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AN ECONOMIC DESIGN GUIDELINE FOR RECTANGULAR MAT FOUNDATION WITH NON-UNIFORM THICKNESS

Sohrabuddin Ahmad[1], Abdul Muqtadir[1], A.S.M. Monzurul Morshed[1] and Alok Sutradhar[1]

ABSTRACT: A linear elastic finite element analysis of rectangular mat with non-uniform thickness is performed. Effects of significant parameters on mat with non-uniform thickness have been studied. Based on these studies a simplified design guideline for mat with non-uniform thickness is presented. A design example is given to show the use of the guideline. It is found that 20% to 30% savings in material cost can be achieved by the proposed design guideline.

KEYWORDS: Mat foundation, finite element, thick shell, non-uniform thickness.

INTRODUCTION

Mat foundations are generally used with uniform thickness all over. Economy can be attained by reconfiguring the mat in different ways (Bowles 1988). One type of such reconfiguration is by thickening the mat below the columns as shown in Fig. 1. This kind of mat with non-uniform thickness saves a large volume of concrete as well as reinforcement. Such mats can only be analysed by finite element method. There is no guideline on how to reshape the mat. This paper is aimed at presenting a guideline for designing rectangular mat foundation with non-uniform thickness. A linear, elastic finite element analysis of the mat resting on Winkler medium is carried out using thick shell elements (Ahmad 1970). In order to obtain a design approach, the effects of varying the geometrical parameters on the behaviour of the mat have also been studied. A simple design guideline for mat with non-uniform thickness has been proposed. To demonstrate the effectiveness and efficiency of the guideline, a design example is presented.

FINITE ELEMENT ANALYSIS

In this study mat has been idealized as an assemblage of eight noded thick shell elements [Ahmad et. al., 1970]. This element has been previously used successfully in other structures such as stair slabs supported at landing levels [Ahmed, et. al., 1995 and 1996].

1 Department of Civil Engineering, BUET, Dhaka-1000, Bangladesh

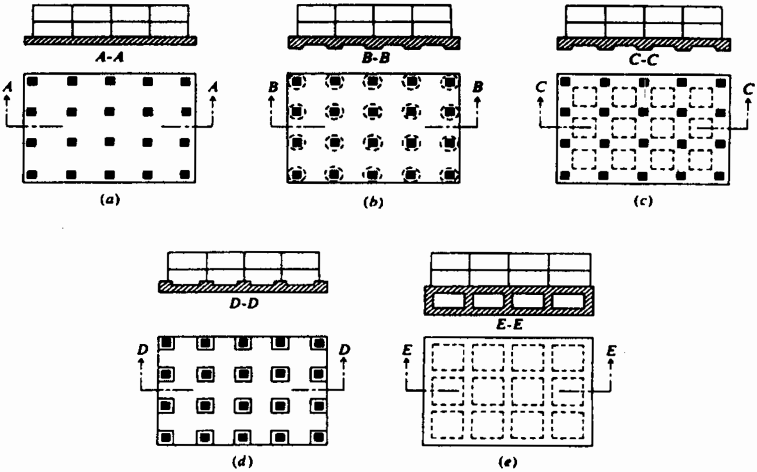


Fig. 1. Non uniform mat foundation

Assumptions

The study was carried out with the following assumptions:

1. The material of mat has been taken to be concrete and it has been assumed to behave as linearly elastic, homogeneous and isotropic.
2. Supporting soil has been replaced by elastic Winkler medium.
3. To simulate the high rigidity of column elements with respect to other portions of the mat, modulus of elasticity of these elements has been increased eight times. This has been found to be adequate from analysis (Morshed,1996).

Material Properties

1. For concrete ultimate strength of 27.6 MPa and Poisson's ratio of 0.18 were assumed.
2. Modulus of subgrade reaction was taken as 15.7 GN/m^3.
3. Steel of 414 MPa strength has been considered.

PERFORMANCE OF MAT WITH NON-UNIFORM THICKNESS

To examine the performance of mat with non-uniform thickness a square six storied, three bay building frame in both direction has been selected. A loading of live load 2.9 kN/m^2, dead load for a material unit weight of 23.6 kN/m^3, floor finish of 1.5 kN/m^2 and partition wall of 1.5 kN/m^2 were considered. The mat is extended beyond the column lines at the boundary by 1.52 m. Due to double symmetry only quarter of the mat is analysed. The mat is discretized into 361 elements with

1160 nodes. A sensitivity study on the mesh refinement has been carried out before selecting the finite element mesh. In the transition zone closer meshes have been taken. Typical finite element mesh is shown in Fig. 2.

