer charts of these soils prepared by Yasin (1990) with diffraction data for quartz (Mitchel, 1976) confirmed that these sands are predominantly quartz.

PARTICLE CHARACTERISTICS

Wadell (1932a,b) was the first to attempt to quantify the particle shape. '*Sphericity*' and 'degree of roundness' were used as shape indices. These were defined as:

Sphericity = (Surface area of a sphere having the same volume as the particle) / (Actual surface area of the particle) (1)

Degree of roundness (in one plane) = (Sum of roundness values of the individual corners in a plane) / (No. of corners in that plane) (2)

Roundness of a corner = r/R,

where, r = radius of curvature of the corner and R = radius of curvature of the largest inscribed circle in the plane of measurement.

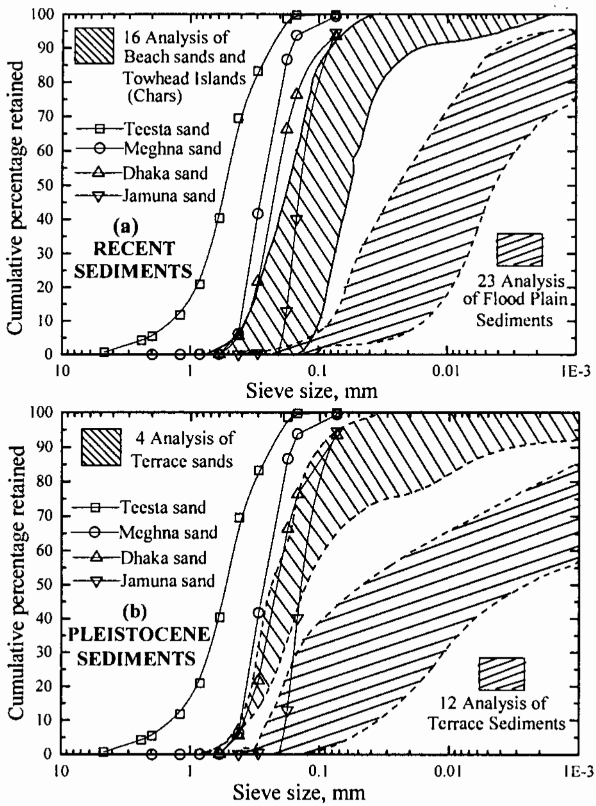

Fig. 3. Composite grain size curves of (a) Recent and (b) Pleistocene Sediments of Bengal (After Morgan and McIntire, 1959)

The maximum possible value of *sphericity* of a particle is 1.0, which is the value when the particle is a sphere. For shapes other than a sphere, the *sphericity* of a particle lies between 0 (zero) and 1.0. The '*degree of roundness*' of a sphere is 1.0 and for other shapes the value of this index is less than 1.0.

Zingg (1935) classified pebble shapes based on the *b/a and c/b* ratio; where *a, b, c* are the long, intermediate and short diameters, respectively (*a*>*b*>*c*). *a* is the longest dimension of the particle, *b is* measured perpendicular to the longest axis in a plane which gives the maximum projected area of the particle, and *c is* measured in a plane passing through the long axis and perpendicular to the plane containing long and intermediate diameters. A particle is then classified as one of the following: "Disc" (if $b/a > 2/3$ and $c/b < 2/3$), "Spherical" (if $b/a > 2/3$ and $c/b > 2/3$), "Bladed" (if $b/a < 2/3$ and $c/b < 2/3$), and "Rod like" (if $b/a < 2/3$ and $c/b > 2/3$).

Based on the definition of *sphericity* given by Wadell (1932b), Krumbein (1941) established the following relationship among long diameter *a*, intermediate diameter *b*, short diameter *c* and sphercity Ψ of a particle.

$$\left(\frac{b}{a}\right)^2 = \frac{\psi^3}{c/b} \quad (3)$$

Curves were provided as a means of rapid determination of *sphericity* values when *a, b, c* are known. Also standard images were provided so that roundness of pebbles could be obtained by visual comparison. Lees (1964) represented particle shape by '*degree of angularity*'. Degree of angularity of a single corner measured in one plane as is defined as: $(180° - \theta_1)\, x/r_m$, where θ_1 = angle subtended at the center by the part of periphery that defines a corner (measured angle), x = distance of the tip of the corner from the center of the maximum inscribed circle, and r_m = radius of the maximum inscribed circle. The total degree of angularity is the sum of all such values for all corners measured in three mutually perpendicular planes. Ehrlich and Weinberg (1970) quantified grain shape by expressing the periphery radius as a function of the angle about the grain's center of gravity using a Fourier series. The radius was thus given by, $R_\theta(\theta) = R_0 + \sum_n R_n Cos(n\theta - \phi_n)$ where θ = the polar angle measured from an arbitrary reference line, R_0 = a constant equivalent to the average radius of the grain in the plane of interest, n = the harmonic order, R_n = the harmonic amplitude, and θ_n = *the phase angle.*

In the present study, *sphericity, flakiness* and *elongation* were adopted as indices for grain shape. The definition of *sphericity* is adopted from Wadell (1932b) with slight modification. According to Wadell's definition, actual surface area of a particle is required to determine *sphericity,* whereas in this study the surface area of a perallelepiped made by three mutually perpendicular projected planes is used. *Flakiness* is defined as the ratio of width to thickness of the particle, and *elongation* is defined as the ratio of the length of a particle to its width. The definitions of *flakiness* and *elongation* are adopted from Allen (1975). Here length is taken as the largest dimension of the particle on a projected plane. The width is taken as the largest dimension perpendicular to the length in the same projected plane. The thickness is taken as the volume of the particle divided by the length and width. Details of the procedure followed for determining these shape quantities were given by Yasin (1990), and are described below. Each of *sphericity*, *elongation* and *flakiness* represents a ratio of similar quantities (i.e. either area or length), and thus is a dimensionless quantity.