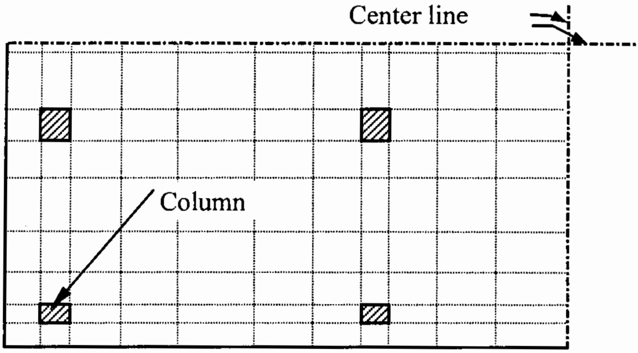


Fig. 2 Typical finite Element Mesh for mat foundation

Plan area of mat can be divided into two distinct zones. The first of these includes the neighbourhoods of the columns where high shears and positive moments occur. The second region consists of the areas away from the columns which are subject to high negative moments but low shear forces. Since thickness of mat is usually determined from shear criteria, it is obvious that thickness requirement in the second zone will be much lower than that of the first zone. So it is possible to provide lower thickness in the second zone resulting in mat of non-uniform thickness.

Typical cross-sections along LINE 1 and 2 of mats with uniform and non-uniform thickness are shown in Fig. 3. It is noted that sections *A*, *B* and *C*, referred to as *neck sections*, must be given due attention when checking flexural shear for mats with non-uniform thickness. For the design, strips with Line 1 and Line 2 as their centre are designated as column strip 1 and column strip 2 respectively. For comparing the design of MAT I and II (Fig. 3) a special term, *weighted average thickness of mat* (t_{avg}), is used which is calculated as follows:

$$t_{avg} = \sum t \times A_t / \sum A_t \qquad (1)$$

where, t = mat thickness,
A_t = plan area of mat having thickness t.

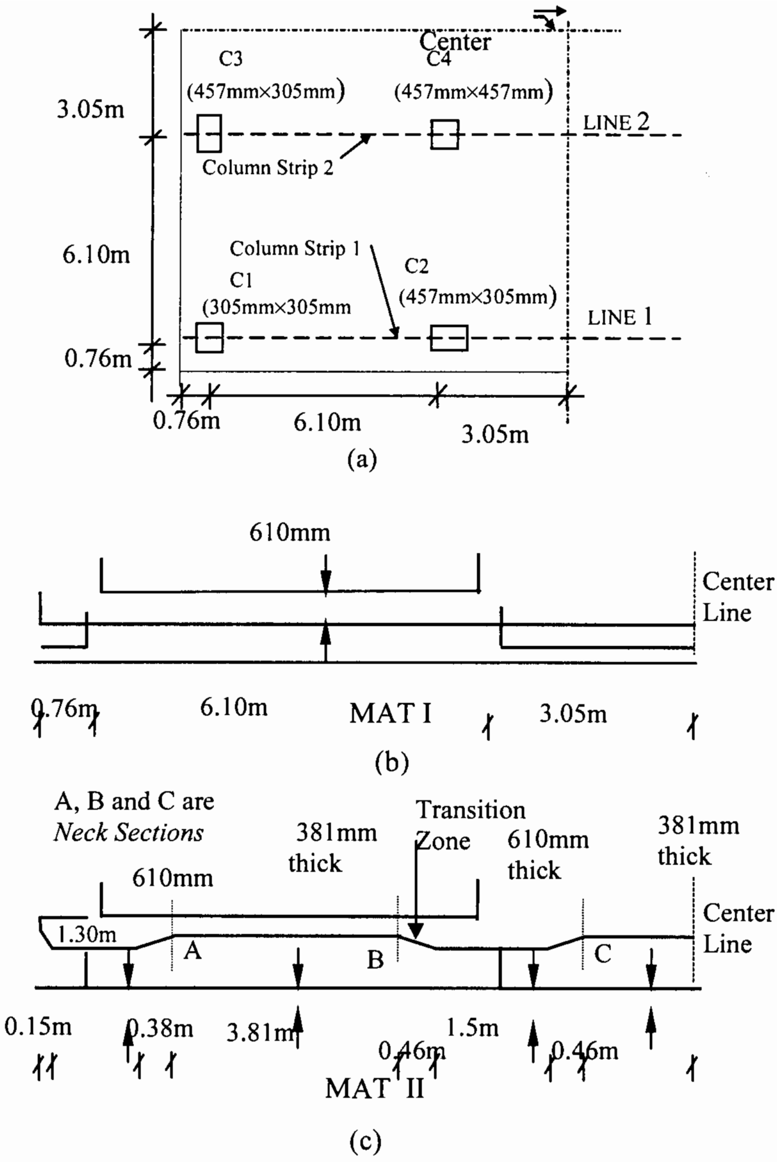


Fig. 3. Example Mat (a) Plan, (b)Cross section of mat with uniform thickness (c) Cross section of mat with non-uniform thickness

In performing the design of mat, Ultimate Strength Design (USD) method of ACI code of practice has been followed. Deflection, bending moment and shear force diagrams of MAT I and II under the loading are depicted in Fig.4, 5 and 6 respectively and their design is summarized in Table 1.

Table 1. Design requirements for MAT I and II

Mat	t_{punch} (mm)	t_{flex} (mm)	t_{neck} (mm)	A_s^{+ve} col. face (mm²/m)	A_s^{-ve} C1-C2 (mm²/m)	A_s^{-ve} C2-C3 (mm²/m)	A_s^{-ve} C5-C6 (mm²/m)	A_s^{-ve} C6-C7 (mm²/m)
I	1064	777		3218	3218	3218	3218	3218
II	1064	742	528	3218	2921	1694	3260	1694

Here A_s is the required cross-section area of steel reinforcement. In Table 2 designs of MAT I and II are compared. It is evident that mat with non-uniform thickness offers a considerable amount of economy. To reconfirm these conclusions some more examples have been examined (Sutradhar, 1999).

Table 2. Relative economic evaluation of MAT I and II

Mat	t_{avg} (mm)	+ve A_s^{avg} (mm²/m)	-ve A_s^{avg} (mm²/m)	Concrete Saving In MAT II	Steel Saving In MAT II
I	1067	3218	3218	31%	19%
II	737	2604	2625		

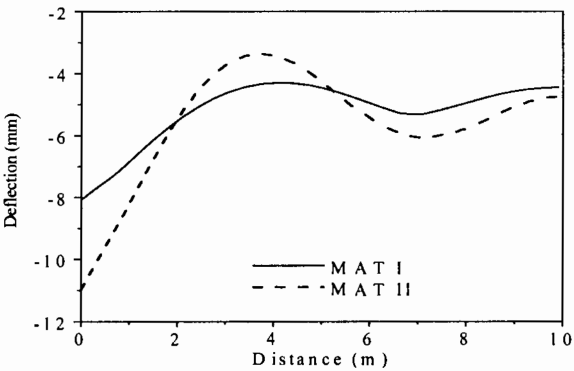


Fig. 4. Comparison of deflection along line 2 of Mat I and Mat II

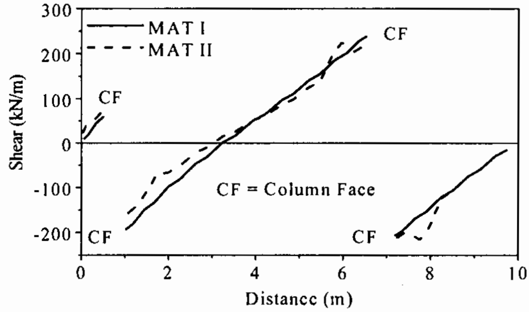


Fig. 5. Comparison of shear force along column strip 2 of Mat I and Mat II

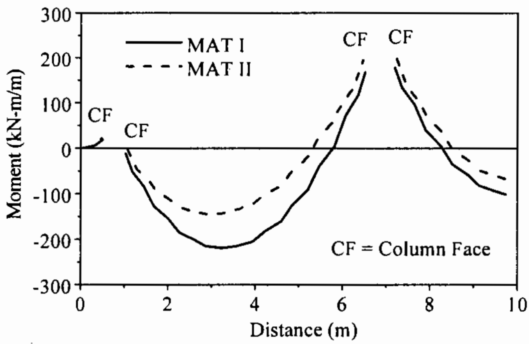


Fig. 6. Comparison of bending moment along column strip 2 of Mat I and Mat II

Recommended design differential settlement varies from 62.5 to 75 mm. Column deflections found are within the allowable limits (Fig. 4) for both the mats considered with the modulus of subgrade reaction value of 15.7 GN/m^3. For structural design, column to column deflection is important. Differential settlement between columns is increased for non-uniform mat foundation, but it is well below the allowable limit. It can be concluded that differential settlement will not cause any problem when thickness is non-uniform. Punching shear does not change significantly due to the reduction of mat thickness away from the columns. Flexural shears change mainly near the columns. At the neck sections *A*, *B* or *C* (Fig. 3) practically there is no change in flexural shear which means that the magnitude of thickness reduction can be estimated from the solution of mat with uniform thickness. In regions where transition from higher to lower thickness takes place (referred to as *transition zones*), shear magnification is observed in the central regions of the column strips (Fig. 6). Reduction of negative moments due to lower thickness in between the columns is as high as 32% for MAT II (Fig. 5). Increase in positive moment near the columns is also significant and is found to be up to 31%. However, with higher thickness at these locations, positive moments remain below the minimum moment capacity despite amplification according to ACI code.