During grain size analysis (ASTM D422, 1972), the fraction of soil retained on each sieve was preserved separately for grain characterization. To determine the average volume of particles in a fraction, a maximum of 500 particles were counted from each fraction and weighed by a balance sensitive to four decimal points of a gram. The average volume of the particles of a sieve fraction is determined by dividing this total weight by the total number of particles and the specific gravity of the soil solids, i.e.,

$$V = (W/N) / G_s \qquad (4)$$

where, V = *Average volume of the particles (cm³)*

W = *Weight of N number of particles (gm)*

G_s = *Specific gravity of the soil solids*

The radius of the equivalent sphere having the same volume as that of the particle is obtained by:

$$R_1 = (3V/4\pi)^{1/3} \qquad (5)$$

Due to non-availability of an appropriate measuring device, the actual surface area of the particle could not be measured. Instead, each particle was conceived as a rectangular parallelepiped and the dimension of such parallelepiped was determined as follows:

i) Sand particles of a sieve fraction were placed on a glass slide. A particle was focused and it was observed that the grain rested on its most stable position.

ii) The longest dimension of the particle, *L*, was measured in the plane of the image when examined under the microscope. A microscope fitted with a sliding micrometer, which allowed measuring a minimum of 20μm, was used for this purpose. The longest dimension thus measured was taken as the length of the particle.

iii) The width, *B*, was measured as the distance between two extreme points on the boundary of the image perpendicular to the direction of the measured length.

iv) The thickness of the particle was computed from the average volume, length and width as follows: $T = V / (BL)$, where V was obtained from equation (4).

v) Steps (ii) to (iv) were repeated for 10 particles selected at random from the sieve fraction.

vi) Values of L, B and T were averaged for the ten particles.

vii) Steps (i) to (vi) were repeated for each sieve fraction of a sand sample.

From the measured values of L and B, and computed values of T and R_1, the *sphericity, elongation* and *flakiness* of the particles of each sieve fraction were computed by using the expressions:

$$\text{Sphericity, } \psi = \frac{4\pi R_1^2}{2(L.B + B.T + T.L)} \tag{6}$$

$$\text{Elongation, } \varepsilon = L/B \tag{7}$$

$$\text{Flakiness, } f = B/T \tag{8}$$

The values of Ψ, ε, f thus obtained represent a particular sieve fraction only. To make them representative of the whole soil sample, the weighted average of all the sieve fractions are calculated by the expressions,

$$\psi_{avg} = \frac{\sum_{i=}^{n} \psi_i w_i}{\sum_{i=1}^{n} w_i}, \tag{9}$$

$$\varepsilon_{avg} = \frac{\sum_{i=}^{n} \varepsilon_i w_i}{\sum_{i=1}^{n} w_i}, \text{ and} \tag{10}$$

$$f_{avg} = \frac{\sum_{i=}^{n} f_i w_i}{\sum_{i=1}^{n} w_i}, \tag{11}$$

where, w_i = the weight of the ith sieve fraction of the soil, and Ψ_i, ε_i and f_i are the corresponding values of *sphericity, elongation* and *flakiness,* respectively. The above measurements of particle characteristics are limited to soil fractions coarser than sieve #100 (0.149mm).

It is found that the particle characteristics of the tested sands vary significantly. The values of *sphericity, elongation* and *flakiness* of all these sands are given in Table 2. The *sphericity* of Teesta sand, Meghna sand, Dhaka sand and Jamuna sand were 0.30, 0.55, 0.63 and 0.40, respectively. Among the river sands, Teesta sand was less round than Meghna and Jamuna sands. Dhaka sand was found to be the most spherical of these four sands, and Teesta sand the least. Teesta sand was very much flaky (as indicated by the *flakiness* index of 9.18), which was followed by Jamuna sand with a *flakiness* index of 7.37. Dhaka sand had the lowest *flakiness* index (2.25) among these sands.

Table 2. Grain shape parameters of the sands

Soil type	Teesta sand	Meghna sand	Dhaka sand	Jamuna sand
Sphericity	0.30	0.55	0.63	0.40
Elongation	1.55	1.35	1.12	1.48
Flakiness	9.18	4.86	2.25	7.37

MAXIMUM AND MINIMUM DENSITY/ VOID RATIO

The prevailing void ratio of a granular soil mass plays an important role in its behaviour under load. Range of void ratio that may prevail in a granular soil mass depends on the grain size distribution as well as the shape of the particles. Density is an important consideration in ground improvement problems, and the improved condition is usually specified as a percentage of the maximum density. Also comparison of strength parameters for different sands is thought to be appropriate when relative density is taken as the base parameter instead of void ratio. Therefore, the maximum and minimum density and the corresponding minimum and maximum void ratio of the tested sands were determined, and their relationships to particle characteristics were studied. The method specified by Bowels (1986) was used to determine the minimum and maximum density. These limiting densities and void ratios are presented in Table 3, and are plotted against particle shape parameters in Fig. 4. Dhaka sand, which is the most spherical, has the least minimum and maximum density, and Teesta sand, which is the least spherical, has the largest maximum and minimum density. Of the other two sands having the same uniformity coefficient of 1.88 but different F.M. and d_{10}, Meghna sand, which has greater *sphericity*, higher F.M. and higher effective grain diameter d_{10}, produced higher values of γ_{max} and γ_{min}.