PARAMETRIC STUDY OF MAT WITH NON-UNIFORM THICKNESS

To formulate a general guideline for the solution of mat with non-uniform thickness a number of geometrical parameters are considered and their effects on the mat is studied (Fig. 7). These are the distance over which the change in thickness occurs (d_s), the lateral extent of greater thickness around the columns (d_g) and the amount of reduction in mat thickness away from the columns (Δt), which can be expressed in terms of t_g and t_s . For the sake of clarity, these items are named as follows :

d_g	=	*Width of greater thickness,*
d_s	=	*Slope width,*
t_g	=	*Greater thickness,*
t_s	=	*Smaller thickness,*
Δt	=	*Change in thickness,* ($t_g - t_s$)

Effect of Slope Width (d_s)

To examine the effect of slope width, MAT II (Fig. 3) is analysed by varying its d_s from 0.23 m to 0.61 m. As the slope width increases

- deflection of column decreases by very small amount (5% at C4),
- column face moment increases by very small amount (6.7% at C4),
- negative moment decreases slightly (14.3% at centre of strip 2),
- punching shear does not change at all, and
- high flexural shear stress concentration occurs in the transition zone near the centre of the column strip.

Effect of slope width on flexural shear is noticeable. Fig. 8 shows that high shear concentration occurs in the transition zones near the center of the column strips. Near the ends of the column strips, flexural shears decrease in the transition zones. A plot of the ratio of the high shear concentration termed the *magnified shear* to the shear at the corresponding points in case of uniform thickness, against slope width (Fig. 9) shows that the ratio, which is designated as *kink shear magnification,* tends to unity as slope width is increased. When considered in terms of the angle of the slope (Fig. 9), the ratio comes close to unity as the slope flattens out. Final selection of the slope depends on the optimization between the flexural shear capacity of the

transition zone and the degree of kink shear magnification. Sharp change in thickness should however, be avoided.

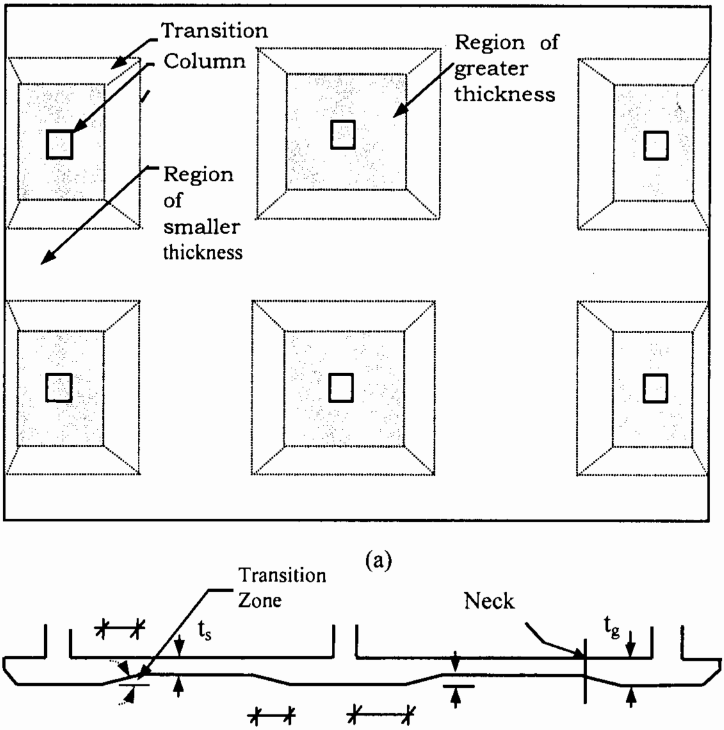


Fig. 7. Typical mat with non-uniform (b) *ess (a) Plan (b) Cross section*

Effect of slope width on flexural shear is noticeable. Fig. 8 shows that high shear concentration occurs in the transition zones near the center of the column strips. Near the ends of the column strips, flexural shears decrease in the transition zones. A plot of the ratio of the high shear concentration termed the *magnified shear* to the shear at the corresponding points in case of uniform thickness, against slope width (Fig. 9) shows that the ratio, which is designated as *kink shear magnification*, tends to unity as slope width is increased. When considered in terms of the angle of the slope (Fig. 9), the ratio comes close to unity as the slope flattens out. Final selection of the slope depends on the optimization between the flexural shear capacity of the transition zone and the degree of kink shear magnification. Sharp change in thickness should however, be avoided.

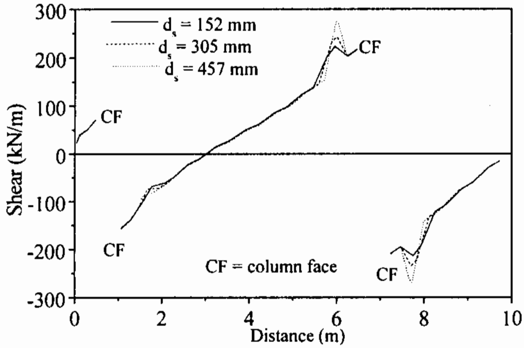


Fig. 8. Shear force diagrams of column strip 2 for different slope widths (Mat II)

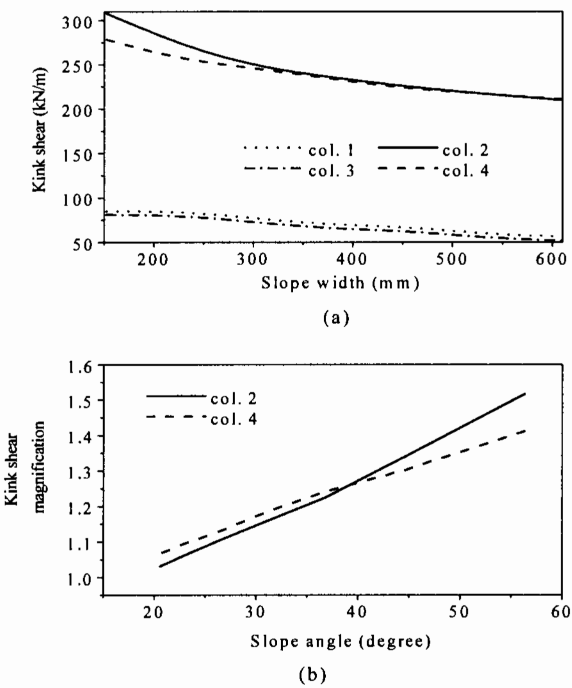


Fig. 9. Effect of slope width on kink shear as function of (a) slope width and (b) slope angle(Mat II)

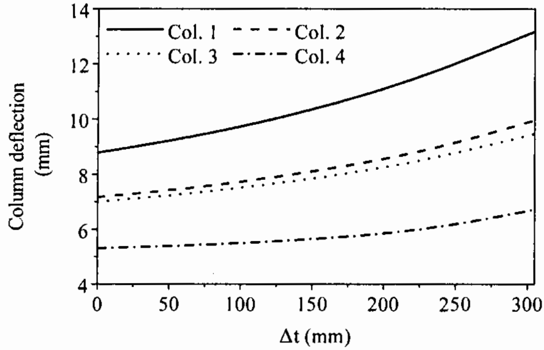
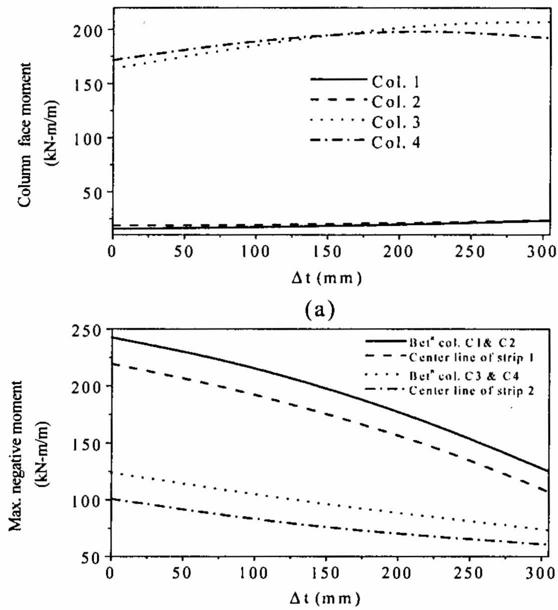


Fig. 10. Effect of change of thickness, Δt on column deflection of Mat II



(b)

Fig. 11. Effect of change of thickness, Δt on (a) positive and (b) negative bending moment

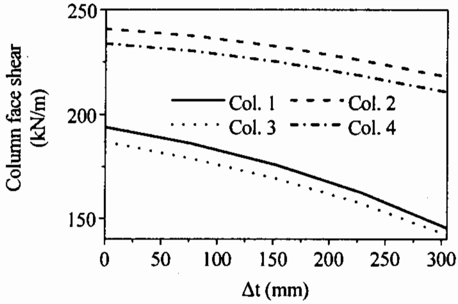


Fig. 12. Effect of change in differential thickness on column face shear (Mat II of Fig. 3)

Effect of Change in Thickness (Δt)

For MAT II change in thickness (Δt) is varied from 0.00 to 0.33 m, keeping the greater thickness t_g fixed at 0.61 m. Variation of different items thus observed are depicted in Fig. 10 through Fig. 13.

As Δt increases

- deflection increases rapidly but differential settlement remains within tolerable limits,
- reduction of negative moments occur in regions away from the columns,
- column face moment is increased moderately, and kink shear increases. A plot of kink shear magnification versus slope angle shows that kink shear magnification becomes less prominent as the slope flattens out.(Fig. 13b)

Effect of Width of Greater Thickness (d_g)

MAT II is analysed by varying d_g in the range of 381 mm to 686 mm. The variation of moments and shears are shown in Fig. 14 through Fig. 16.