Table 3. Limiting density and void ratio of the soils

Sand type	Teesta	Meghna	Dhaka	Jamuna
Maximum dry density, pcf	106.12	101.62	87.52	97.54
Minimum dry density pcf	90.14	85.35	70.40	78.37
Maximum void ratio	0.92	0.97	1.36	1.14
Minimum void ratio	0.57	0.66	0.89	0.72

1 pcf=0.157 kN/m^3

Teesta sand, having the least spherical particles, has the lowest minimum void ratio of 0.57 and the lowest maximum void ratio of 0.92 among the four sands. On the contrary, Dhaka sand, consisting of relatively spherical particles, has the largest minimum and maximum void ratio of 0.89 and 1.36, respectively. Kirkpatric (1965) reported that the maximum and minimum porosity of different sieve fraction of Leighton Buzzard sand (particles having regular shape) and glass bead were the same. Therefore, the difference in the limiting void ratios of the sands studied may be attributed to the difference in particle shape apart from gradation. From Fig. 4 it is confirmed that the limiting void ratios increase with increasing roundness of sand particles, i.e., there are more voids in the packing of sand consisting of spherical particles than in sand consisting of elongated and flaky particles. Similar results were also presented by Koerner (1970).

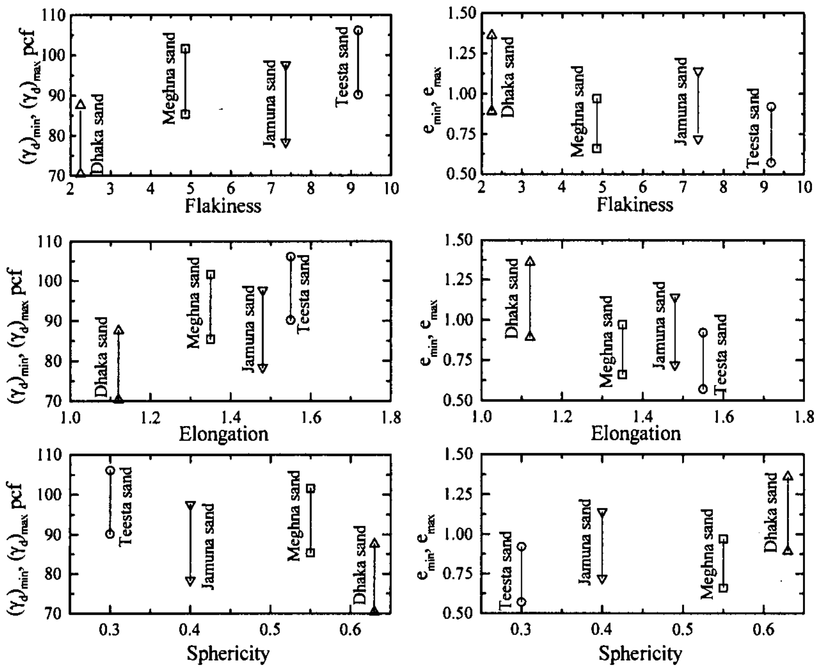

Fig. 4. Effect of particle shape on limiting density and limiting void ratio of the tested sands

TRIAXIAL TEST

The engineering properties, such as angle of internal friction, volume change characteristics and critical void ratio of the sands, were determined by strain-controlled drained triaxial compression tests on 2.8 inch diameter and 5.6 inch high specimens having different initial void ratios. The specimens were prepared by placing sand in a mold using a tea-spoon. The side of the mold was tapped after placing a layer to control the density. The specimens were saturated by flow of water from the bottom drainage to the top drainage for a period of at least 6 hours. A cell pressure of 14.5 psi (100 kPa) was maintained by a constant pressure device during the saturation period and application of deviator stress. During shearing, deviator stress was applied at an axial displacement rate of 0.76 mm/min. A schematic diagram of the triaxial apparatus used is shown in Fig. 5.

The angles of internal friction of the tested soils were determined from the peak stress data on the deviator stress vs. axial strain curves. These friction angles along with initial void ratio, relative density, deviator stress at peak and major principal stress at peak are shown in Table 4. The peak friction angles range between 30 and 38 degree. Fig. 6 shows variation in peak friction angle with porosity for the tested sands with those of other river borne sands reported by Lambe and Whitman (1969) for confining stresses less than 100 psi (690 kPa). For a given initial porosity, the peak frictional angle of the Bangladeshi sands is higher. The observed variation in 'peak friction angle' for a given 'initial porosity' might have resulted primarily from different degrees of interlocking of the particles in the shear zone which, in turn, is dependent on particle shape.