As the width of greater thickness increases

- column deflection decreases slightly (5.5% at C4 in Fig. 14a),
- column face positive moment does not change appreciably,
- negative moment at centre of strips decreases moderately (to 13% in Fig. 14b), and
- column face shear decreases slightly (by 3-8% in Fig. 15).

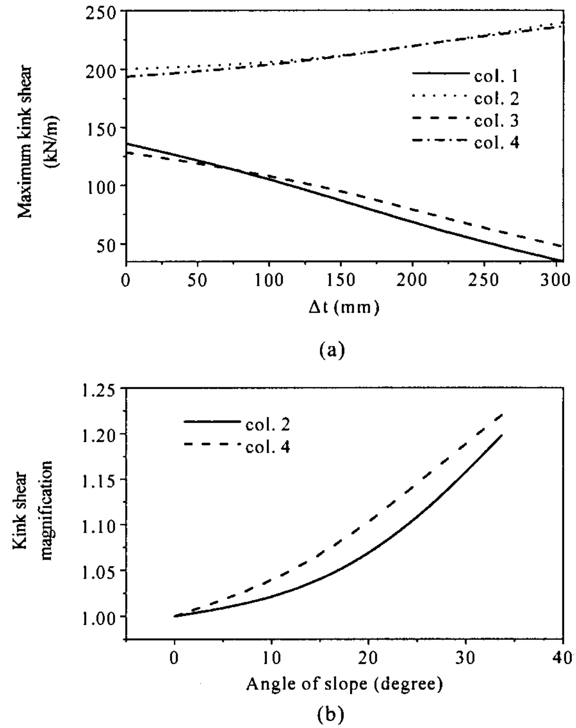


Fig. 13. Effect of change of thickness, Δt on kink shear as a function of (a) Δt and (b) slope angle (MaI II)

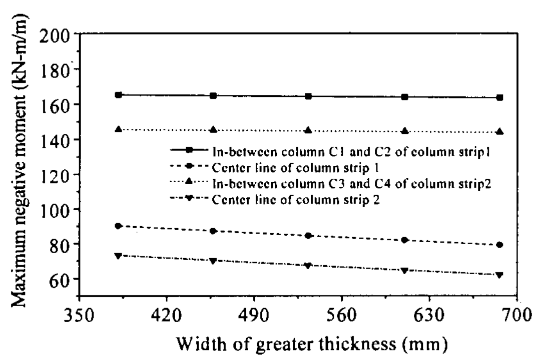
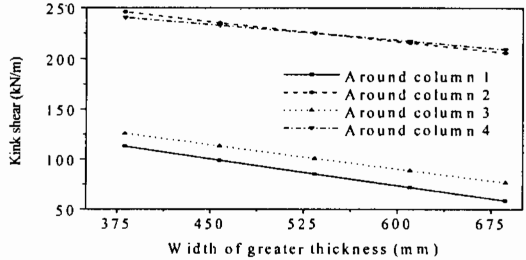
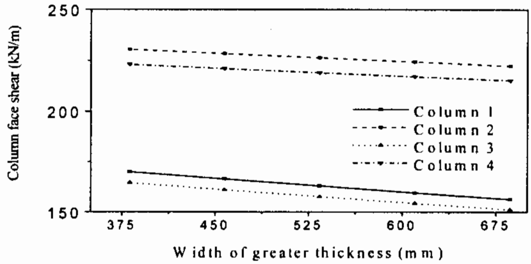


Fig. 14. Effect of width of greater thickness on negative moment (Mat II)



(a)



(b)

Fig. 15. Effect of width of greater thickness on (a) kink shear and (b) column face shear (MAT II)

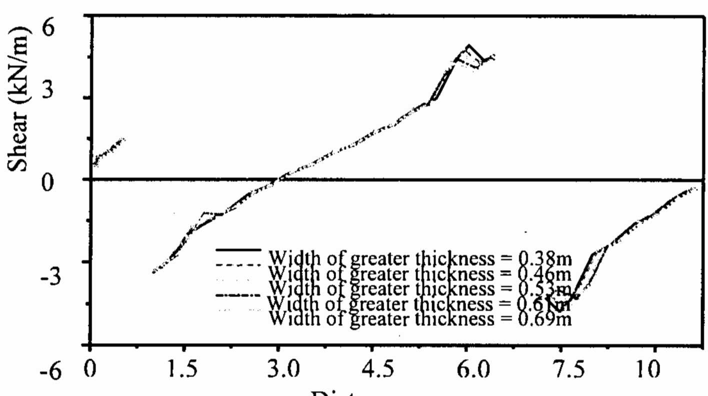


Fig. 16. Shear force diagrams of column strip 2 for different widths of zones of greater thickness (Mat II)

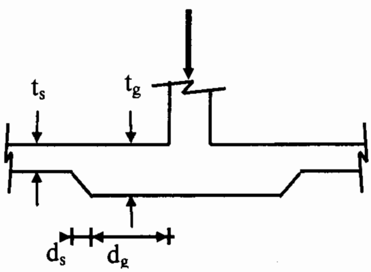
DESIGN GUIDELINE FOR MAT WITH NON-UNIFORM THICKNESS

The parametric study reveals that the design of regular shaped mats with non-uniform thickness follow some well defined trends. Based on these findings, a design approach for mat with non-uniform thickness is recommended as follows:

(i) First the geometry of a mat with non-uniform thickness is selected from the guideline shown in Fig. 17. Column punching shear should be estimated as 0.95 x maximum column axial force. (Many problems have been solved in order to get this value, Sutradhar 1999).

t_g = *Greater thickness,* to be calculate from the punching shear
t_s = *Smaller thickness,* = 2/3 of t_g
d_g = *Width of greater thickness,*
= effective depth corresponding to t_g
d_s = *Slope width,* = $t_g/\sqrt{3}$
Slope angle ≅ 30^0

$P_{DL+LL\,(max)}$ = Maximum column axial load due to live load and dead load



t_g = thickness required to encounter 0.95 $P_{DL+LL\,(max)}$ as punching shear,
t_s = 2/3 of t_g
d_g = effective depth corresponding to t_g
d_s = $t_g / \sqrt{3}$.

Fig. 17. Guideline for selecting cross sectional geometry for mat with non-uniform thickness

(ii) A mat analysis is performed with this geometry. Now the Greater thickness (t_g) under the columns should be designed from the column punching shear found from analysis.

(ii) The greater thickness (t_g) should be provided around the column peripheries over a distance equal to the effective depth corresponding to the greater thickness itself.

(iii) Smaller thickness should be calculated from the maximum flexural shear, found from the solution, at the neck sections.

(iv) Width of the transition zones (for gradual reduction of thickness) should be such that slope angle of the underside of mat in these zones remain within 30^0 to 35^0.

(v) The problem should be analyzed again with the new geometry just selected in order to calculate reinforcements. Finally a check is done on the adopted geometry using the shear forces diagrams and punching shears found from the *new solution.*

(vi) Minimum reinforcement required for the zone of greater thickness should be provided as bottom reinforcement under the columns across the entire widths of column strips. Again, the new solution may be used to check the adequacy of these reinforcements.

(vii) Reinforcements for negative moments in-between the columns should be designed using the new solution.

(viii) Top reinforcements in between the columns (negative moment zones) and bottom reinforcements under the columns (positive moment zones) should be calculated as per minimum requirements specified by ACI code.

Case studies reveal that mat thickness away from the column faces can be reduced by about 40%. But to be on the conservative side 1/3 reduction is suggested in the guideline.

DESIGN EXAMPLE

A 10 storied building with 5 bay each of 5.5 m and story height of 3 m is selected for the study. A 30.5 m square mat is selected with 1.5 m overhanging portion (Fig. 18). There are 36 columns arranged in a 6 x 6 grid, each 732 mm square in cross-section and spaced 5.5 m apart. The loads considered are :

Dead Load	= 3 kN/m^2
Live load	= 4 kN/m^2
Partition wall	= 1.5 kN/m^2

DESIGN LOADING = 1.4 x Dead Load + 1.7 x Live Load

Estimated Punching shear = 0.95 x 5280 = 5016 kN
If uniform mat is designed then,
thickness, t_g = d + clear cover = 615 mm + 89 mm = 704 mm
The geometry selected by following the guideline in Fig. 17 is shown in Fig. 19. For mat with non-uniform thickness

∴ tg = Greater thickness = 704 mm

t_s = *Smaller thickness,* = 2/3 of t_g = 470 mm

d_g = *Width of greater thickness,* = 622 mm

d_s = *Slope width,* = $t_g/\sqrt{3}$ = 406 mm

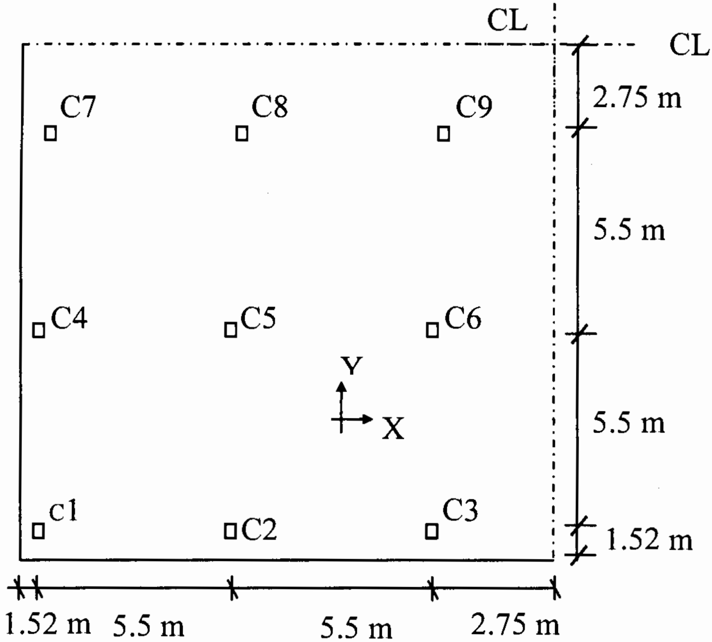


Fig. 18. Plan of mat of design example

Now with this geometry, the mat is analysed. Punching shear around columns, flexural shear and neck flexural shear from the analysis are as shown in Table 3.