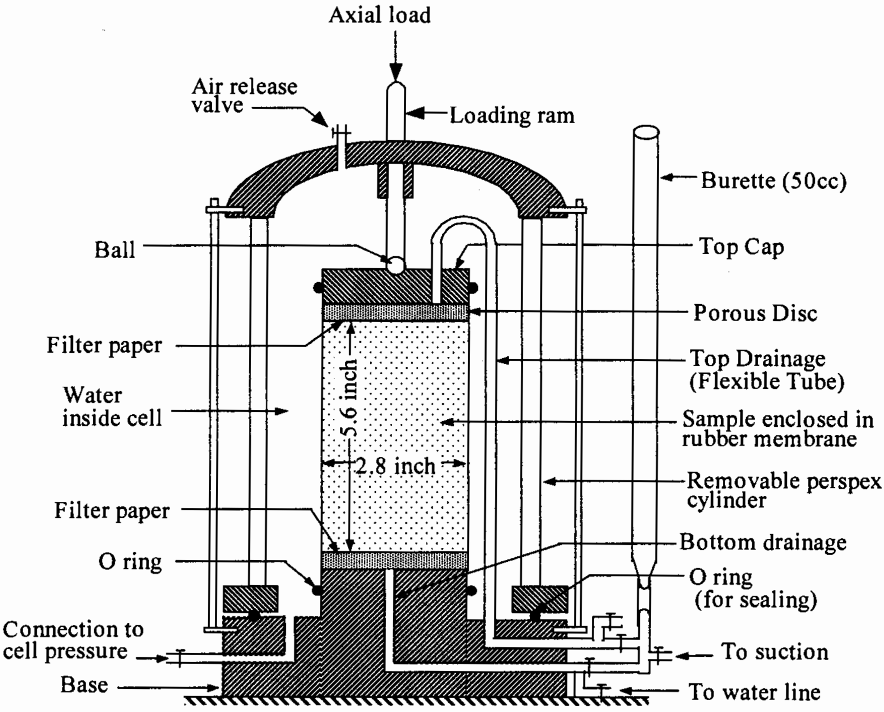

Fig. 5. Schematic diagram of the triaxial apparatus

Table 4. Angle of shearing resistance at failure for various initial void ratio and relative density

Sample	Initial void ratio	Relative density %	Deviator stress psi	Major principal stress, psi	Angle of internal friction at failure (degree)
Teesta	0.76	46	43	57.5	36.67
	0.89	8	38.8	53.5	34.91
	0.93	0	37	51.5	34.10
Meghna	0.81	52	38.8	53.3	34.90
	0.86	35	37.4	51.9	34.28
	0.97	15	34.1	48.6	32.71
Dhaka	0.92	77	33.2	47.7	32.26
	1.12	42	32.1	46.6	31.69
	1.28	14	29.5	44.0	30.28
Jamuna	0.80	81	47.0	61.5	38.20
	0.93	50	42.4	56.9	36.43
	1.03	26	38.2	52.7	34.65

1psi=6.9 kPa

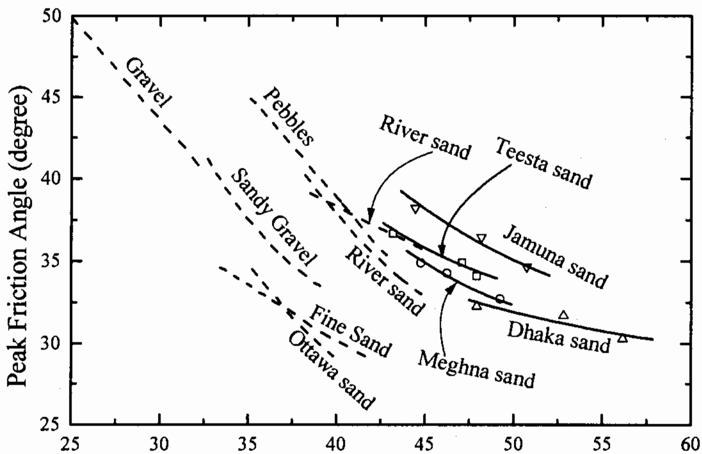

Fig. 6. Angle of internal friction versus initial porosity (results reported by Lambe and Whitman (1969) are included

EFFECT OF SHAPE PARAMETERS ON SHEAR STRENGTH

The trend of increasing friction angle with decreasing *sphericity* of the particles were reported by Koerner (1970), Chen (1948), Terzaghi and Peck (1948), and Sower and Sowers (1951). This trend is also confirmed in this study. Fig. 7 shows the variation in ϕ_d, the drained angle of internal friction, with relative density, D_r. Soils having smaller *sphericity* and greater *elongation* and *flakiness* have higher angle of internal friction at peak, i.e., greater shear strength. For example, at 50% relative density, Dhaka sand having a *sphericity* of 0.63, *elongation* of 1.12 and *flakiness* of 2.25 has a ϕ_d of 31.4° whereas at the same relative density, Teesta sand having a *sphericity* of 0.30, *elongation* of 1.55 and *flakiness* of 9.18 has a ϕ_d of 37°. Also, the rate of change of friction angle with *sphericity, elongation* or *flakiness* is higher at higher relative density, i.e., the effect of shape is greater at higher relative density. For the soils tested, the relationship between friction angle and the relative density for each sand, i.e., for a particular *sphericity, elongation* or *flakiness,* is approximated by a straight line.