Maximum punching shear is 4781 kN. Thickness has already been calculated using 5016 kN. So thickness is adequate.

Maximum Flexural shear is at C8 = 350 kN/m
Effective Depth required = 470 mm,
Thickness = 470 mm + 89mm = 559 mm < 704 mm
Maximum Neck Flexural shear at C8 = 278 kN/m,
Effective Depth required = 373 mm
Thickness = 373 mm + 89 mm = 462 mm < 470 mm t_{s+} is adequate.

As per guideline reinforcement for positive moment in the column region will be governed by minimum reinforcement
A_s (+ve) = 2075 mm^2 / m
Maximum positive moment for case II = 313 kN-m/m
Steel ratio 0.0022 < (ρ_{min} = 0.0033)
Hence, minimum reinforcement governs.

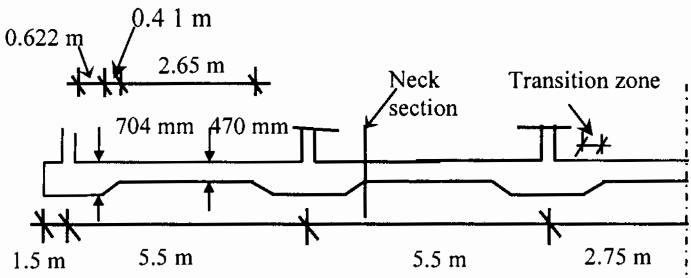


Fig. 19. Cross section of the mat with non-uniform thickness

Reinforcement for negative moment in between column region will have to be calculated as per guideline. Maximum Negative Moment = 154.45 kN-m/m (In between C7 and C8)

Steel ratio ρ = 0.0029 < ρ_{min} (= 0.0033)

$\therefore$ A_s (-ve) = 1270 mm^2 /m

Table 3. Punching, Flexural, and Neck flexural shears, Column face +ve moment, in betn. Column –ve moment of Design Example

Col	Axial Load kN	Punch shear kN	% of col. load	Flex. shear (kN/m)	Neck Flex. Shear (kN/m)	Col. face +ve moment (kN-m/m)	Loc.	In betn. Col.-ve moment (kN-m/m)
C1	1498	1231	82	66	100	50	C1-C2	116
C2	2609	2288	88	231	183	214	C2-C3	72
C3	2640	2289	87	220	172	195	C4-C5	142
C4	2993	2622	88	166	147	87	C5-C6	92
C5	5218	4754	91	337	271	308	C7-C8	154
C6	5281	4781	91	333	255	285	C8-C9	100
C7	2993	2594	87	167	151	84		
C8	5218	4723	91	350	278	313		
C9	5281	4745	90	337	259	288		

Design requirement and relative economic evaluation of non-uniform mat with respect to uniform mat are presented in Table 4 and Table 5 respectively. From the results it is seen that the column strip C7-C9 is the controlling strip. The example problem demonstrates the effectiveness of the proposed guideline.

Table 4. Design requirements for design example mat with non-uniform thickness

Case	$t_{punching}$ (mm)	$t_{flexural}$ (mm)	t_{neck} (mm)	A_s^{+ve} col. Face (mm^2/m)	A_s^{-ve} In between columns (mm^2/m)
Non-uniform	682	559	462	2075	1270
Uniform	682	526		2075	2075

A computer software MATFEA has been developed to design and analyse mat foundation with both uniform and non-uniform thickness. The program seeks only material properties and column loads. Given these, the program automatically fixes the geometry and calculates the reinforcements.

Table 5. Relative economic evaluation of design example mat with non-uniform thickness

CASE	t_{avg} (mm)	+ve A_s^{avg} (mm^2/m)	-ve A_s^{avg} (mm^2/m)	Concrete Saving w.r.t. uniform thickness mat	Steel Saving w.r.t. uniform thickness mat
Non-uniform	508	1397	1270	29%	38%
Uniform	711	2075	2075		

CONCLUSION

Use of the design guideline for mat with non-uniform thickness proposed in this paper, offers about 20% to 30% saving of material cost compared to mat with uniform thickness. Reshaping the geometry for a number of times will ensure even more economy.

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