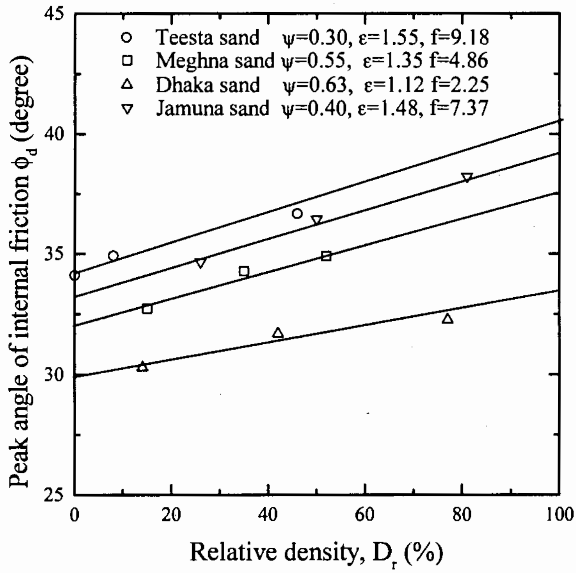

Fig. 7. Variation of friction angle with relative density for different sands

The trend of increase in friction angle with decrease in void ratio for five different types of soils, obtained by Chen (1948), is shown in Fig. 8. Also, data for the soils studied are included in this figure for comparison. Figs. 9a, 9b and 9c indicate the variation in ϕ_d with average *sphericity, elongation* and *flakiness,* respectively, at different relative density. These plots suggest that the variation in ϕ_d with *sphericity, elongation* and *flakiness* can be approximated by a straight line. For a certain relative density, ϕ_d decreases with increasing *sphericity,* but increases with increasing *elongation* and *flakiness.* The greater the *sphericity* the smaller the change in friction angle, and the greater the *elongation* or *flakiness* the greater the change in friction angle for the same change in relative density. For example, Fig.9a shows that Teesta sand having a *sphericity* of 0.30 exhibits an increase of 6° (from 36° to 42°) in friction angle due to a change in relative density from 0.25 to 0.80 whereas Dhaka sand having a *sphericity* of 0.63 show an increase of 2° in the friction angle for the same change in relative density. Such results may be interpreted by the fact that the more the particles approach a spherical shape the less the interlocking, and consequently the less the change in shear strength with an increase in relative density. Conversely, the more the *elongation* and *flakiness* the more the degree of interlocking, resulting in a greater change in shear strength with an increase in relative density. From direct shear tests on river sands Hossain (1984) also observed that for a particular normal stress and void ratio, peak shear strength decreased linearly with increase in *sphericity*, and increased with increasing *elongation* and *flakiness* whereas the ultimate shear strength was independent of particle characteristics.

	Type of sand	Shape	Uniformity coefficient, c_u
Chen (1948)	A - Gravelly sand	Subangular	10.0
	B - Well graded sand	Subangular	10.0
	C - Ottawa standard sand	Rounded	1.20
	D - Screened ottawa sand	Subrounded	1.10
	E - Screened sand	Subangular	1.40

	Type of sand	Sphe-ricity	Elonga-tion	Flaki-ness	Uniformity Coefficient c_u
This study	F - Teesta sand	0.30	1.55	9.18	2.46
	G - Meghna sand	0.55	1.35	4.86	1.88
	H - Dhaka sand	0.63	1.12	2.25	2.56
	I - Jamuna sand	0.40	1.48	7.37	1.88

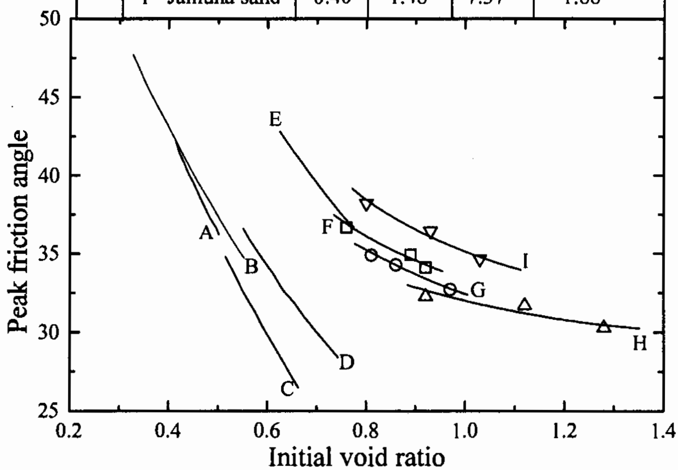

Fig. 8. Comparison of Friction angle of Bangladeshi sands with those reported by Chen (1948)

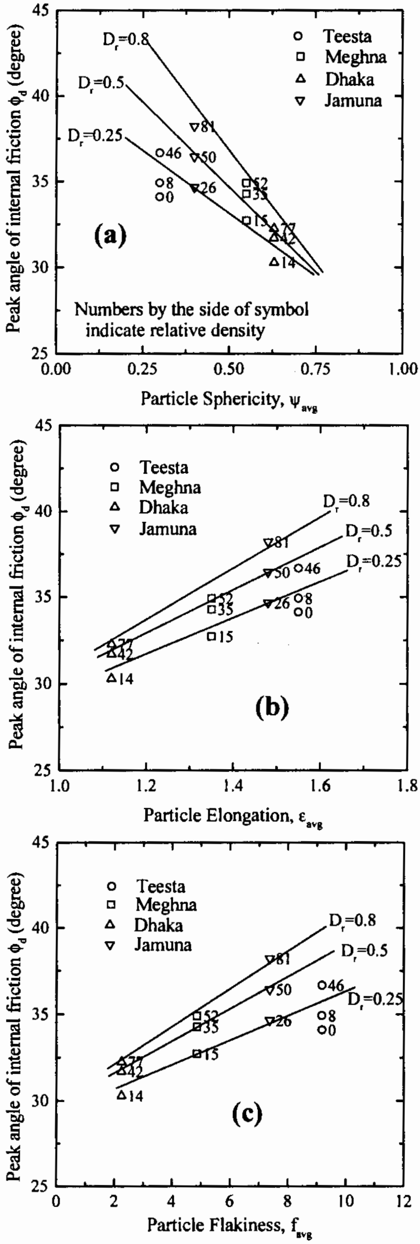

Fig. 9. Effect of particle (a) sphericity (b) elongation and (c) flakiness on the angle of internal friction

NATURE OF DEVIATOR STRESS VS. AXIAL STRAIN AND VOLUMETRIC STRAIN VS. AXIAL STRAIN CURVES

'Stress-strain' curves and the 'volumetric strain-axial strain' curves for the tested sands are shown in Fig. 10. Almost all the stress-strain curves show a peak. The curves conform to the general trend that for loose sands the curves become flatter. The peaks occurr at axial strain levels between 8 and 16 percent. The secant modulus of elasticity at half of the peak stress ranges between 450 psi (3103 kPa) and 2150 psi (14824 kPa). The volumetric strain vs. axial strain curves shows that the denser the sand the greater the tendency to dilate. Teesta, Meghna and Jamuna sands initially show a decrease in volume, and then continue to increase. However, Dhaka sand shows a decrease in volume only.

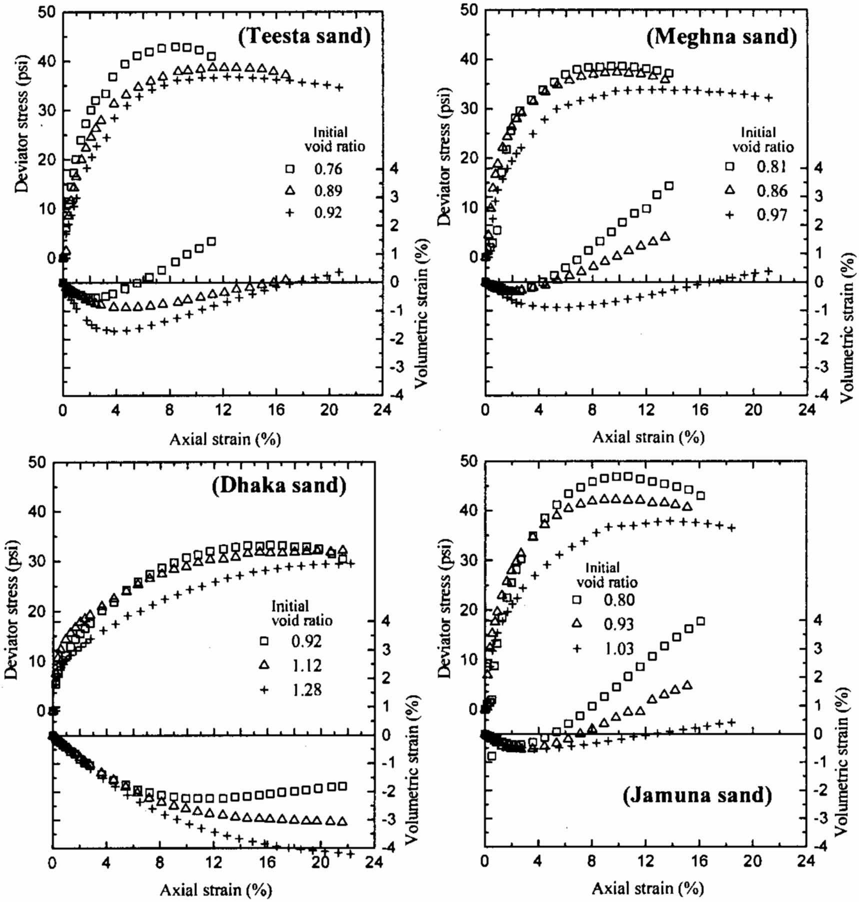

Fig. 10. Stress-strain and volumetric strain characteristics

Fig. 11 presents a plot of volumetric strain at peak stress against initial relative density for the four soils tested. The volumetric strain at failure varies linearly with the initial relative density of the specimen, and the greater the *sphericity* the greater the decrease in volume at peak stress. Also, the greater the *elongation* and *flakiness* the smaller the change in volume at peak.

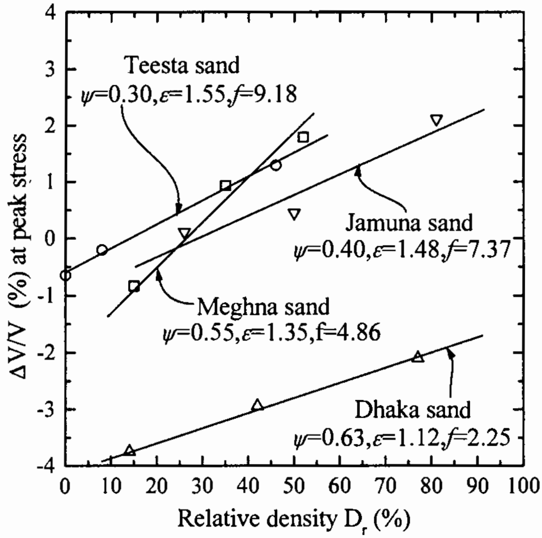

Fig. 11. Variation of volumetric strain at peak with relative density

CRITICAL VOID RATIO

Casagrande (1936) first introduced the concept of critical void ratio in connection with the liquefaction potential of granular materials. The critical void ratio is also important in explaining the volumetric deformation of granular materials associated with shear deformation. Several definitions of critical void ratio exist in literature (Geuze, 1948; Taylor, 1948; Begemann et al., 1977). Casagrande (1938) defined critical void ratio as 'the initial void ratio for which at the instant the peak deviator stress is applied in a triaxial test, no volume deformation occurs with respect to initial volume'. This definition was adopted in this study and the computed critical void ratios of the sands tested are shown in Fig. 12. Apart from Teesta sand, the higher the *sphericity* the lower the critical void ratio, and the higher the *elongation* and *flakiness* the higher the critical void ratio. The reason for this deviation of Teesta sand from the trend remains to be explained. In reality, the volume change mechanism is complex, being associated with random displacements of soil particles in the shear zone. From Fig. 12 it is also evident that the critical void ratio decreases with increase in uniformity coefficient. A study of the critical porosity of the Jamuna sand conducted at Delft Geotechnique (Heijnen, 1988) from a series of stress controlled undrained triaxial tests on saturated specimens reported the critical porosity to be between 45% and 47.5%. The Casagrande critical porosity of the Jamuna sand obtained in this study is slightly higher (50.5%).

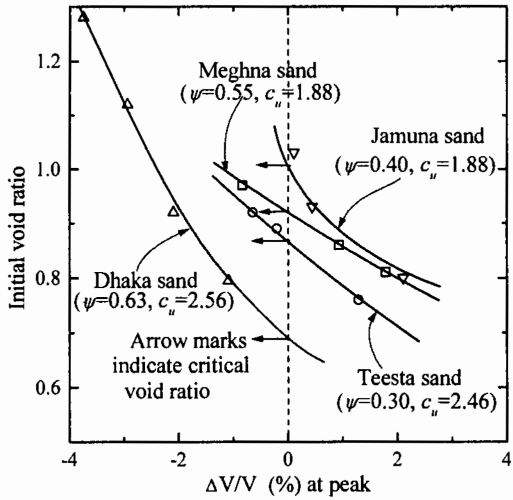

Fig. 12. Casagrande critical void ratio

CONCLUSIONS

Particle shape characteristics, qunatified by *sphericity*, *elongation* and *flakiness*, of sand collected from four locations in Bangladesh were investigated. Based on these shape quantities and other physical properties as well as the results of triaxial compression tests performed on these sands, the following could be concluded:

1) The friction angle of tested sands increased linearly with increase in relative density. However, for any given relative density, the angle of internal friction increased with increase in *elongation* and *flakiness* (i.e. decrease in *sphericity* of the particles).

2) Angle of internal friction for a soil composed of flakey and angular particles increased rapidly with increase in relative density whereas for spherical particles there was little increase in friction angle with increase in relative density. This was evident from ϕ_d versus D_r curve that showed increasing slope for soils having higher *elongation* and *flakiness* (or lower density).

3) Sands with spherical particles experienced greater change in volume during shear than sands with elongated and flakey particles.

4) The Casagrande critical void ratio of sands decreased as the particles approached a spherical shape.

5) The critical void ratio varied inversely with the uniformity coefficient, i.e., sands with higher uniformity coefficient tended to show a lower critical void ratio.

6) Sands composed of particles having higher *sphericity* had lower minimum and lower maximum density. On the other hand, sands composed of elongated and flakey particles yielded higher maximum and higher minimum densities.

REFERENCES

Allen, T., (1975), Chapter 4, Particle size measurement, Chapman and Hall, London.

Begemann, H.K.S.PH., Koning, H.L. and Lindenberg, J. (1977), Critical density of sand, Proc. 9th Int.Conf Soil Mech., Tokyo, vol.1, pp.43-46.

Bowels, J.E. (1986), Engineering properties of soils and their measurement, McGraw-Hill Book Company.

Casagrande, A. (1936), Characteristics of cohesionless soils affecting the stability of slopes and earth fills. Journal of Boston Society of Civil Engineers, Vol.23, p.13, Reported by Leonards.

Casagrande, A. (1938), The shearing resistance of soil and its relation to the stability of earth dams, Proceedings of the Soil and Foundation Conference of the U.S. Engineering Department, Boston, reportrd by Lindenberg and Koning

Chen, L.S. (1948), An investigation of stress, strain and strength characteristics of cohesionless soils by triaxial compression tests, Proceedings, 2nd Int. Conf. on Soil Mechanics and Foundation Engineering, Rotterdam, Vol.5, pp.35-48.

Ehrlich, R. and Weinberg, B. (1970), An exact method for characterization of grain shape, Journal of sedimentary petrology, vol.40, no.1, pp.205-212.

Geuze, E.C.W.A. (1948), Critical density of some dense sands. Proc. 2nd Int. Conf. Soil Mech., Rotterdam, vol.3, pp125-130

Heijnen, W. J. (1988), Liquefaction study, Jamuna Bridge, Field and Laboratory Testing, Delft Geotechniques, June 25, 1988.

Horne, M.R. (1965), The behaviour of an assembly of rotund, rigid, cohesionless particles, Part I & II, Proceedings of the Royal Society, vol.286, pp.62-97.

Hossain, M.A. (1984), Effect of particle shape and angularity on engineering properties of natural granular deposits of Bangladesh, M.Sc. Engg. Thesis, BUET, Dhaka.

Hutchinson, B. and Townsand D. (1961), Some grading density relationship for sands, Proceedings, 5th International Conference on Soil Mechanics and Foundation Engineering, vol.1, pp.159-163.

Kirkpatrick, W.M. (1965), Effect of grain size and grading on the shearing behaviour of granular materials, Proceedings, 6th ICSMFE, vol.1, pp.273-277.

Koerner, R. M. (1970), Effect of particle characteristics on particle strength, Journal of Soil Mechanics and Foundation Division, ASCE, Vol.96, SM4, 1221-1234.

Kolbuszewski, J. and Frederick, M. R. (1963), The significance of particle shape and size on the mechanical behaviour of granular materials, European conference on soil mechanics and foundation engineering, vol.1, Wiesbaden, pp.253-263.

Krumbein, C.W. (1941), Measurement and Geological Significance of shape and roundness of sedimentary particles, Journal of Sedimentary Petrology, vol.11, no.2, pp.64-72.

Lambe, T.W. and Whitman, R.V. (1969), Soil Mechanics, Johm Wiley and Sons, New York.

Lees, G. (1964), A new method for determining the angularity of particles, Sedimentology, vol.3, pp.2-21.

Lindenberg, J. and Koning, H. L. (1981), Critical density of sand, Geotechnique, vol.31, no.2, pp.231-245.

Mitchel, J.K. (1976), Fundamentals of soil behavior, John Wiley and Sons Inc., New York.

Morgan, J. P. and McIntire, W.G. (1959), Quaternary Geology of the Bengal Basin, East Pakistan and India, Bulletin of the Geological Society of America, vol.70, pp.319-342.

Rowe, P.W. (1962), The stress dilatancy relation for static equilibrium of an assembly of particles in contact, Proceedings of the Royal Society, Vol.A269, pp.500-527.

Rowe, P.W. (1973), Stress strain relationships for particulate materials in equilibrium, Proceedings of ASCE Specialty Conference on performance of earth and earth retaining structures, vol.3, pp.327-357.

Sowers, G. B. and Sowers, G. F. (1951), Introductory Soil Mechanics and Foundations, The McMillan Company, New York.

Taylor, D.W.(1948), Fundamentals of soil mechanics, John Wiley & Sons, Inc, New York.

Terzaghi , K. and Peck, R. B. (1948), Soil Mechanics in Engineering Practice, John Wiley and Sons, Inc., New York.

Wadell, H. (1932a), Volume, shape and roundness of rock particles, Journal of geology, Vol.40, pp.443-451.

Wadell, H. (1932b), Sphericity and roundness of rock particles, Journal of geology, Vol.41, pp.310-331.

Yasin, S.J.M. (1990), Effect of particle characteristics on the strength and volume change behaviour of selected granular deposits of Bangladesh, M.Sc.Engg. Thesis, BUET, Dhaka.

Zingg, Th. (1935), Beitrag zur Schotteranalyse: Schweiz, Min.u. Pet. Mitt., vol 15, pp.39-140, Reported by Krumbein.

NOTATIONS

c_c	coefficient of curvature
c_u	uniformity coefficient
d_{10}, d_{30}, d_{60}	effective grain diameters
r	radius of curvature of a corner
R	radius of curvature of the largest inscribed circle in the plane of measurement
a, b, c	long, intermediate and short diameters of a particle, respectively
ψ	sphericity of a particle
ε	elongation of a particle
f	flakiness of a particle
θ_I	angle subtended at the center by the part of periphery that define a corner
x	distance of the tip of a corner from the center of the maximum inscribed circle
r_m	radius of the maximum inscribed circle
θ	the polar angle measured from an arbitrary reference line
R_θ	radius at an angle θ
R_0	a constant equivalent to the average radius of the grain in the plane of interest
n	harmonic order

R_n	harmonic amplitude
ϕ_n	phase angle
V	average volume of the particles
L	longest dimension of a particle
B	width of a particle
T	thickness of a particle
W	weight of particles
G_s	specific gravity of the soil solids
R_l	radius of the equivalent sphere having the same volume as that of the particle
ϕ_d,	the drained angle of internal friction
D_r	relative density